Tutorial Exercises

Optical Detectors

Exercise 1. Basic quantities of photometry

1. What is the wavelength range of light in the visible ? Deduce its frequency domain.
2. The Planck constant h is $6.62 \cdot 10^{-34}$ uSI (units in the International System) and the charge of the electron e is $1.6 \cdot 10^{-19}$ uSI. Deduce the energy of a photon in eV (electron volts) for the light in the visible domain.
3. An isotropic point source emits a total luminous flux (in whole space) of 1257 lm. Calculate the illuminance produced by the source on a screen situated at 10 m from the source. What then is the illuminance if all the luminous flux is confined into an arc angle of cone 15°?
4. A point source emits light to all the space. The emitted light intensity is 950 lm/sr. What is the flux received by a small area of 1 cm^2 located at 20 meters from the source, perpendicular to the light rays? What would be the flux if the normal of surface was tilted 30° relative to the incident rays?
5. Evaluate the luminance of the sun knowing that the irradiance received on the surface of the Earth is 500 W/m^2, the luminous efficiency of the eye to sunlight is about 91 lm/W. We assume that the sun is seen in a solid angle $\frac{\pi}{4}10^{-4}$ sr (corresponding to a cone of angle of 0.5 degree). In this calculation we neglect the attenuation of light by the atmosphere.

Exercise 2. Energy of light captured by a lens

We want to determine the energy of light captured by a lens. Suppose an optical system consisting of a lens of radius $R = D/2$, focal distance f and having a transmission rate τ, illuminated by a Lambertian area S_e of luminance L at a distance l. A receiving surface S_r is placed in the image plane located at a distance a from the focal plane of the lens.

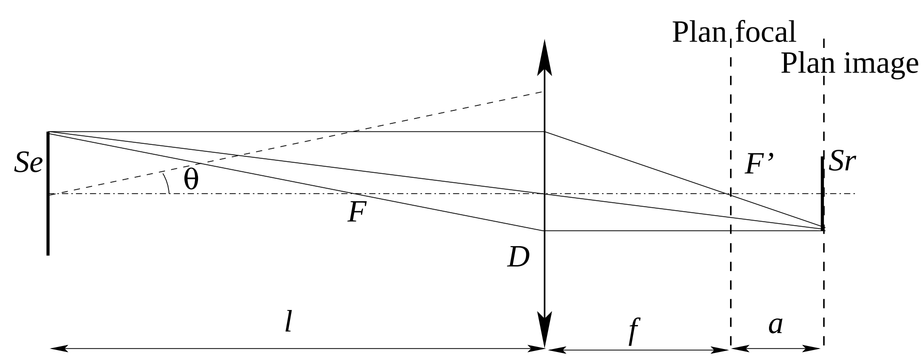

Figure 1: Equivalent schema of the detector.

1. Calculate the energy flux incident on the surface S_r.
2. Calculate the illuminance E_{sr} received by the surface S_r. Give the simplified expression of E_{sr} in the case $l \gg D$.

Exercise 3. Halogen lamp

The characteristics of a halogen lamp are: supplied power $P_e = 500$ W; emitted radiant flux $F_e = 400$ W; emitted luminous flux F_v =8 000 lm. This source satisfies the following laws:

- Stefan's law : $M_e = \sigma T^4$ with $\sigma = 5.67 \cdot 10^{-8}$ W$\cdot$ m$^{-2}\cdot$ K^{-4}, where T is the temperature of the source.
- Wien's law : $\lambda_{max} T = 2\ 898$ μm$\cdot$K, where λ_{max} is the wavelength of maximum emission from the source.

1. Calculate the luminous efficiency and electric efficiency.
2. The filament has a length $l = 5$ cm and its diameter is $d = 0.2$ mm. Calculate the temperature of the filament when the lamp illuminates.
3. Calculate the wavelength of the maximum radiation. In which domain of light is located this wavelength?
4. This lamp is considered punctual and used as a street light that distributes the light in the upper half space (indirect lighting by reflection from the ceiling). Calculate the luminous intensity characterizing the source.

Exercise 4. photoelectric cell

A photoelectric cell is illuminated with a monochromatic light of wavelength 526 nm and beam power of 0.25 W.

1. The work function is 2.2 eV, calculate the speed of photo-emitted electrons.
2. The quantum efficiency of the cell is 0.8 %, what is the intensity of the saturation current?

The useful constants: $q = 1.6{\cdot}10^{-19}$ C ; $h = 6.62{\cdot}10^{-34}$ J.s ; $c = 3{\cdot}10^8$ m/s ; $m_e = 9.1{\cdot}10^{-31}$ kg.

Exercise 5 : Cathode of a photocell

A cathode of a photoelectric cell is characterized by a work function of 2.5 eV. It is illuminated with monochromatic light of wavelength 400 nm.

1. Calculate the kinetic energy of the photo-emitted electrons and the stop voltage .
2. A potential difference $V_A - V_C = 10$ V is now applied between the cathode and anode. Calculate the kinetic energy of the electrons as they arrive at the anode.
3. For this voltage, the cell is saturated ($i = I_S$). Knowing that the power of the light beam is 400 mW and the saturation current is 50 mA, calculate the quantum efficiency of the cell.

Exercise 6 : Photodiode

A photodiode is designed to measure a light output of a source in an inaccessible area, located 1 km from the experimenter with a computer. The emitted power varies as function of time according to $\Phi(t) = \Phi_0 + \phi_1 \sin(\omega t)$. A cable will be used to transmit the signal.

We expect a variation of luminous flux with a maximum frequency $f_{max} = 100$ kHz and a maximum amplitude $\Phi_{1max} = 0.1$ mW. The maximum value of the DC component of the flux is estimated to be $\Phi_{0max} = 1$ mW.

The equivalent schema of the photodiode and its condition of usage is shown in Figure 2. The static sensitivity of the photodiode is $K = I/\Phi = 0.35$ A/W, and its proper capacity is $C_L = 80$ pF.

1. Is-it an active sensor or a passive sensor?
2. Determine the transfer function of the system $S(f) = V_L/\Phi$.
3. Determine the maximum gain $G(f) = |S(f)|$ of the circuit and the cut-off frequency as function of K, R_L and C_L. We recall that the cutoff frequency corresponds to a decrease in the gain of a factor $\sqrt{2}$ from its maximum value.
4. What should the value of the resistance R_L for a sensor cutoff frequency to be 2 times of the maximum frequency of the useful signal?
5. With this resistance value, what is the gain (sensitivity) of the static sensor? What is the value of the dynamic gain when $f = f_{max}$?
6. Give the expression of voltage V_L as function of the flux received by the sensor for a signal of frequency to be 1/10 of the cutoff frequency.

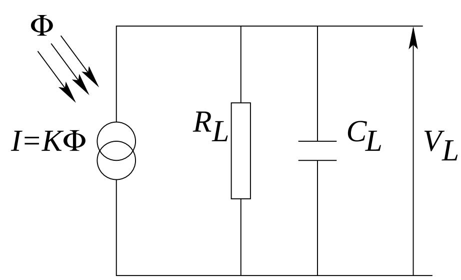

Figure 2: Equivalent schema of the photodiode.

Exercise 7: Characterization of the emission of a source

1. Consider a source of light radiating isotropically a radiant flux F_e.
   (a) What is the form of the emission indicator of this source? Justify the answer.
   (b) What is the radiant intensity I_e of this source (in $\mathrm{W} \cdot \mathrm{sr}^{-1}$)?
   (c) What is the irradiance E_e received at a distance d in far field?
2. Considered a spherical source of radius R, emitting as a black body at temperature T.

(a) What is the total emittance M_e of the source integrated in all direction and for all wavelengths of the spectrum?

(b) Give the expression of the radiant flux F_e as function of R and T .

(c) Give the relation between emittance M_e of the source and the irradiance E_e at the distance d .

(d) At what wavelength λ_{max} does the source radiates the most energy ?

Exercise 8: Efficiency of detectors

For a single-detector, we provides the following the quantum efficiency η as a function of the wavelength λ:

$\lambda(\mu m)$	0.8	1.0	2.2	3.0	3.6	4.2	5.1
η	0.1	0.5	0.6	0.75	0.7	0.5	0.2

1. Draw in a graph the variation of the current response in $R_\lambda(\lambda)$ of this detector, scale in x: 2 cm/μm, scale in y: 2 cm/A$\cdot$ W^{-1}. Then deduce:

   (a) The domain of sensitivity of the detector in the range of the used wavelength.

   (b) The wavelength λ_p at the peak.

   (c) The current response at peak $R_\lambda(\lambda_p)$.

   (d) The cutoff wavelength λ_c of the detector.

2. Draw in the same graph the ideal current response for a unity quantum efficiency of detector.

   Compare the answers with the performance of the InGaAs-PIN photodiode G3476-05 given in the catalog *Hamamatsu* (current Response 0.95 A / W at the peak at 1.55 μm, diameter of the active area $\phi = 0.05$ cm, see the technical sheet attached in the appendix). What is its wavelength and its peak quantum efficiency at this wavelength?

   Show that the NEP (Noise Equivalent Power, 8×10^{-15} W $\cdot$Hz$^{1/2}$) and the detectivity D^* given at λ_p by the manufacturer (5×10^{12} cm.Hz $^{1/2}$ / W) are comparable.

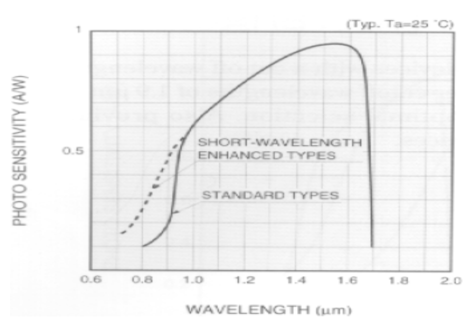

Figure 3: Efficiency of the detector InGaAs-G3476-05 given by the constructor.

Exercise 9: Filter

The transmission curve of a filter is given in Figure 4-a. We observe through this filter a source of emission curve P_λ given in Figure 4-b. We do not concern with the real optical setup.

1. What is the color of the source inspected directly with the naked eye? What does it become the color of the source after passing through the filter?
2. Calculate the power received by the detector after passing through the filter.
3. If the detector response is $R = 5 \times 10^5$ V / W, and noise $\sigma = 6$ μ V rms, what is the signal-to-noise ratio?
4. What is the NEP (Noise-equivalent power) of the detector ?
5. The time constant of the detector is $\tau = 10$ ms, calculate the response observed if the source behind a modulator of 10 blades rotating at $n = 100$ revolutions/min.

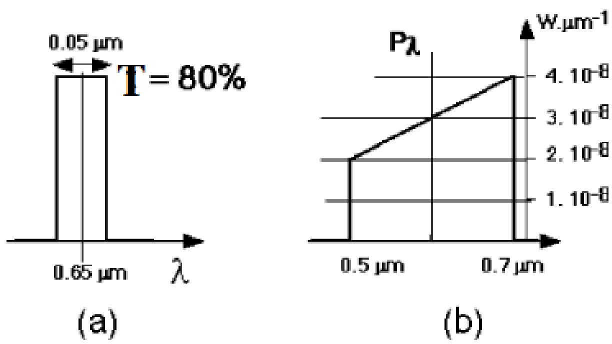

Figure 4: Filter.

Tutorial exercices

Optical Detectors

Correction

Exercice 1. Basic quantities of photometry

1. The wavelength range of light in the visible :

$$\lambda_{min} = 400 \text{ nm}, \qquad \lambda_{max} = 800 \text{ nm}$$

   The relation between the frequency and the wavelength: $\nu = \frac{c}{\lambda}$ with $c = 3 \cdot 10^8$ m/s. So we have

$$\nu_{min} = \frac{c}{\lambda_{max}} = 3,75 \cdot 10^{14} \text{ Hz et } \nu_{max} = \frac{c}{\lambda_{min}} = 7,5 \cdot 10^{14}\text{Hz}$$

2. We know $h = 6,62 \cdot 10^{-34}$ J.s, $e = 1,6 \cdot 10^{-19}$ C, $\lambda_{min} = 400$ nm et $\lambda_{max} = 800$ nm.

   - The energy of a photon in Joule:

$$E_{max} = \frac{hc}{\lambda_{min}} = 5 \cdot 10^{-19} \text{ J}, \; E_{min} = \frac{hc}{\lambda_{max}} = 2,5 \cdot 10^{-19}\text{J}$$

   - The energy of a photon in eV:

$$E_{max}(eV) = \frac{E_{max}}{e} = 3,1 \text{ eV}, \; E_{min}(eV) = \frac{E_{min}}{e} = 1,55\text{eV}$$

3. The luminous flux : $F_v = 1257$ lm

   - Method 1: By the definition, the illuminance :

$$E_v = \frac{F_v}{A} = \frac{F_v}{4\pi d^2} = 1\text{lux}$$

   - Method 2: Luminous intensity : $I_v = \frac{dF_v}{d\Omega}$, $dF_v = I_v \cdot d\Omega$ et $dA = R^2 d\Omega$.

$$E_v = \frac{dF_v}{dA} = \frac{I_v}{d^2}$$

     On the other hand $I_v = \frac{F_v}{4\pi} = 100$lm/sr, so the illuminance :

$$E_v = \frac{I_v}{d^2} = 1 \text{ lux}$$

   If all the flux is confined in a cone:

$$E_{v,15^\circ} = \frac{F_v}{d^2 \cdot 2\pi(1-\cos\theta)} = E_v \frac{2}{1-\cos 15^\circ} = 58.7\text{lux}$$

4. The luminous intensity : $I_v = \frac{dF_v}{d\Omega} = \frac{F_v}{\Omega}$=950 lm/sr.
   The illuminance at 20 m:

$$E_v = \frac{dF_v}{dA} = \frac{I_v d\Omega}{d^2 d\Omega} = \frac{I_v}{d^2} = 2,375 \text{ lux}$$

and the received luminous flux:

$$dF_v = E_v dA = 2,4 \times 10^{-4} \text{ lm}$$

The illuminance at 20 cm:

$$E_v = \frac{I_v}{d^2} = \frac{950}{0,2^2} = 2,375 \cdot 10^4 \text{ lux}$$

The received luminous flux:

$$F_v = E_v dA = 2,4 \text{ lm}$$

The received flux when the detector is inclined at 30° :

$$F_v(30°) = F_v \cos 30° = 2,05 \times 10^{-4} \text{ lm (à 20 m)}, \qquad 2,05 \text{ lm (à 20 cm)}$$

5. The definition of the luminance :

$$L = \frac{dI}{dA\cos\theta} = \frac{d}{dA_e \cos\theta}\frac{dF_r}{d\Omega_e}$$

The definition of the solid angle $d\Omega_r = \frac{dA_e}{d^2}$ and $d\Omega_e = \frac{dA_r}{d^2}$, so

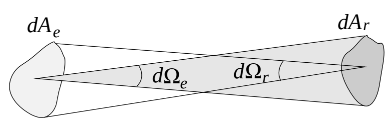

Figure 5: Relation entre les angles solides et les aires.

$$dA_e \cdot d\Omega_e = dA_r d\Omega_r$$

We deduce

$$L = \frac{dE_r}{d\Omega_r}$$

The radiance of the sun ($\theta = 0°$):

$$L_e = \frac{500}{\frac{\pi}{4}10^{-4}} = 0,63 \cdot 10^8 \text{ W/(sr.m}^2)$$

The luminance of the sun :

$$L_v = L_e V = 56.6 \cdot 10^8 \text{ lx.sr}^{-1} \text{ (ou cd.m}^{-2})$$

Exercice 2: Energy of light captured by a lens

1. Lambertian source : $I = I_0 \cos\theta$
Knowing that the solid angle of a cone with angle θ is $\Omega = 2\pi(1-\cos\theta)$, $\Rightarrow d\Omega = 2\pi\sin\theta d\theta$ et $I = \frac{dF}{d\Omega}$, the received total flux:

$$F = \int_\Omega I(\theta)d\Omega = \int_0^{\theta_{max}} I_0\cos\theta \cdot 2\pi\sin\theta d\theta = \pi I_0 \sin^2\theta_{max}$$

Since $\sin\theta_{max} = \frac{D/2}{\sqrt{l^2+D^2/4}}$ and $I_0 = LS_e$

This flux is restricted in S_r with an attenuation τ:

$$F_r = \pi\tau LS_e \frac{D^2}{4l^2 + D^2}$$

2. The emittance on S_r ($l \gg D$):

$$E_{sr} = \frac{F_r}{S_r} = \pi\tau L \frac{S_e}{S_r} \frac{D^2}{4l^2 + D^2} \simeq \pi\tau L \frac{S_e}{S_r} \frac{D^2}{4l^2}$$

However, $\frac{S_e}{S_r} = \frac{l^2}{(f+a)^2} \simeq \left(\frac{l}{f}\right)^2$, we obtain finally:

$$E_{sr} = \pi\tau \frac{L}{4} \frac{D^2}{f^2}$$

Exercice 3. Halogen lampe

The electric power: $P = 500$ W

The radiant flux : $P_e = 400$ W

The luminous flux: $F_v = 8000$ lm

1. The luminous efficiency: $K = \frac{8000}{400} = 20$ lm/W

   The ectrical Efficiency: $\epsilon = \frac{400}{500} = 80\%$

2. The area of the filament: $S = \pi dL$

   The filament temperature according to Stefan's law:
   $T = \left(\frac{P}{\sigma S}\right)^{1/4} = 3871$ K

3. The maximum emission wavelength according to Wien's law:
   $\lambda_{max} = \frac{2898}{T} = 0,749$ μm (in the visible range)

4. The luminous intensity: $I_v = \frac{F_v}{\Omega} = \frac{8000}{2\pi} = 1273$ cd

Exercice 4: Photocell

1. The kinetic energy of the electron excited by a photon:

$$E_c = h\nu - W_s = \frac{1}{2} m_e v_e^2$$

   We know $W_s = 2,2$ eV$= 2,2 \times 1,6\ 10^{-19} = 3,52\ 10^{-19}$ J
   $h\nu = \frac{hc}{\lambda} = \frac{6,62\cdot 10^{-34}\times 3\cdot 10^8}{526\cdot 10^{-9}} = 3.78 \cdot 10^{-19}$ J.
   We have therefore:

$$v_e = \sqrt{\frac{2(h\nu - W_s)}{m_e}} = 0,24 \cdot 10^6 \text{ m/s}$$

2. The saturation current:

   The number of photons of the light beam: $n_p = \frac{P}{h\nu} = 6,62 \cdot 10^{17}$ photons/s.

   The number of electrons emitted: $n_e = n_p\eta = 6,62 \cdot 10^{17} \times 0,008 = 5,3 \cdot 10^{15}$ electrons/s.

   The saturation current: $I_{sat} = n_e e = 8,4 \cdot 10^{-4}$ A=0,84 mA.

Exercice 5: Performance of a photocell

1. The energy of a photon: $E_p = \frac{hc}{\lambda} = 4.965 \cdot 10^{-19}$ J=3.1 eV.
   The energy of the electron photo-emitted : $E_e = E_p - W_s = 3,10 - 2,5 = 0,6$ eV.
   The stop potential: $U_a = \frac{E_c}{e} = 0,6$ V.
2. The kinetic energy of the electron arrived on the anode :
   $E_c = E_e + eU = 10,6$ eV$= 1,7 \cdot 10^{-18}$J.
3. The number of photons per second: $n_p = \frac{P}{h\nu} = \frac{400 \cdot 10^{-3}}{4.965 \cdot 10^{-19}} = 8.06 \cdot 10^{17}$ ph/s.
   The number of electrons per seconde: $n_e = \frac{I_{sat}}{e} = \frac{50 \cdot 10^{-3}}{1.6 \cdot 10^{-19}} = 3.25 \cdot 10^{17}$ electrons/second.
   The quantum efficiency:
   $$\eta = \frac{n_e}{n_p} = \frac{3.25}{8.06} = 38.8\%$$

Exercice 6: Photodiode

1. This is an active sensor: transformation of light energy into electrical energy by PN junction in the photodiode.
2. A capacitor is equivalent to a complex resistance of $R_C = 1/j\ omega C_L$, the total resistance of two resistors in parallel
   $$R = \frac{R_1 R_2}{R_1 + R_2}$$
   So we have
   $$V_L = I \frac{R_L / j\omega C_L}{R_L + 1/j\omega C_L} = K\Phi \frac{R_L}{1 + j\omega R_L C_L}$$
   We know: $\omega = 2\pi f$, o the transfer function:
   $$S(f) = \frac{V_L}{\Phi} = \frac{KR_L}{1 + j2\pi f R_L C_L}$$
3. The gain:
   $$G(f) = |S(f)| = \frac{KR_L}{\sqrt{1 + (2\pi f R_L C_L)^2}}$$
   Obviously the gain is maximum when $f = 0$:
   $$G_{max} = KR_L$$
   The cutoff frequency f_c is obtained by::
   $$G(f_c) = \frac{G_{max}}{\sqrt{2}} = \frac{KR_L}{\sqrt{2}} = \frac{KR_L}{\sqrt{1 + (2\pi f_c R_L C_L)^2}}$$
   where
   $$f_c = \frac{1}{2\pi R_L C_L}$$
4. We see that the cutoff frequency depends on R_L and C_L, we now want the cutoff frequency to be $f'_c = 2f_{max} = 200$ kHz, donc
   $$R_L = \frac{1}{2\pi f_c C_L} = \frac{1}{2\pi 200 \times 10^3 \times 80 \times 10^{-12}} \simeq 10k\Omega$$

5. The static Gain (Sensitivity) is $G(f=0)$, so

$$G_{stat} = KR_L = 0,35 \times 10^3 = 3500V/W$$

The dynamic sensitivity is $G(f_{max})$ donc

$$G_{dyn} = \frac{KR_L}{\sqrt{1+(f_{max}/f_c')^2}} = \frac{KR_L}{\sqrt{1+1/4}} = 3500\sqrt{\frac{4}{5}} = 3130V/W$$

6. $\Phi = \Phi_{0max} + \Phi_{1max}\sin(2\pi ft)$
$V_0 = G_{stat}\Phi_{0max} = 3500 \times 0,001 = 3,5$ V
$V_1 = G_{dyn}(20kHz)\Phi_{1max} = \frac{3500\times 0,1\times 10^{-3}}{\sqrt{1+(1/10)^2}} = 0,35$ V
$V = 3,5$ V$+0,35\sin(2\pi ft)$.

Exercice 7: Emission Characterization of a Source

1. Isotropic Source:
   (a) Isotropic Source $\Rightarrow$ the emission indicator is constant: $I(\theta) = 1$, because the radiation is homogeneous in all directions.
   (b) P_0 radiated in 4π steradians gives an intensity $I = P_0/4\pi$ (W/sr).
   (c) At a distance of d, the illuminance is $E = F_e/(4\pi d^2)$ (W/m^2)
2. Spherical source:
   (a) The emittance (exitance) of the source is $M_e = \sigma T^4$.
   (b) The total flux radiated by the source is $F_e = 4\pi R^2\sigma T^4$.
   (c) At a distance of d, the irradiance is $E = P_0/4\pi d^2 = MR^2/d^2$.
   (d) The source radiates as a black body, it emits at most for $\lambda_{max} \approx 3000/T$(K)

Exercice 8: Performance of a detector

1. The relationship between current response and quantum efficiency is: $R = \eta \cdot e\lambda/hc$. The curve of $\eta(\lambda)$ and corresponding $R_\lambda(\lambda)$ are indicated in the figure. If λ is in micron, this relation gives

$$R \approx 0.8056\eta\lambda(\text{A/W})$$

This is a straight line 0.8056λ (approximatively 0.8λ), Theoretical response of a unit efficiency.

$\lambda(\mu m)$	0.8	1.0	2.2	3.0	3.6	4.2	5.1
η	0.12	0.61	0.75	0.89	0.83	0.63	0.25
$R(\lambda)$	0.08	0.50	1.33	2.15	2.40	2.13	1.03

   (a) The sensitivity domain is determined from the area where $R_\lambda > R(max)/2 \approx 1.2$ A/W, ce which is in the range of $2-5$ μm (cutoff wavelength $\approx 5\mu$m).
   (b) The peak wavelength: $\approx 3.6\mu$m.
   (c) The max response is of the order 2.4 A/W.

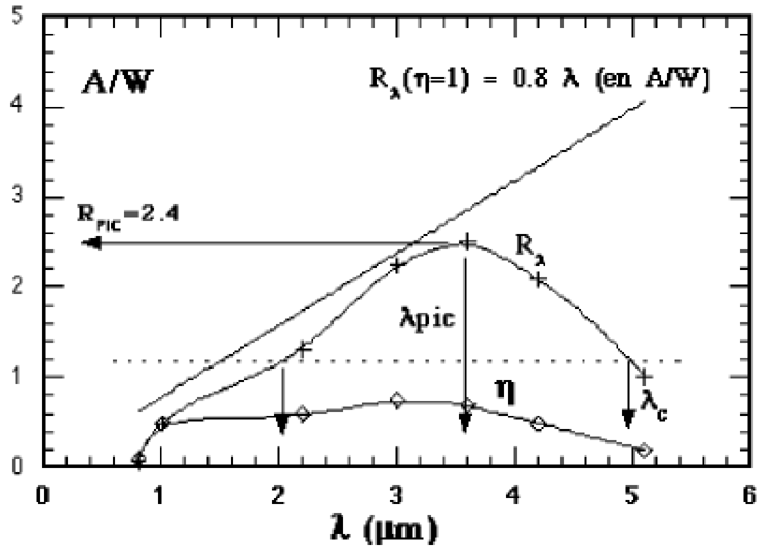

Figure 6: Tracé et mesures pour exercice Rendement.

(d) The cutoff wavelength is $\lambda_c = 5.1\ \mu$m.

2. $R_\lambda(\lambda)$ is a straight line: $R_\lambda(\eta = 1) = \frac{e\lambda}{hc} = 0.8\lambda$ (A/W) (λ in μm). he commercially available detector has a current response of 0.95 A/W at peak to 1.55 μm. which corresponds to a quantum yield of $\eta = 0.95/(0.8 \times 1.55) = 77\%$, which is very correct. The detector surface is $A = \pi 0.05^2/4 \approx 1.96 \times 10^{-3}$ cm^2. $D* = \sqrt{A}/NEP$. For $NEP = 8 \times 10^{-15}$ W/Hz$^{1/2}$, we find $D^* \approx 5.5 \times 10^{12}$ cm.Hz$^{1/2}$/W, CQFD.

Exercice 9: Filter

1. This source is rather orange without filter (slope of the rising spectrum when *lambda* increases) and red with the filter.

2. The color The filter is square, which allows to calculate the total power received from the source: for $\lambda = 0.65\ \mu$m, $P_\lambda = 3.5 \times 10^{-8}$ W/μm^{-1}. The power selected by the filter (transmission 80% est donc $P = 3.5 \times 10^{-8} \times 0.80 \times 0.05 = 1.4 \times 10^{-9}$ W.

3. The answer in Volts will then be: $R_v = 1.4 \times 10^{-9} \times 5 \times 10^5 = 700\ \mu$V, give a signal on noise: $S/N = 117$.

4. The NEP of the detector is $\sigma/R = 1.2 \times 10^{-10}$ W.

5. The cutoff frequency of the detector is $f_c = 1/2\pi\tau = 15.9$ Hz. When the detector is modulated at $f = 1000/60 = 16.7$ Hz, the response is reduced by a factor $1/\sqrt{1 + (f/f_c)^2} = 0.69$, and passes at 3.4×10^4 V/W.