Optical diagnostics in fluid mechanics
Metrology of particles

Part 3: measurement techniques

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Measurement techniques

- **Measurement of individual particles**
  - LDV and PDA
  - Rainbow refractometry
  - Imaging
- **Measurement of cloud particles**
  - Extinction spectrometry
  - Refractometry (Malvern)
  - Global rainbow

Measurement techniques

Techniques and measured parameters

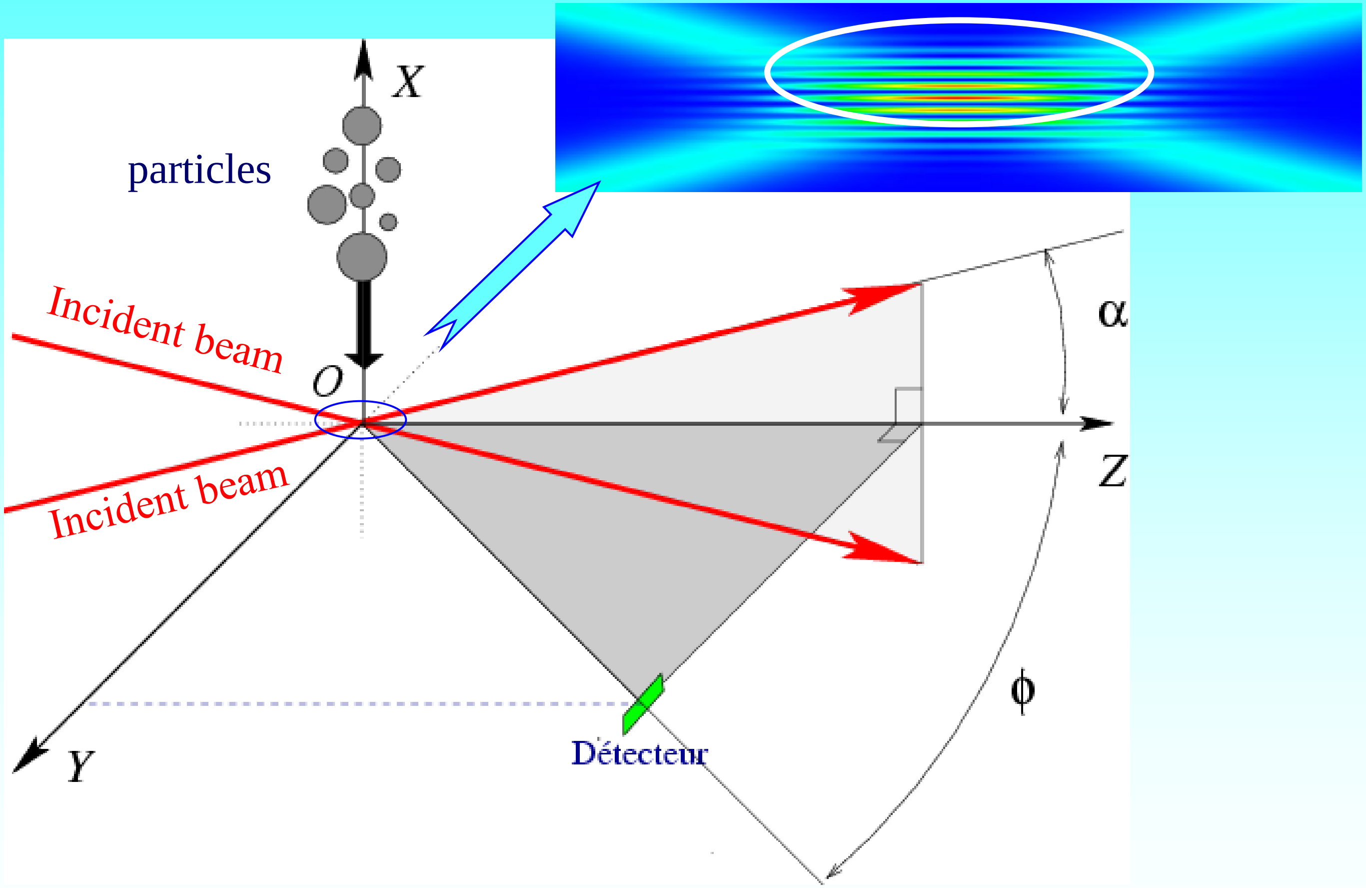
	Detector		Particles		Measured parameters					
	Points	**CCD**	**Individual**	**Cloud**	**V**	**D**	**C**	**T/m**	**x, y, z**	**Time**
LDV	X		X		X		?		X	y
PDA	X		X		X	X	difficulty	possible	X	y
Rainbow		X	X			X		X	?	?
Global rainbow		X		X		X		X	?	?
Extinction spectroscopy		X		X		X	X	?		Y
PIV		X		X	X					Y

V : velocity
D : diameter
C : concentration

T/m : temperature or refraction index
x,y,z : positions of particles
Time : temporal evolution

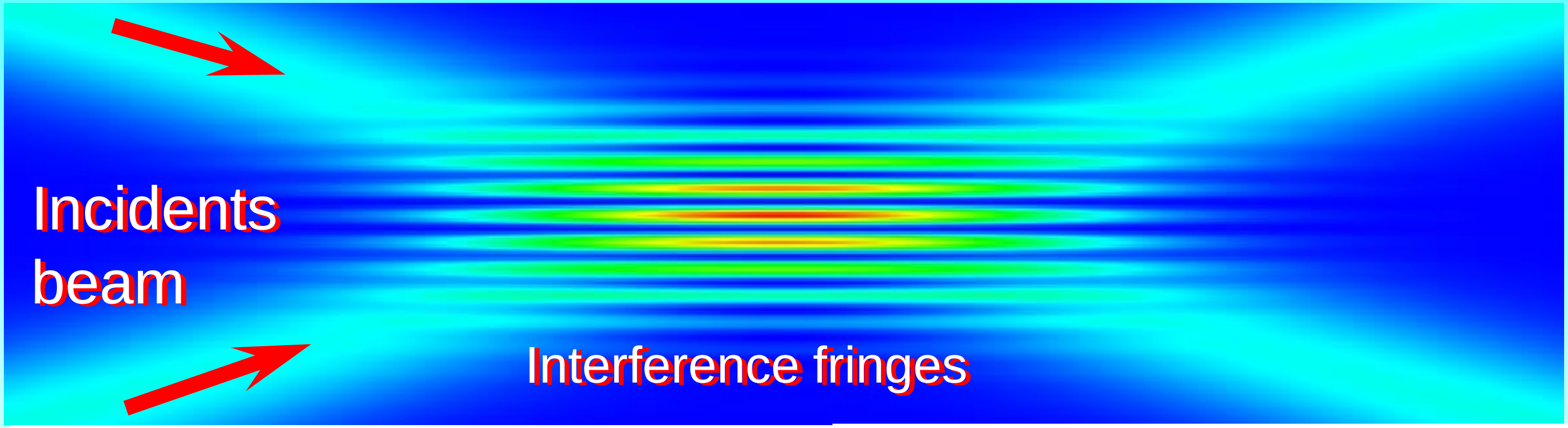
LDV - Laser Doppler Velocimetry

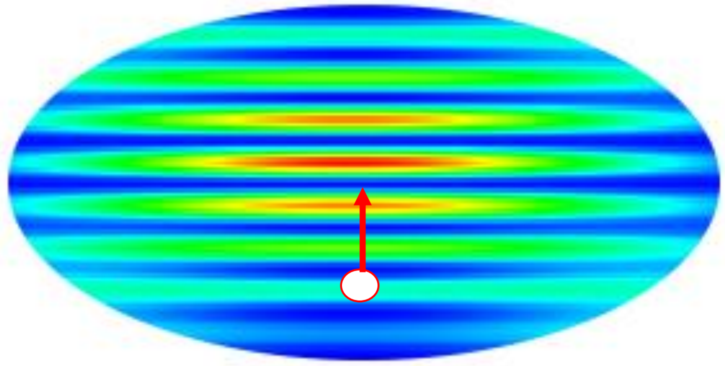
Configuration

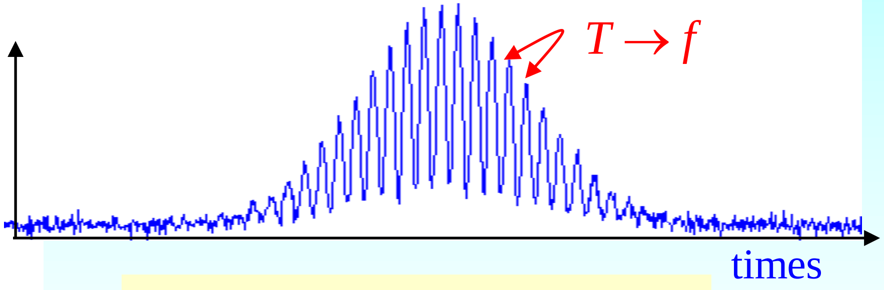


LDV - Laser Doppler Velocimetry

Principle of measurement







Measurement of velocity:

$$\delta_f = \frac{\lambda}{2\sin\alpha} \qquad v_x = \frac{f\lambda}{2\sin\alpha}$$

LDV - Laser Doppler Velocimetry

Principle of measurement

Demonstration of the fringe :

$$E_1 = E_0 e^{i\vec{k}_1\cdot\vec{r}} = E_0 e^{ik_z z + ik_x x}$$

$$E_2 = E_0 e^{i\vec{k}_2\cdot\vec{r}} = E_0 e^{ik_z z - ik_x x}$$

$$E = E_1 + E_2 = E_0 (e^{ik_z z - ik_x x} + e^{ik_z z + ik_x x}) = 2E_0 e^{ik_z z} \cos(k_x x)$$

$$I = 4E_0^2 \cos^2(k_x x)$$

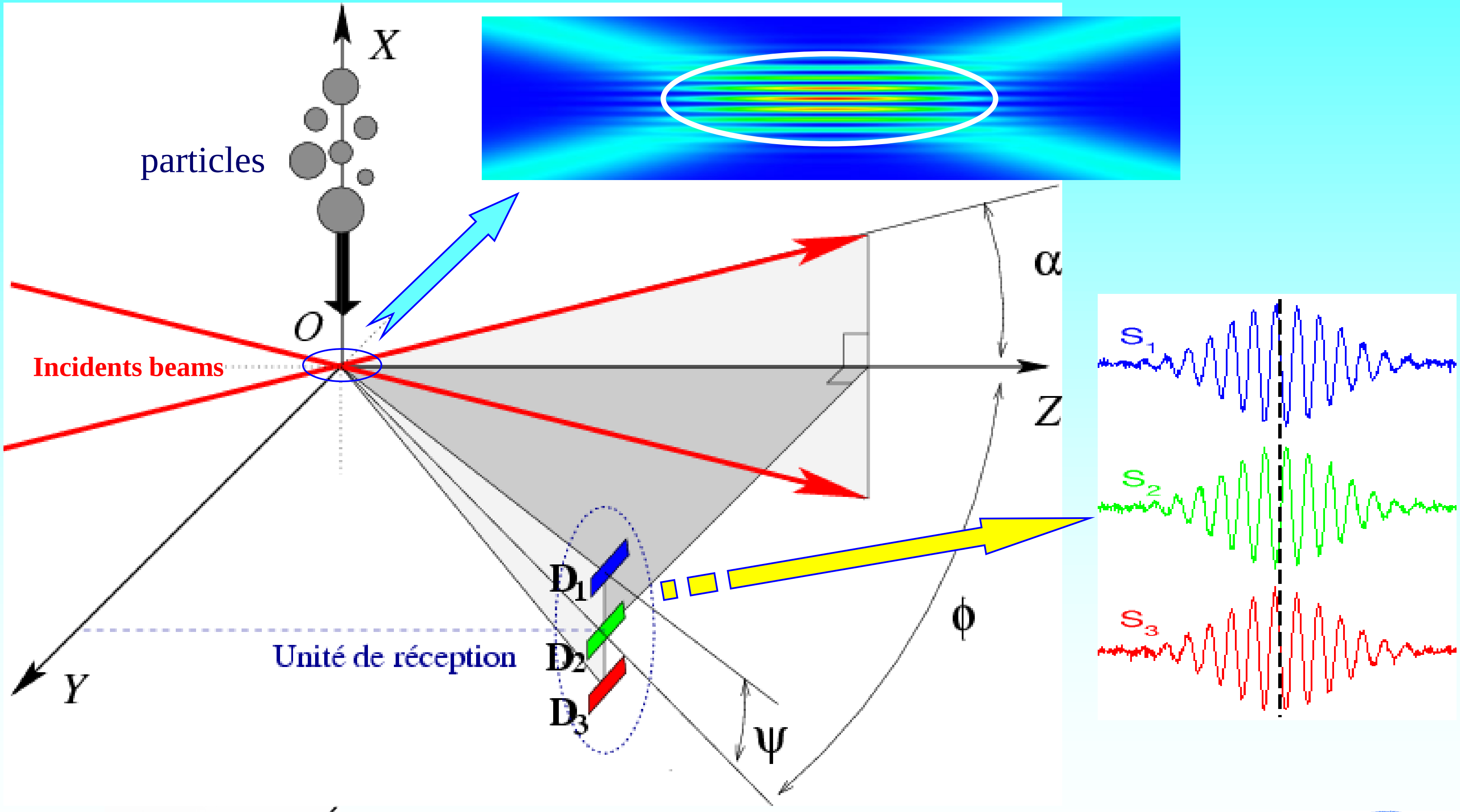
The intensity is maximum:

$$k_x x = k \sin\alpha x = p\pi \qquad x_p = \frac{p\pi}{k\sin\alpha} = \frac{\lambda p}{2\sin\alpha}$$

$$\delta_f = x_p - x_{p-1} = \frac{\lambda}{2\sin\alpha} \qquad v_x = \frac{f\lambda}{2\sin\alpha}$$

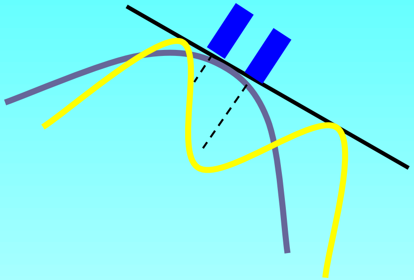
PDA - Phase Doppler Anemometry

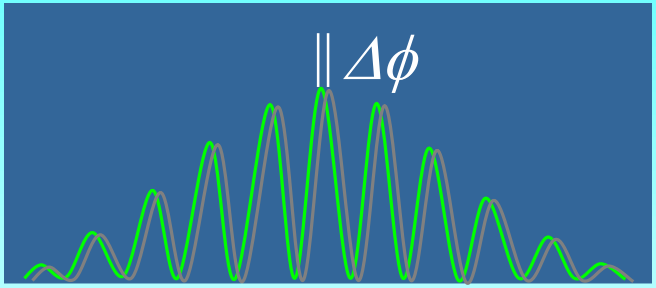
Principle of measurement

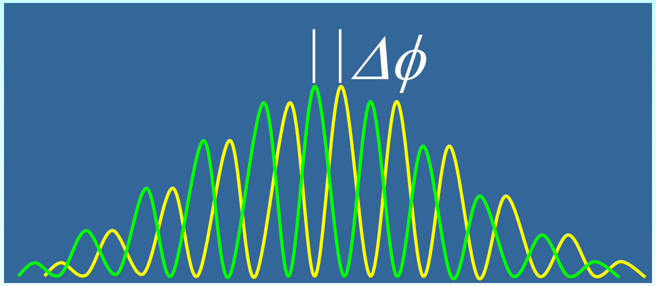


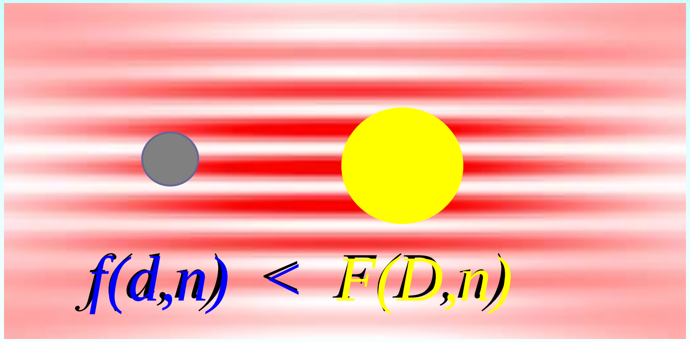
PDA – Model of fringes

Principle of measurement









$f(d,n) < F(D,n)$

Volume of measurement

PDA – Model of geometrical optics

Principle of measurement

Relation phase-diameter:

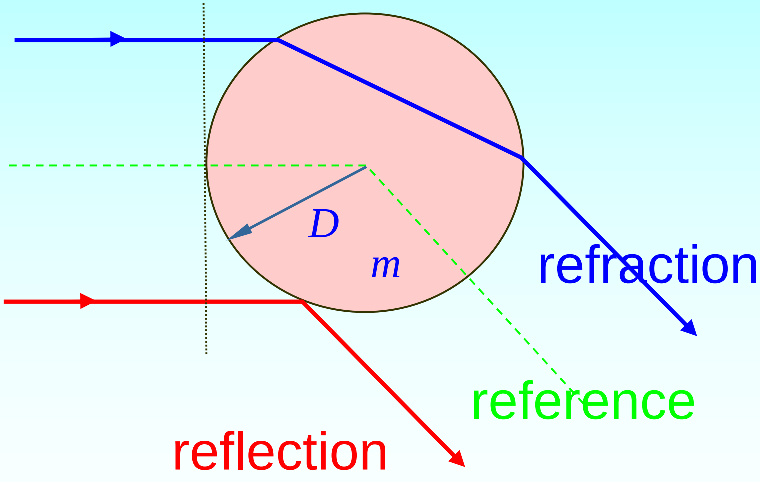
$$\boxed{\varphi_0^i = \frac{2\pi \sin\alpha \sin\psi^i}{\sqrt{2(1 - cos\alpha \cos\psi^i \cos\phi^i)}}\frac{D}{\lambda} + \varphi_0}$$

$$\boxed{\varphi_1^i = \frac{-2\pi m \sin\alpha \sin\psi^i}{\sqrt{2(1 - cos\alpha \cos\psi^i \cos\phi^i)[1 + m^2 - m\sqrt{1 - cos\alpha \cos\psi^i \cos\phi^i)}}}\frac{D}{\lambda}}$$

Relation phase difference - diameter:

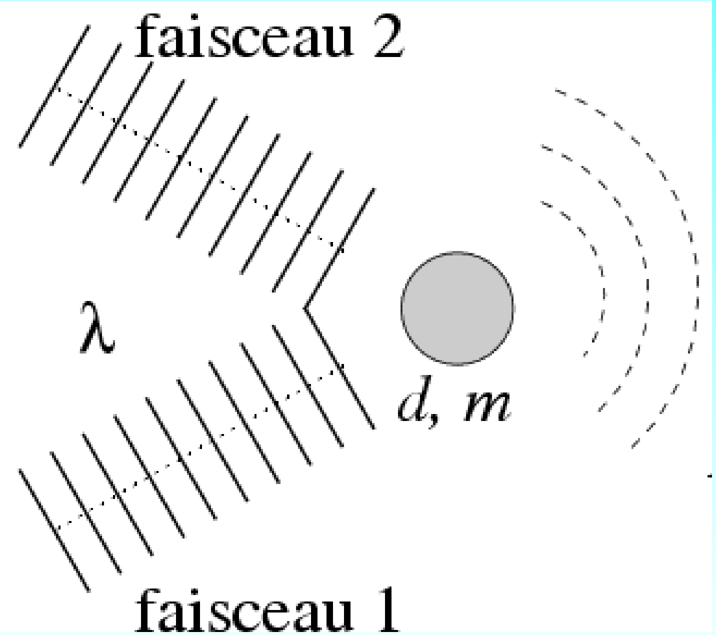
$$\Delta\varphi_{refl} = \varphi_{refl}^2 - \varphi_{refl}^1 = C_{refl} D$$

$$\Delta\varphi_{refr} = \varphi_{rerl}^2 - \varphi_{refr}^1 = C_{refr}(m) D$$



PDA – model of waves

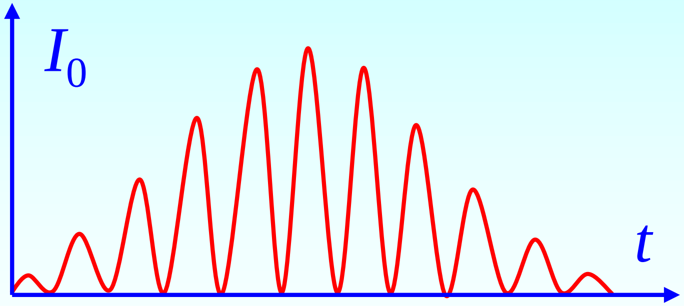
Principle of measurement



Rigorous simulation

$$E_s = E_1(d,m,\lambda,\alpha) + E_2(d,m,\lambda,\alpha)$$

$$I = I_0\left[1 + V\cos(2\pi\upsilon\, t + \phi)\right]$$



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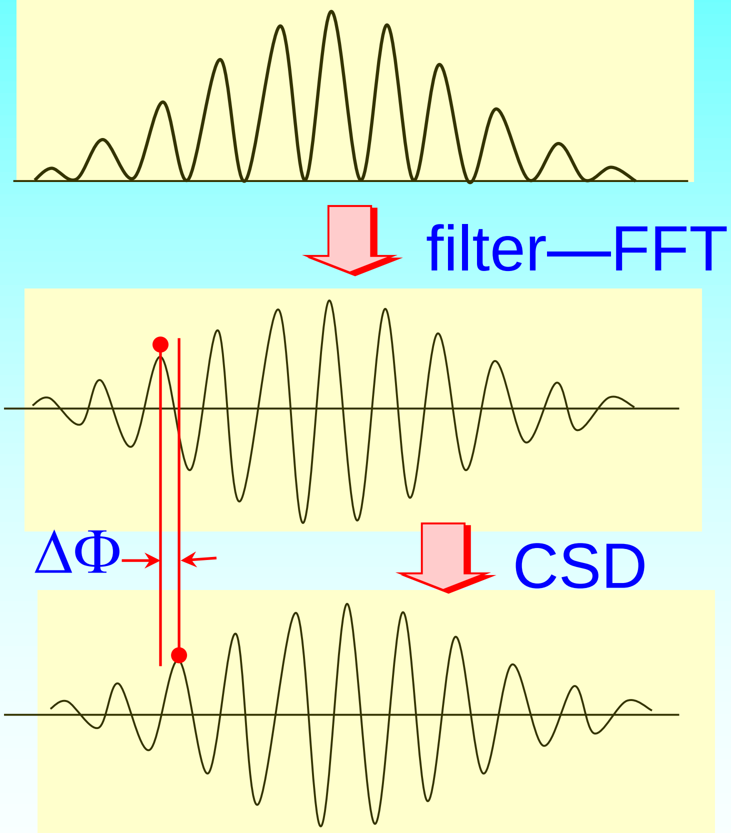
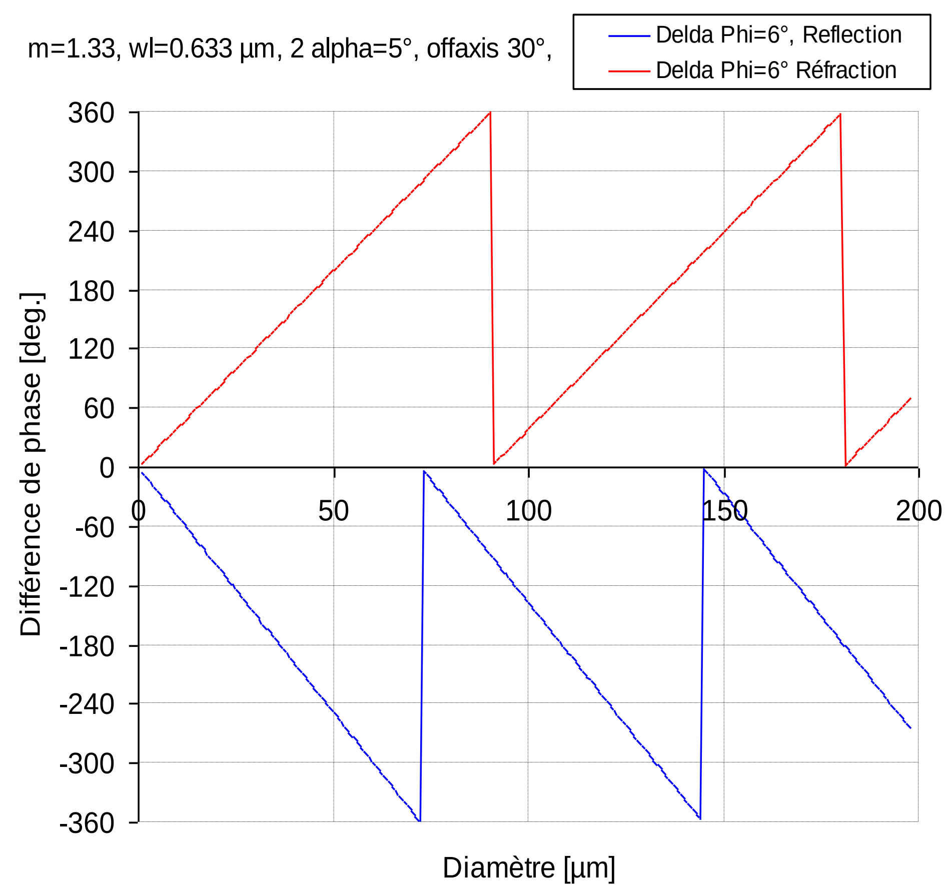
PDA – signal processing

Principle of measurement

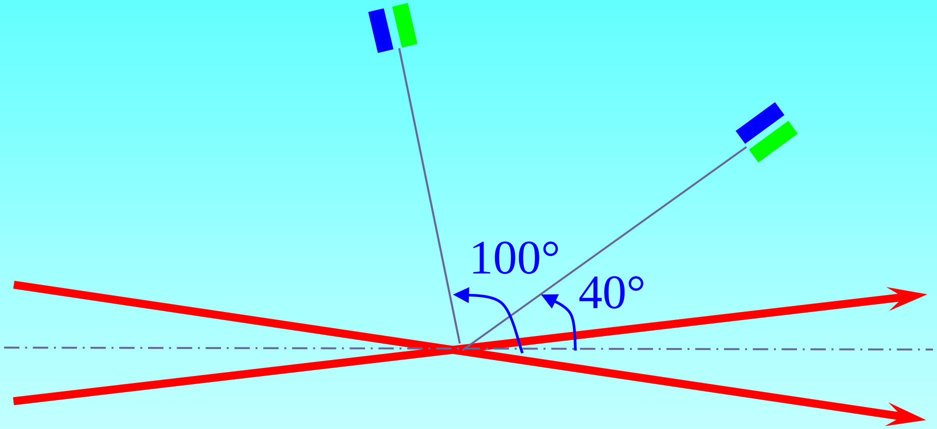
PDA signal

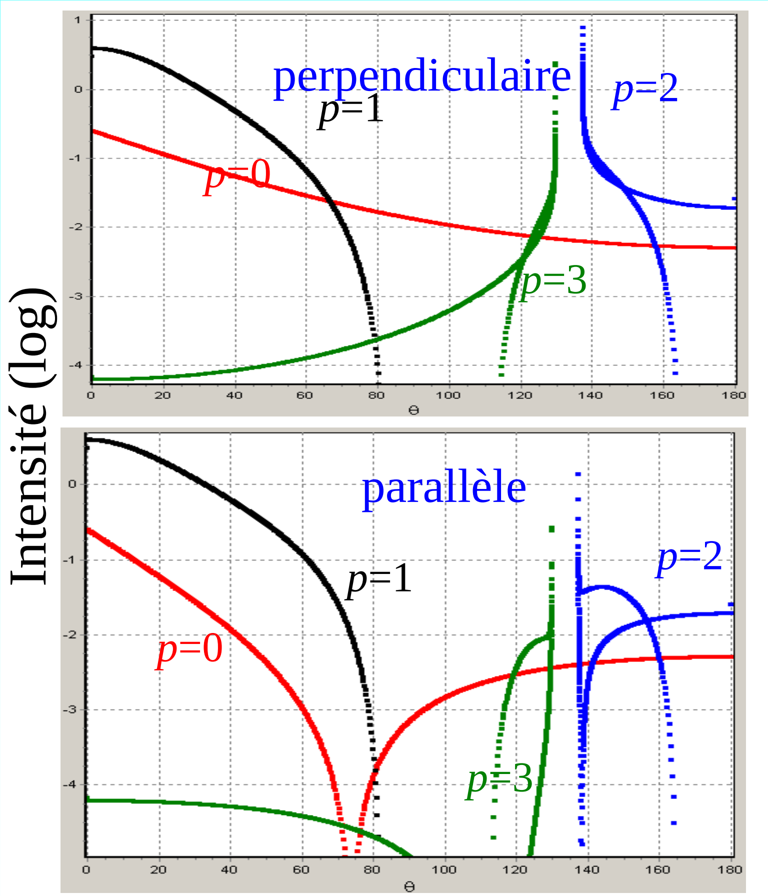




PDA – trajectory effect

Effect of scattering mode





Reflection dominant

$$\Delta\varphi_{refl} = C_{refl} D$$

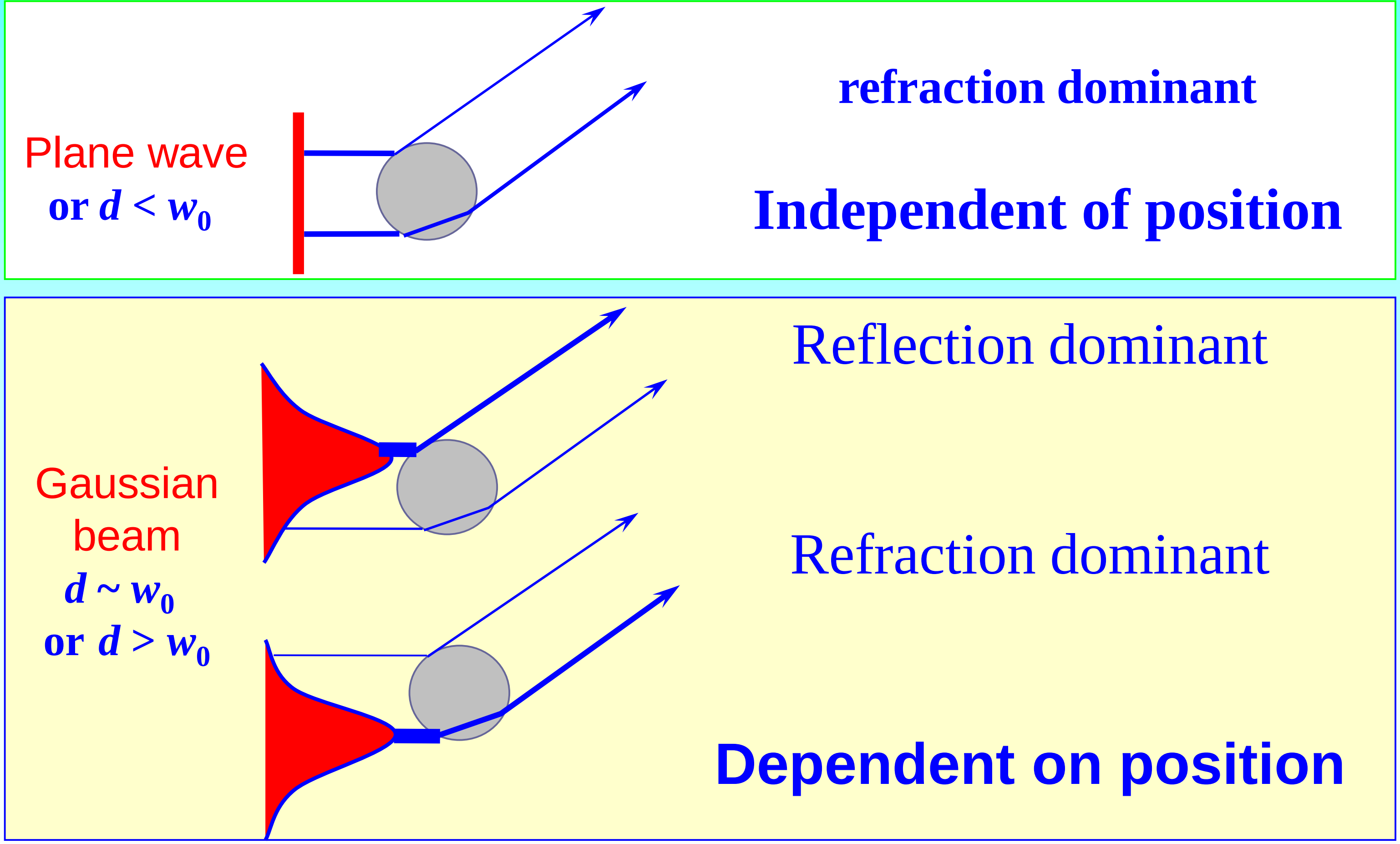
?

or refraction dominant

$$\Delta\varphi_{refr} = C_{refr}(m) D$$

PDA – trajectory effect

trajectory effect



Plane wave

or $d < w_0$

refraction dominant

Independent of position

Gaussian beam

$d \sim w_0$

or $d > w_0$

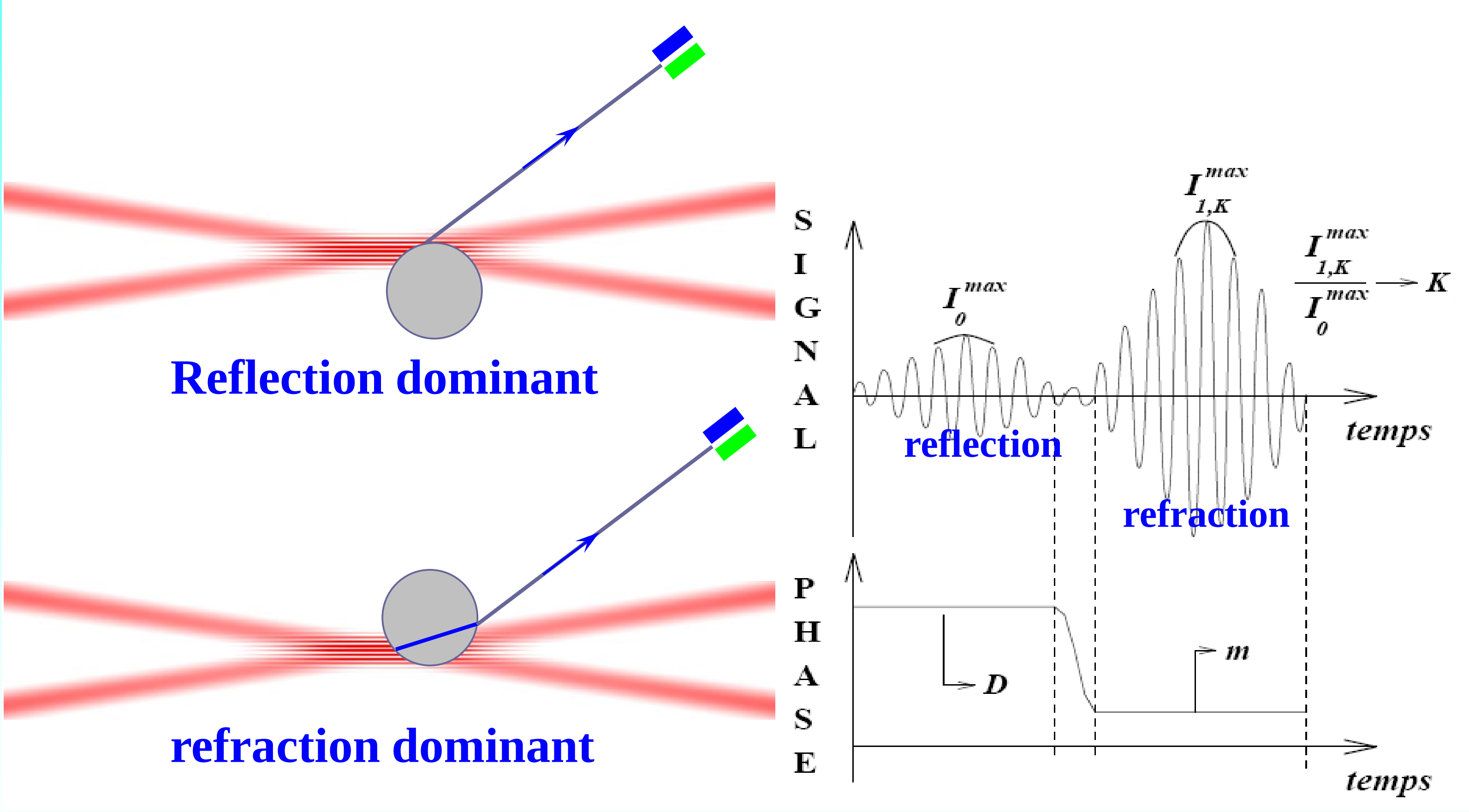
Reflection dominant

Refraction dominant

Dependent on position

PDA – trajectory effect

trajectory effect



ADL and PDA

Dynamics of measurement

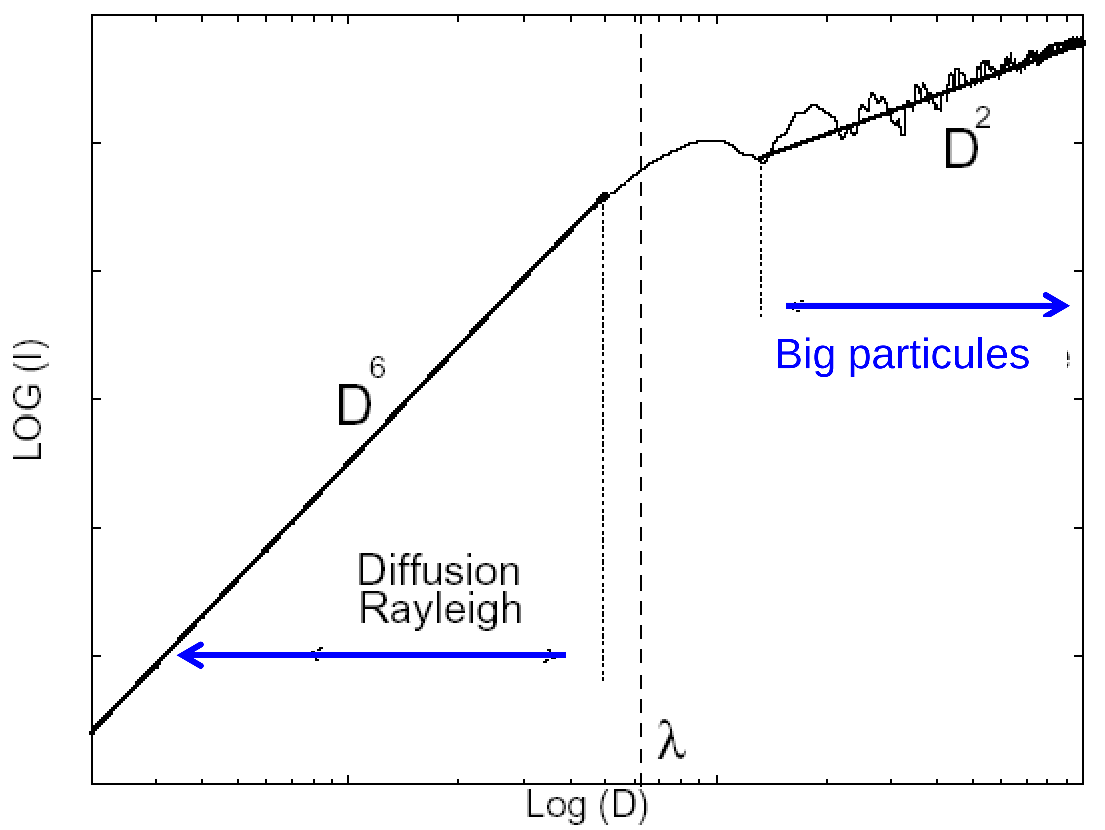
For small particles

(Rayleigh scattering regime):

$$I \propto D^6 \quad \rightarrow \quad \log(I) \propto 6\log(D)$$

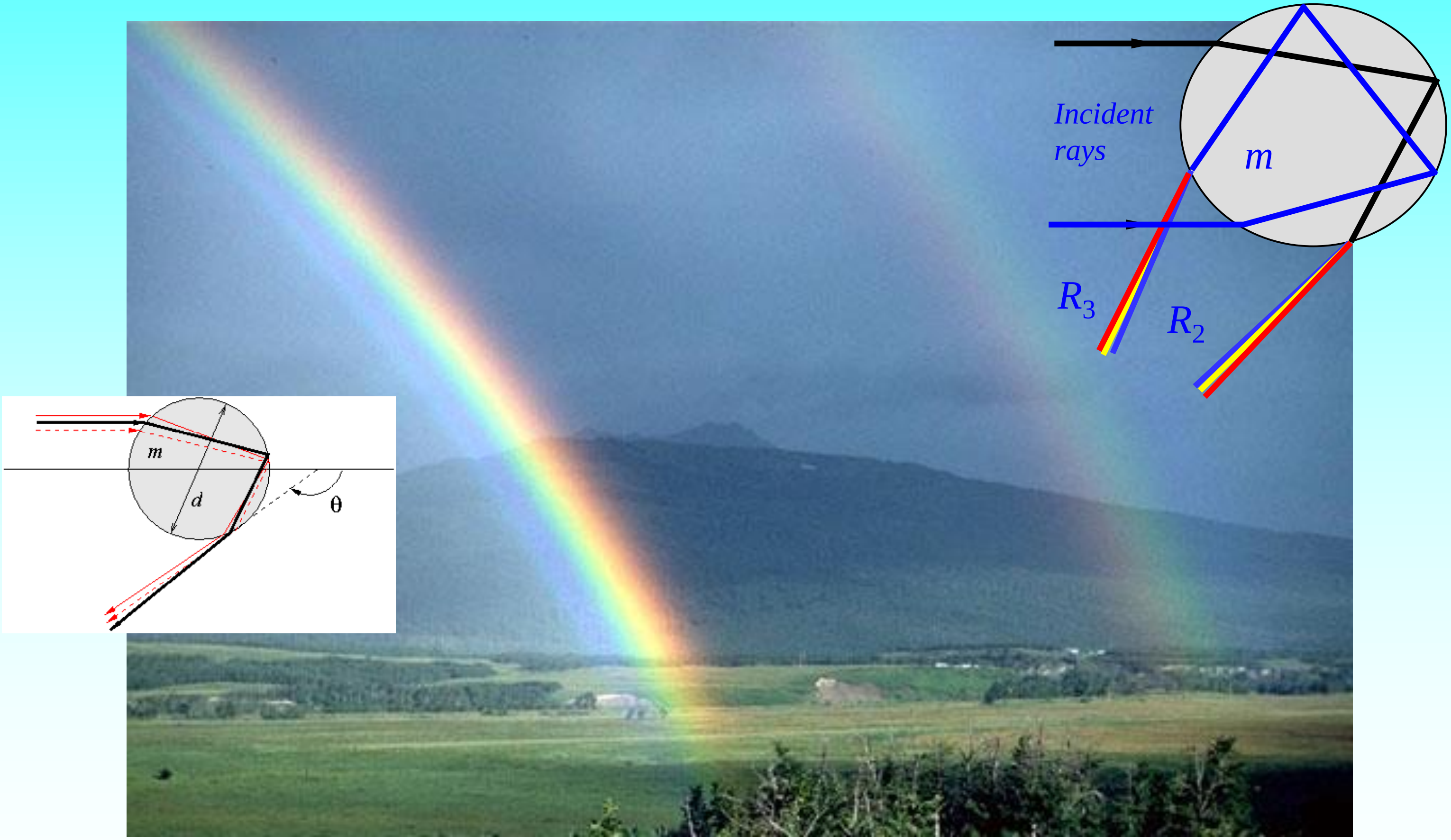
For big particles:

$$I \propto D \quad \rightarrow \quad \log(I) \propto \log(D)$$



Rainbow

Natural phenomenon



Rainbow – geometrical optics

GO model of rainbow

Geometrical optics:

$$\theta_0 = \pi - 2i, \quad \theta_1 = -2(i - i')$$

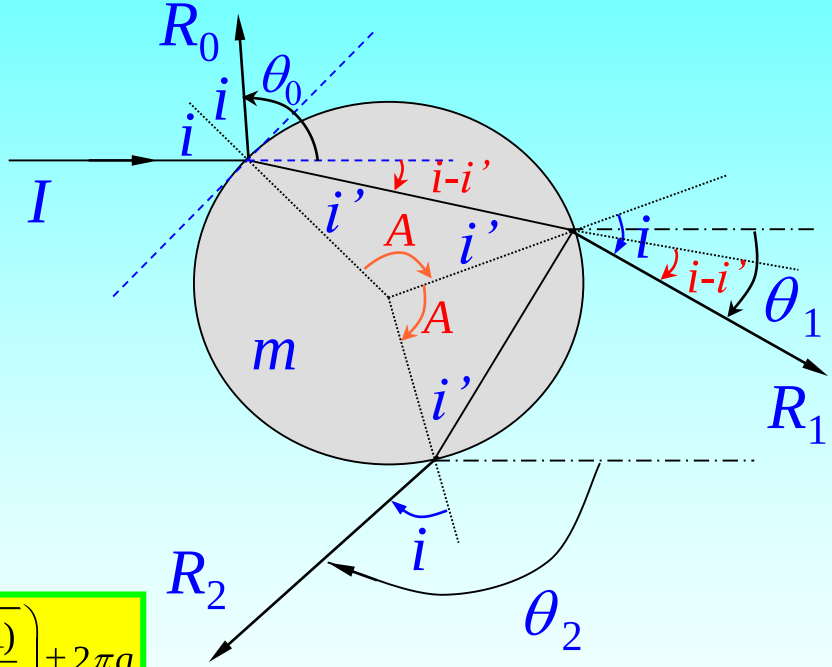
$$\theta_p = 2\tau - 2p\tau' = 2pi' - 2i - (p-1)\pi$$

$$\frac{d\theta_p}{di} = 2p\frac{di'}{di} - 2 = 0$$

$$\theta_p^{go} = 2p\cos^{-1}\left(\frac{p\cos i}{m}\right) - 2i \pm 2\pi q$$

$$\text{avec} \;\; \sin i = \sqrt{\frac{p^2 - m^2}{p^2 - 1}}$$

$$\boxed{\theta_p^{go} = 2\arctan\left(\sqrt{\frac{m^2-1}{p^2-m^2}}\right) - 2p\arctan\left(\sqrt{\frac{p^2(m^2-1)}{p^2-m^2}}\right) \pm 2\pi q}$$



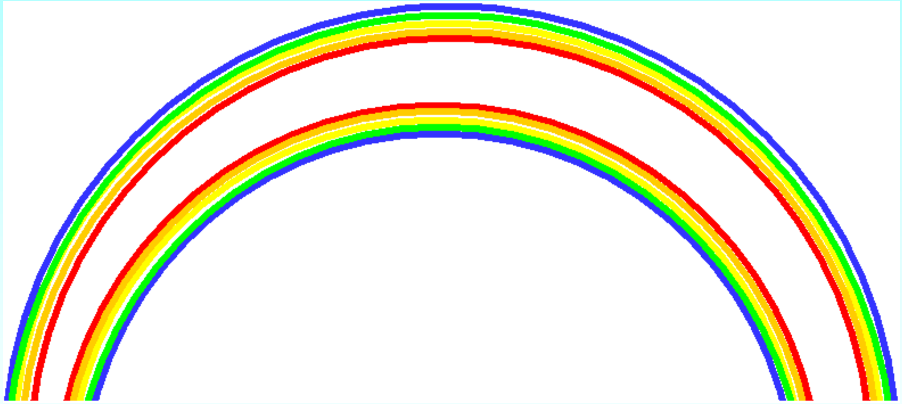
Rainbow of 1st order : p=2

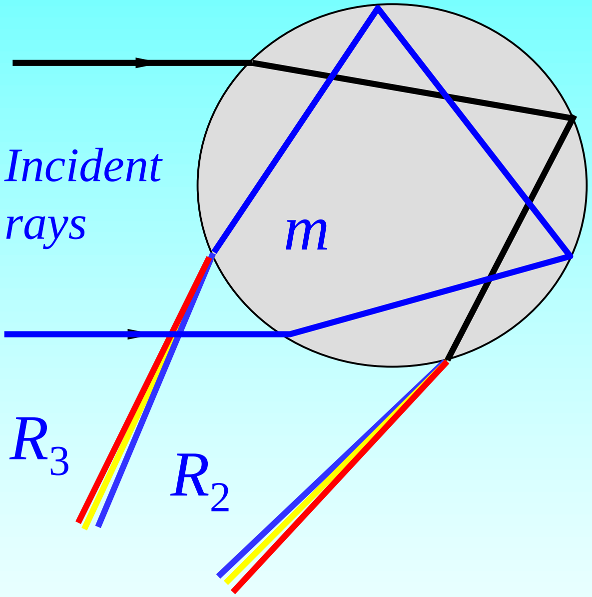
Rainbow of 2nd order : p=3

Rainbow – geometrical optics

GO model of rainbow

λ	n	θ_2	θ_3
486.1	1.3371	138.5°	128.0°
589.3	1.3330	137.9°	129.1°
656.3	1.3311	137.6°	129.6°





Rainbow – in the nature

Can we observe personally a whole rainbow?

Why ?

How?

Photo of a rainbow taken by drone in *Guangzhou Sept. 20, 2020*

Rainbow – Airy theory

Airy theory (1838)

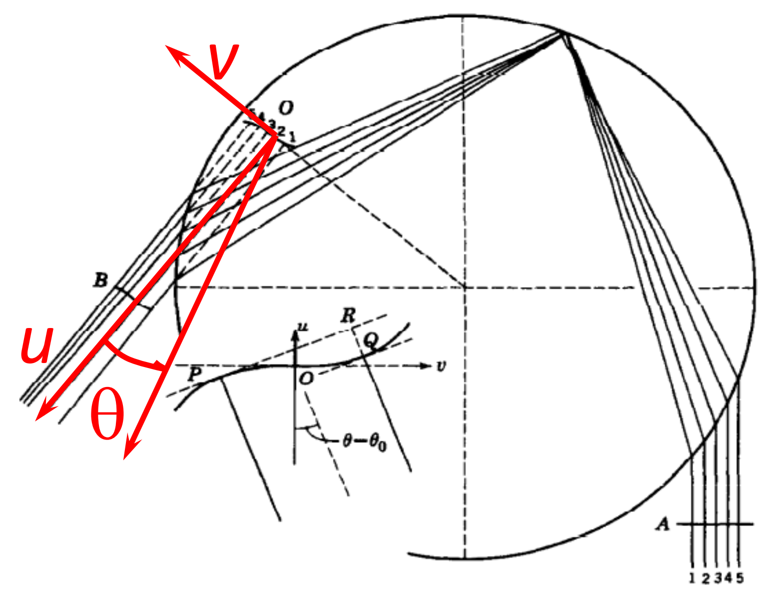
Sir George Airy
(1801-1892)

- Phase difference is:

$$\Delta\Psi = khv^3/3a^2$$

- Amplitude is constant for all emergent rays
- Amplitude of scattered field in θ direction:

$$\int_{-\infty}^{\infty} e^{-ikv(\theta-\theta_0)+ikhv^3/3a^2}\,dv$$



- The version of van de Hulst (1957) permits to predict the profile,
- But the absolute intensity was corrected later.

Rainbow – Airy theory

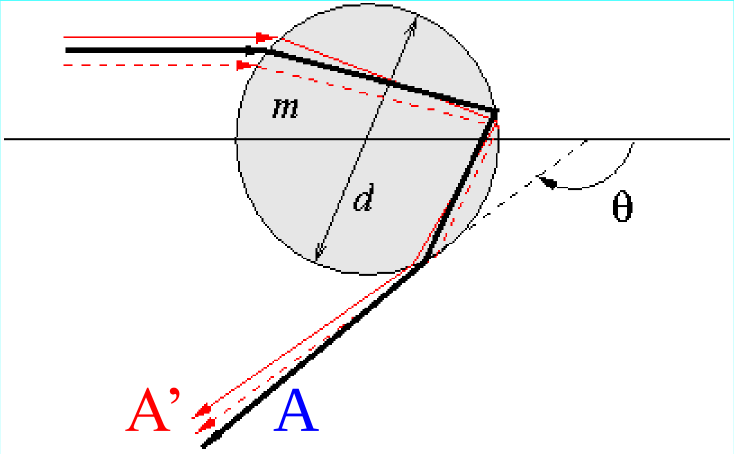
Peak positions according to Airy theory :

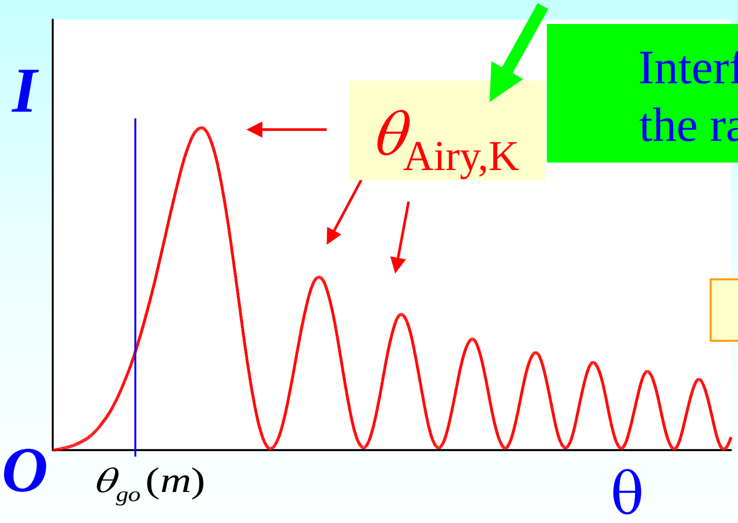
$$\theta_{Airy,i}(m,d)$$

$$\left|\theta_{Airy,1}-\theta_{go}(p,m)\right| = 1.087376\left(\frac{\pi^2}{12\alpha^2}H\right)^{1/3}$$

$$\left|\theta_{Airy,K}-\theta_{go}(p,m)\right| = \left[\frac{9\pi^2}{4\alpha^2}\left(K+\frac{1}{4}\right)^2 H\right]^{1/3}$$

$$H = \frac{\left(p^2-1\right)^2}{p^2}\left[\frac{p^2-m^2}{\left(m^2-1\right)^3}\right]^{1/2}$$





$\theta_{Airy,K}$

Interference b/t the rays A et A’

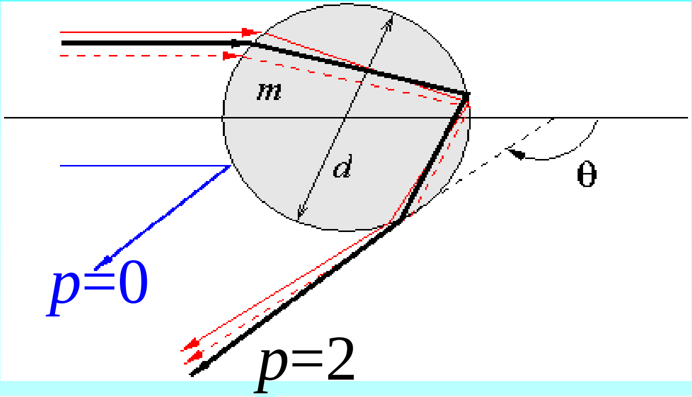
Measurement of *m* et *d*

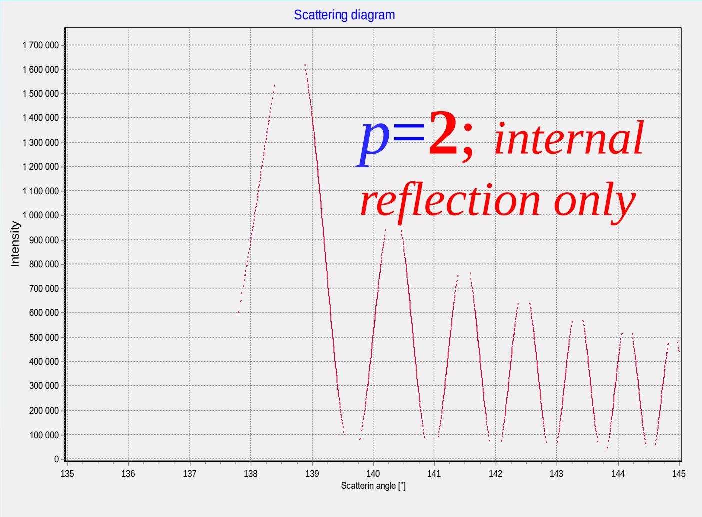
A_i: constants

Ref: Wang & van de Hulst
Appl. Opt. 30(1):106-117 (1991).

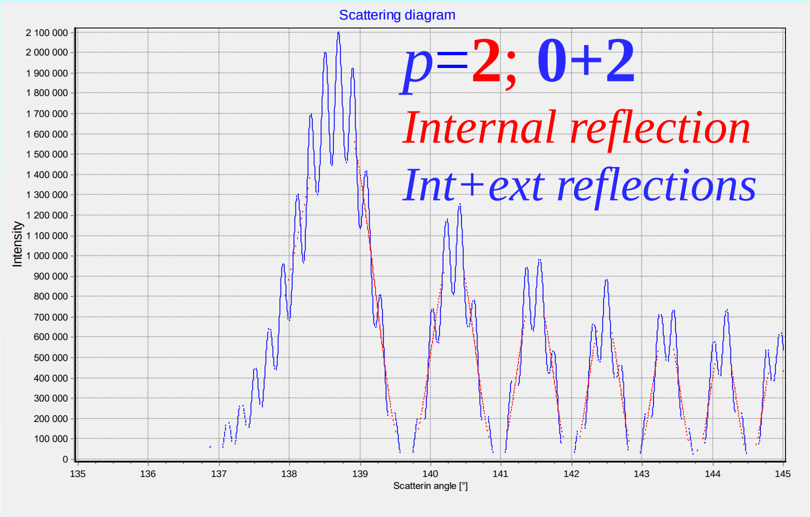
Rainbow – Lorenz-Mie theory

Lorenz-Mie theory and Debye theory





p=**2**; *internal reflection only*



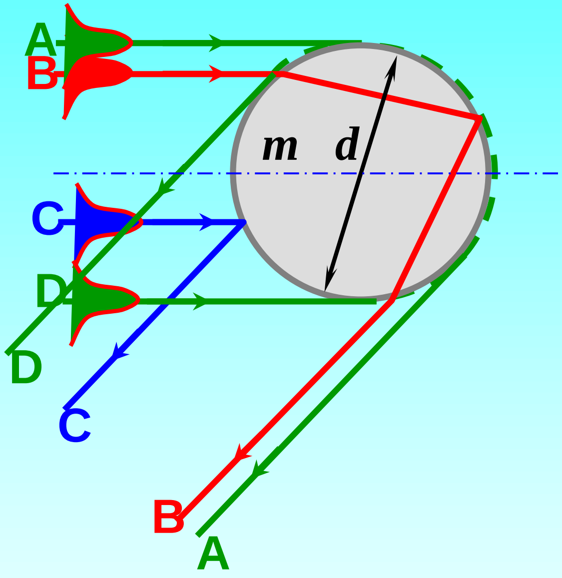
p=**2**; **0+2**

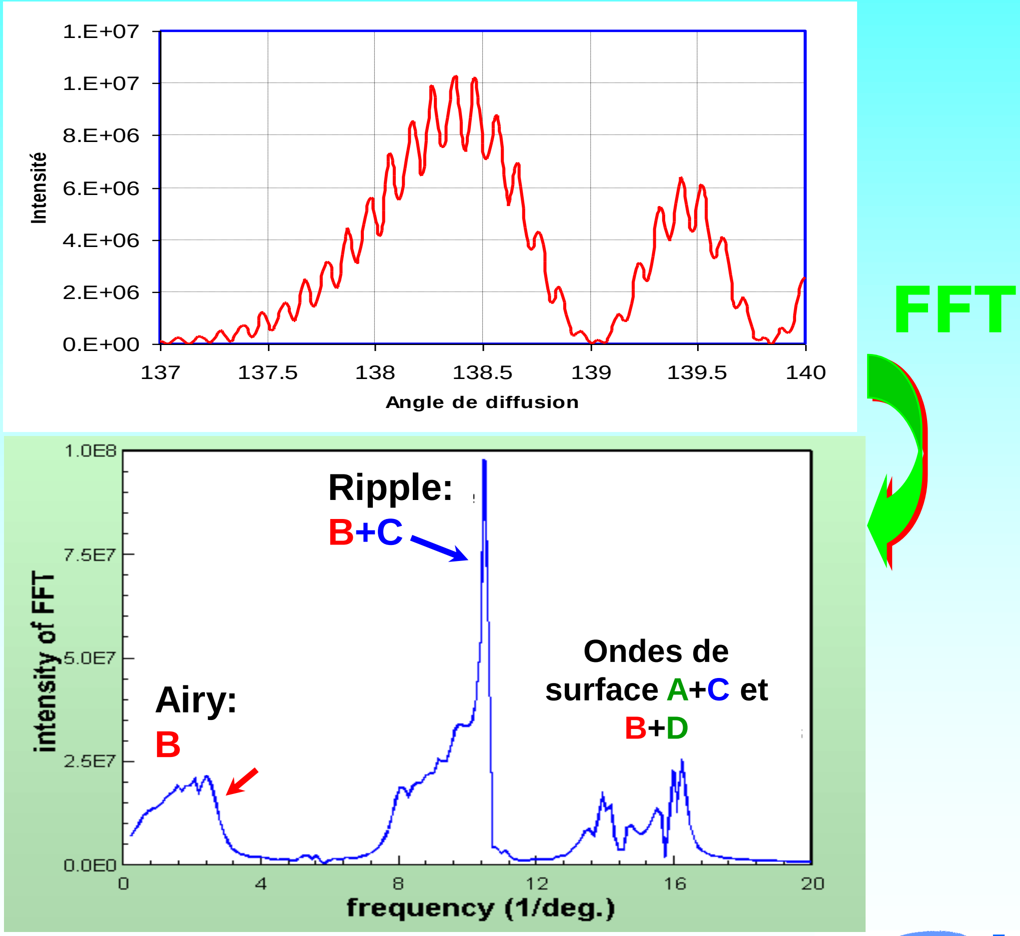
Internal reflection

Int+ext reflections

Rainbow – GLMT

GLMT and Debye theory





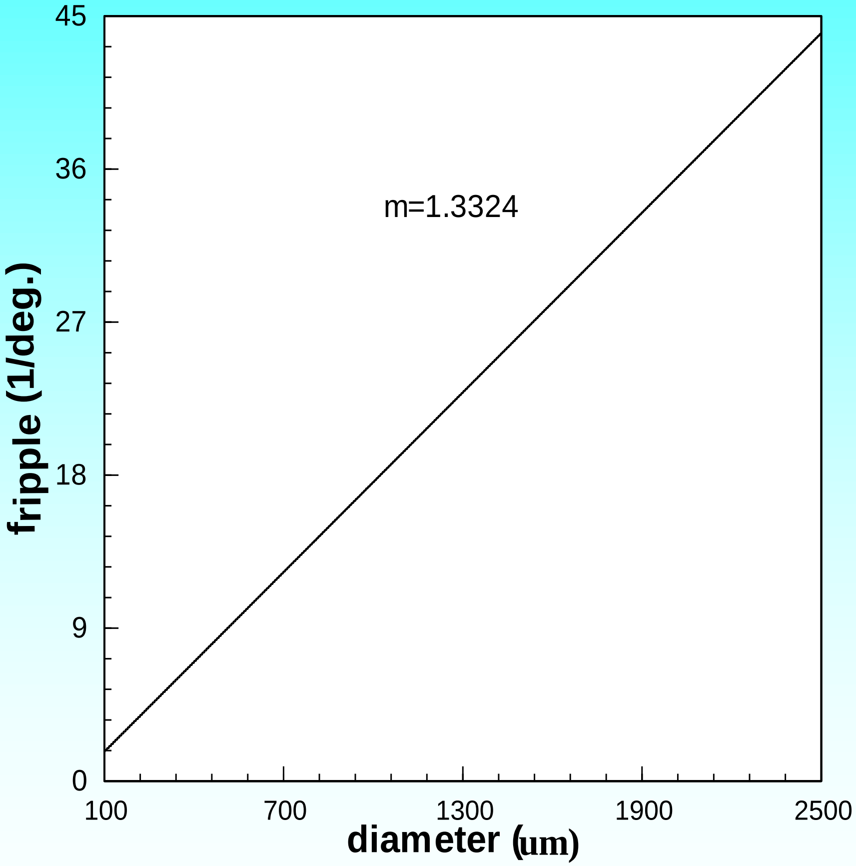
Debye theory/series:

Separation of all the modes

- Diffraction
- Reflection
- Refraction of order *p*

Rainbow – metrology

Measurement of diameter



Relation

diameter- ripple frequency

An example of simulation

$$d = 57\lambda f_{ripple}$$

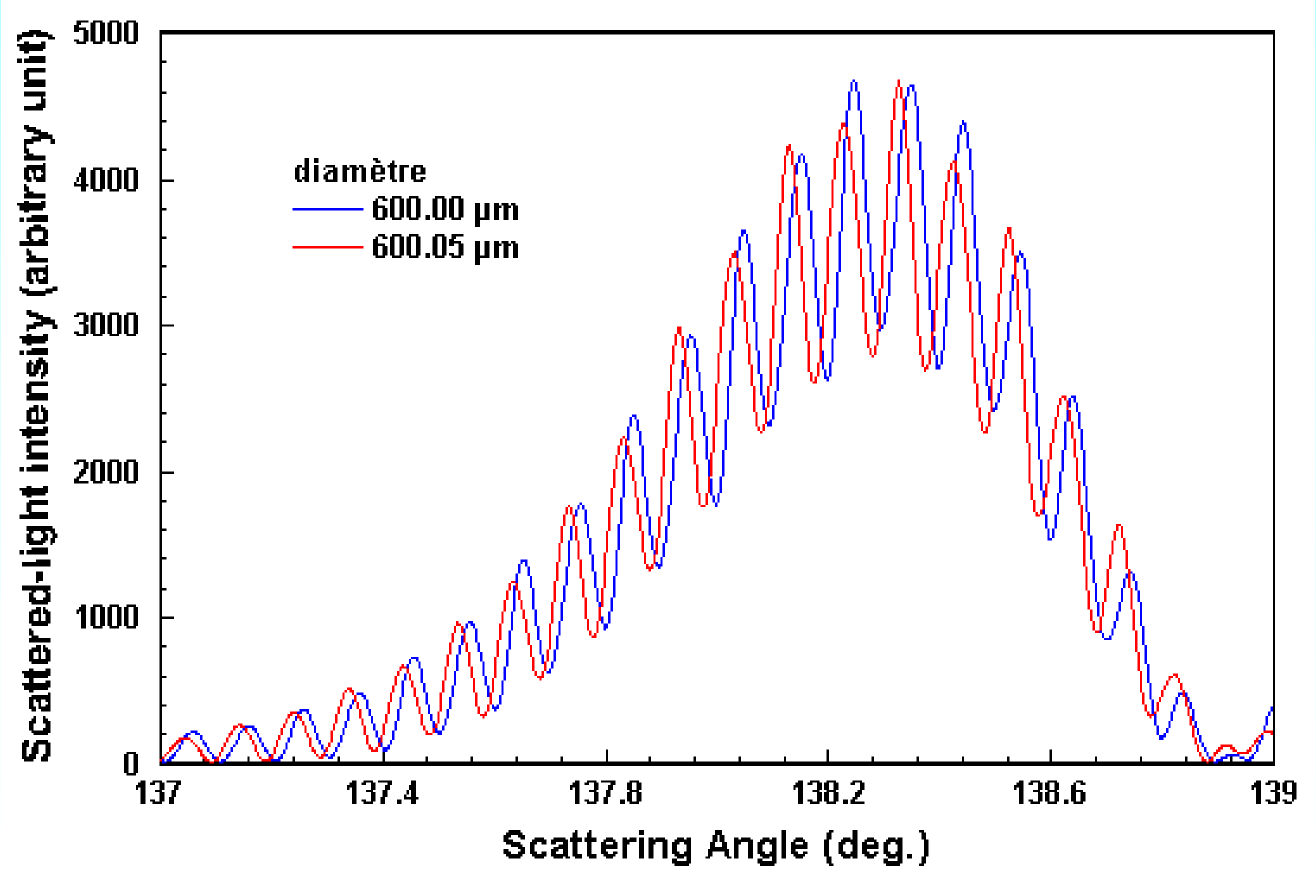
Typical error: 600 μm ± 2μm

For you to reply:

- Does the factor depend-on the configuration?
- Suppose that the number of lobes ≈ α, find the relation between *f* et *d*.

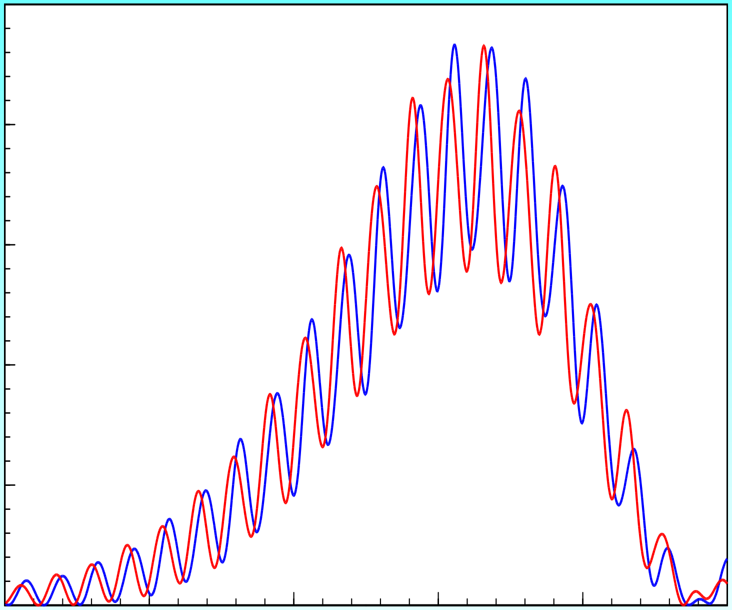
Rainbow – metrology

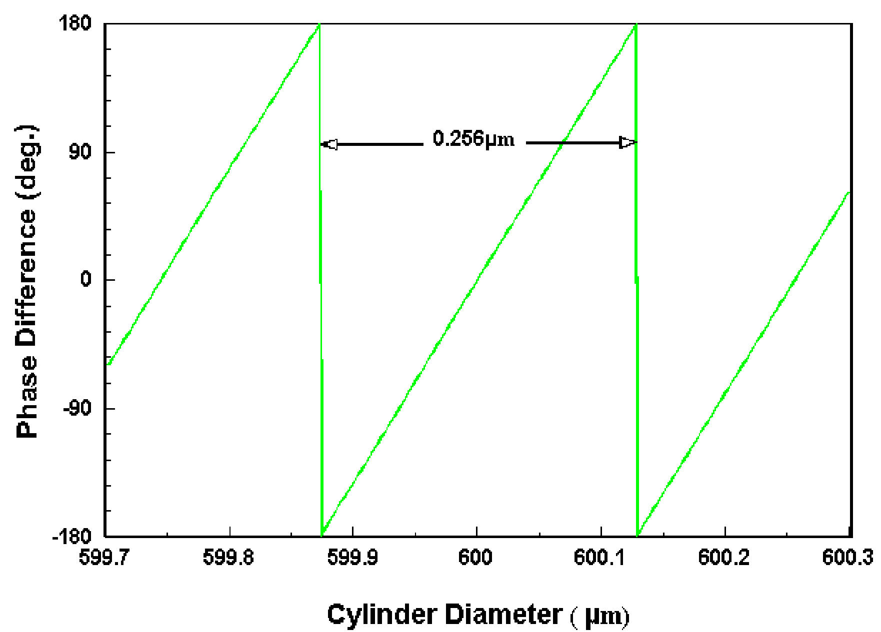
Measurement of diameter variation



Rainbow – metrology

Measurement of diameter variation



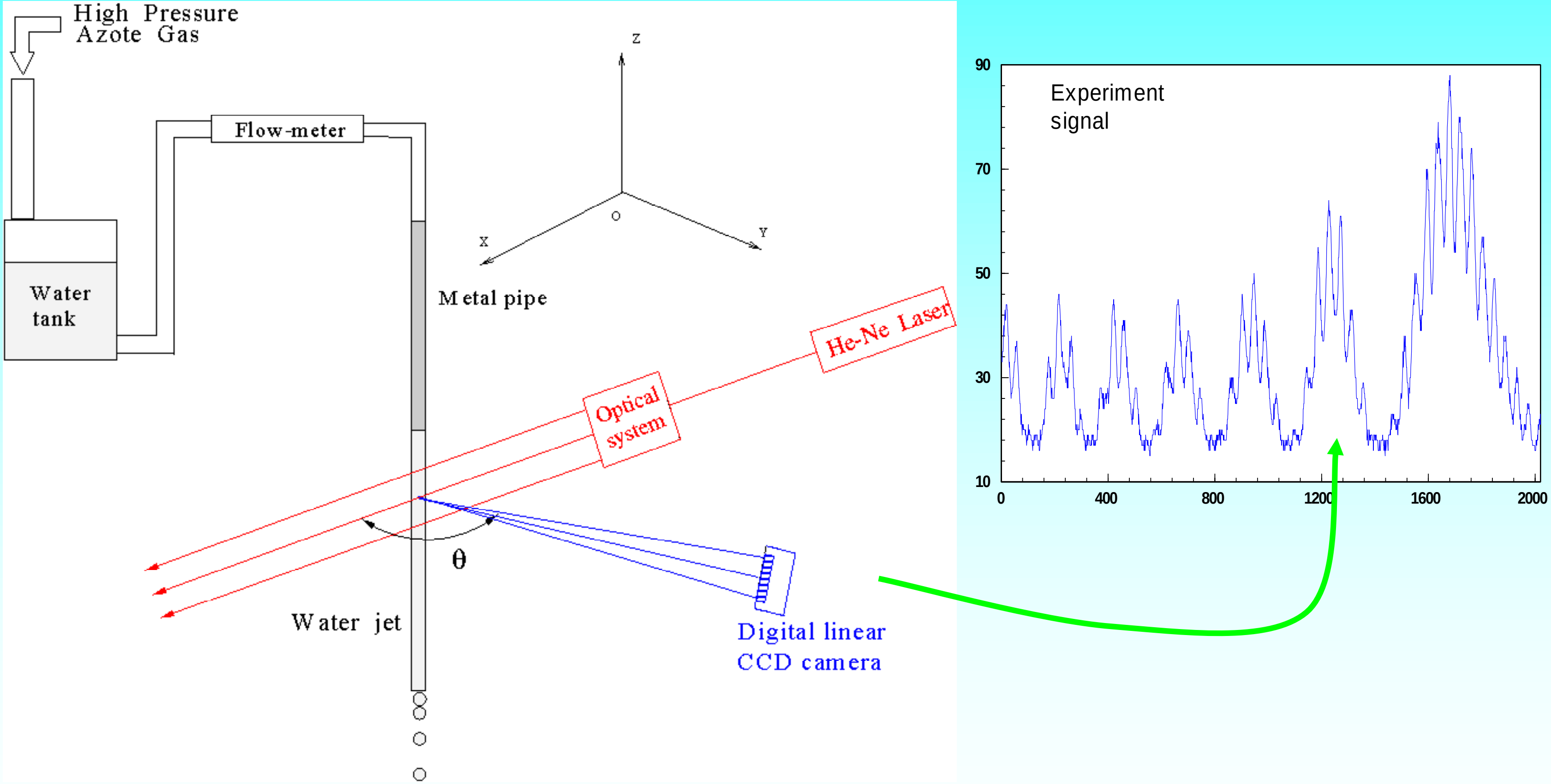


CSD

Precision : ~ 10 nm for d=600 μM

Rainbow – metrology

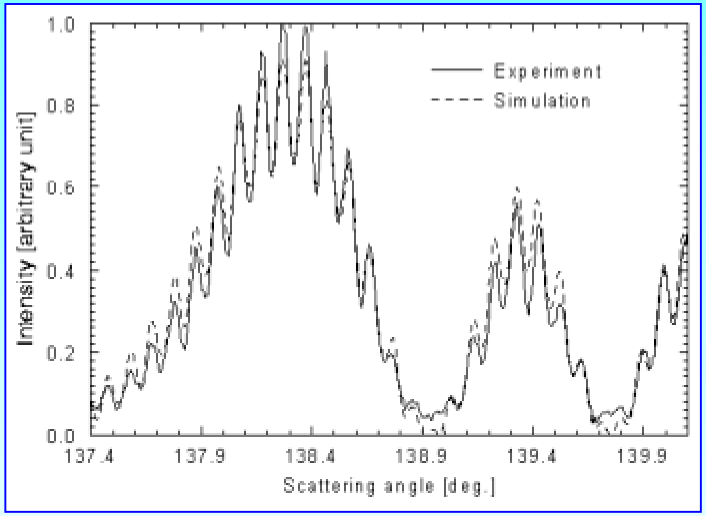
Experimental setup

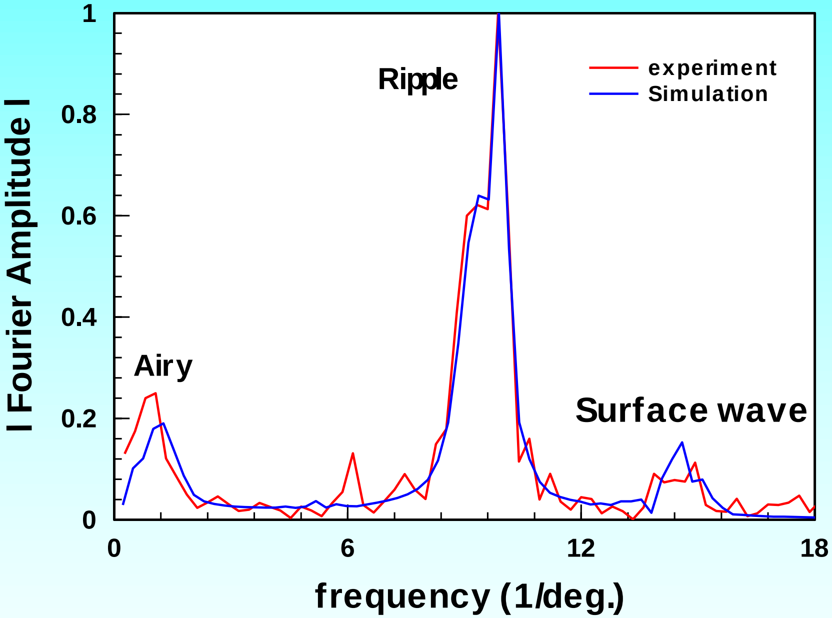


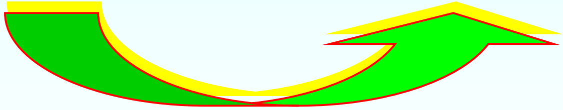
Rainbow – metrology

Experimental results

Comparison between measured and calculated signals



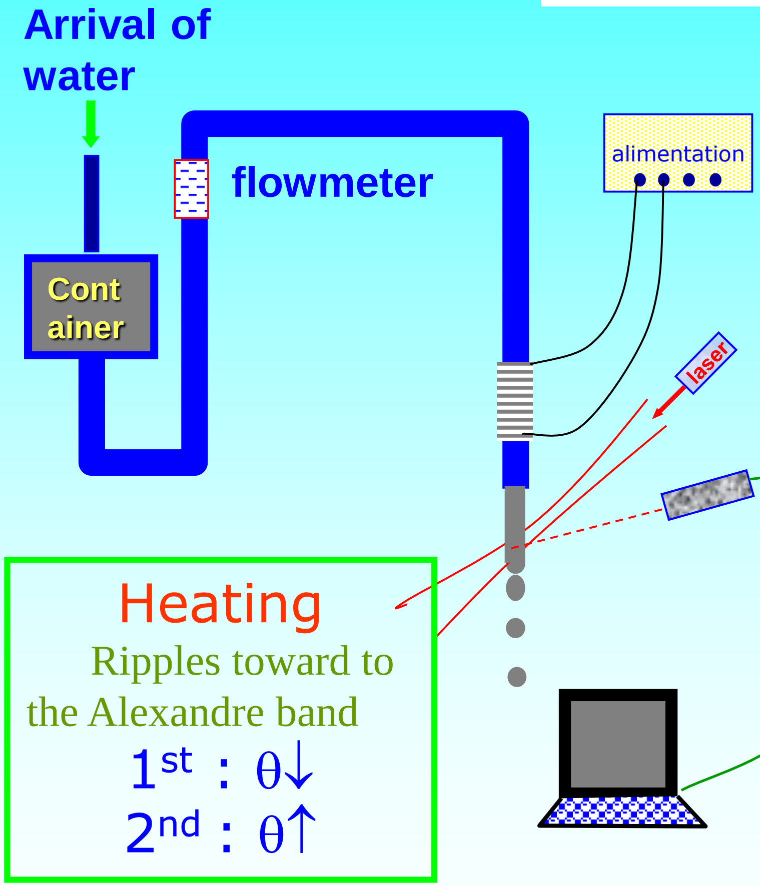




FFT of the derivative

Rainbow – metrology

Experimental results

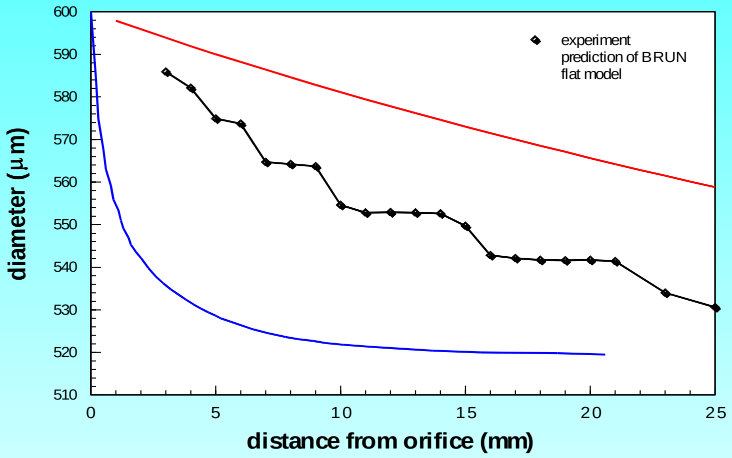


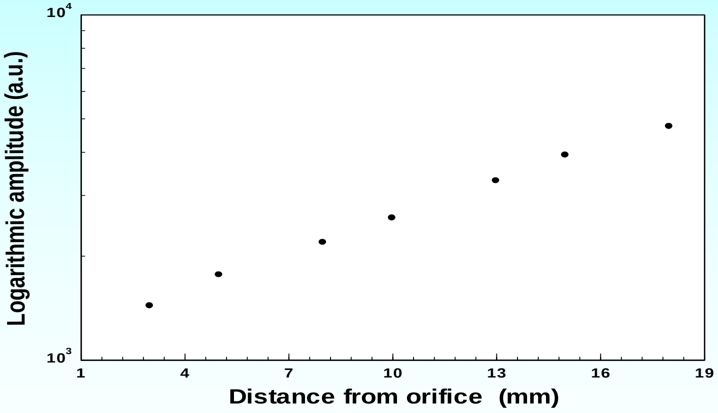
Heating

Ripples toward to the Alexandre band

1st : θ↓

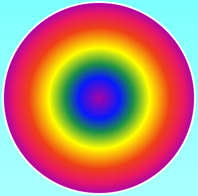
2nd : θ↑

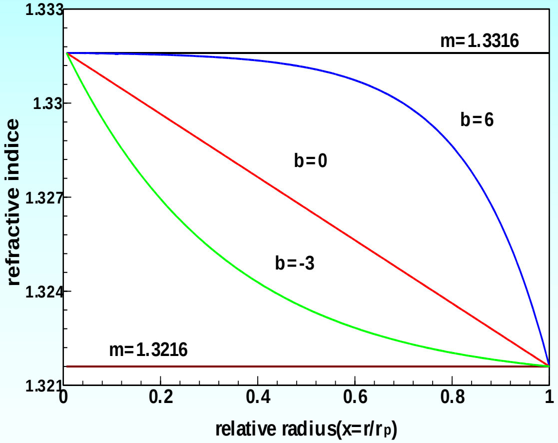


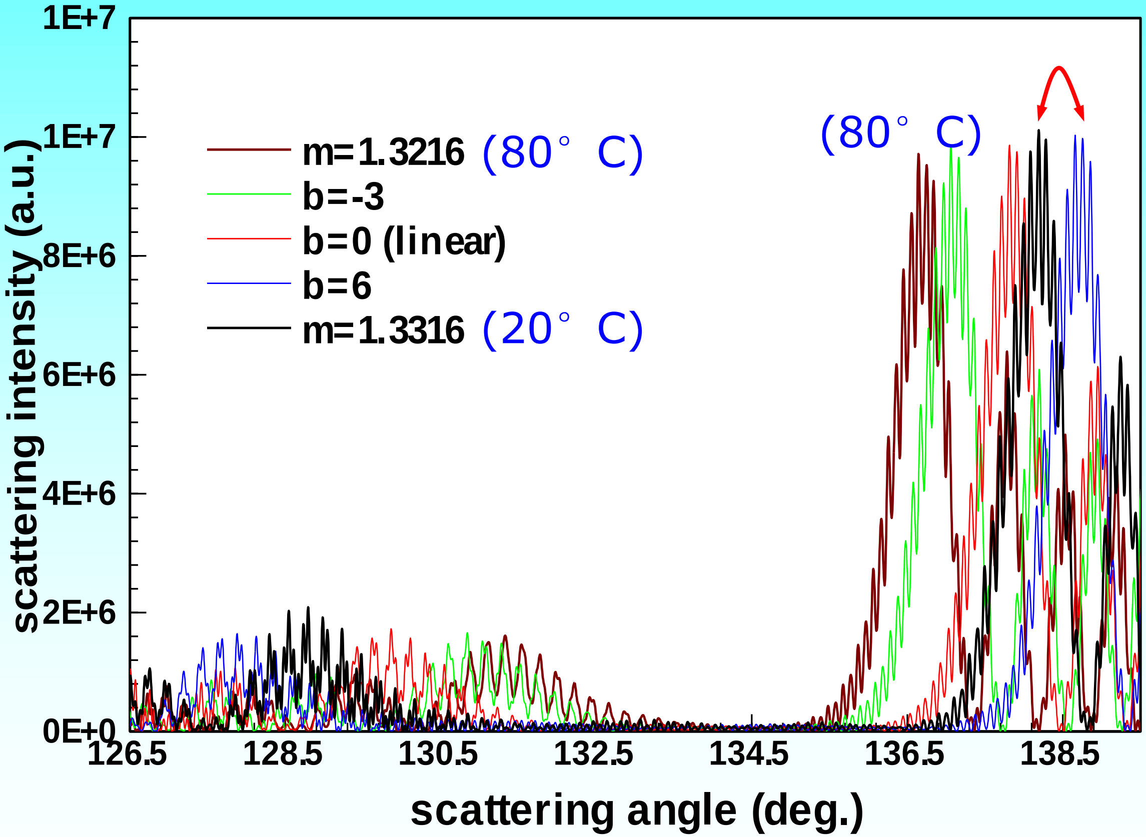


Rainbow – metrology

Measurement of temperature gradient





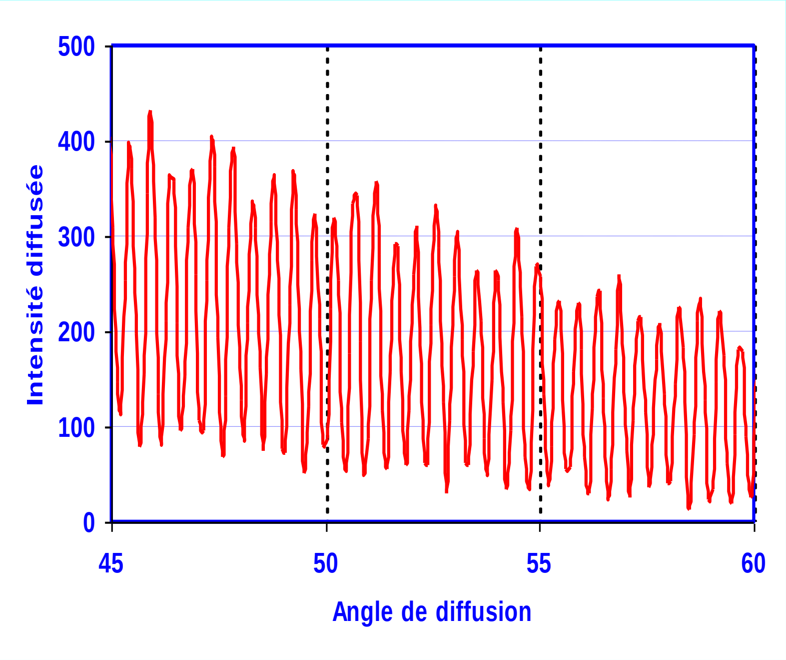


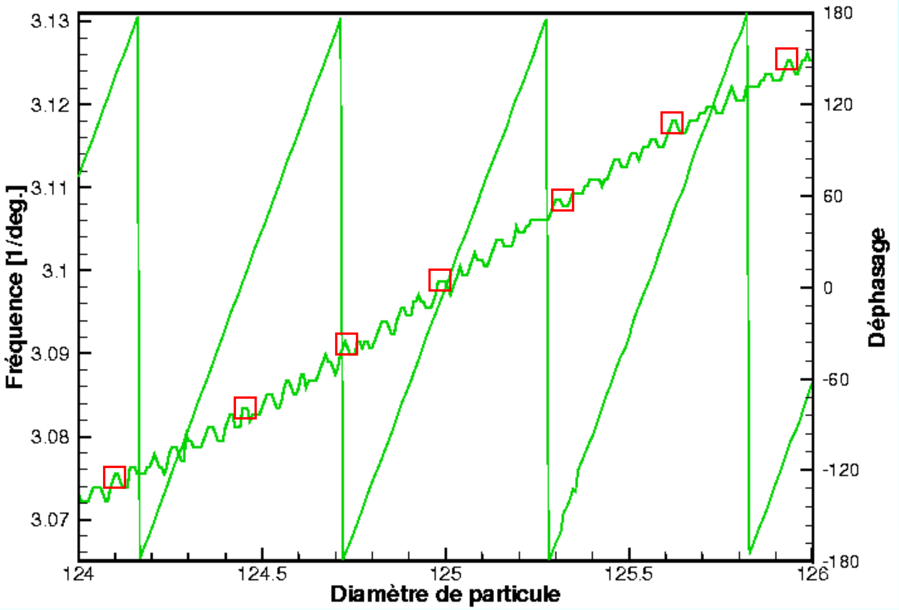
Rainbow – metrology

Variant of rainbow refractometry

FFT & CSD

Ripple frequency : Δ*d* ~ 0,1 µm
Phase difference: Δ*d* < 0,01 µm

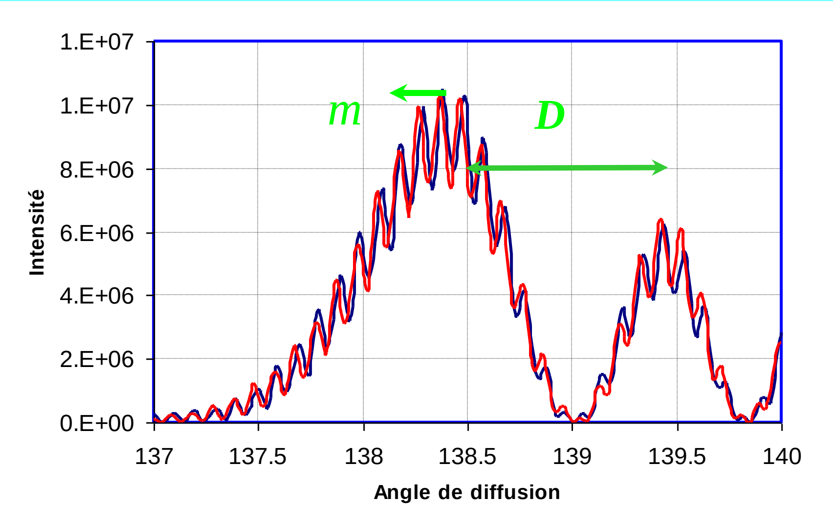




Rainbow – metrology

Summary of rainbow refractometry

- ➢**Refraction index**
- ➢**Particle size**



- ✓**Gradient of refraction index or of temperature**
- ✓**Precise size and sphericity**

FFT

$f_{rip} \rightarrow d$ **(600 µm)**

δd < 5 µm

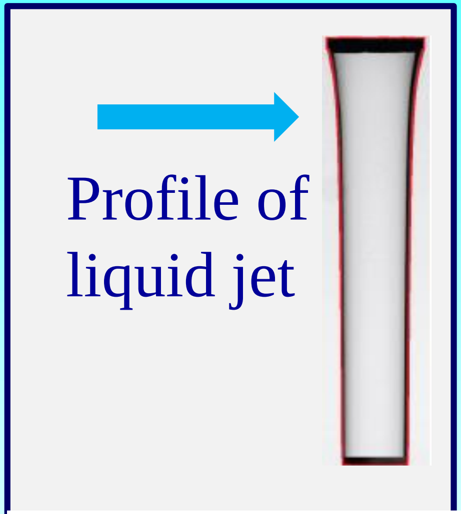
CSD

$\Delta\phi \rightarrow \Delta d$

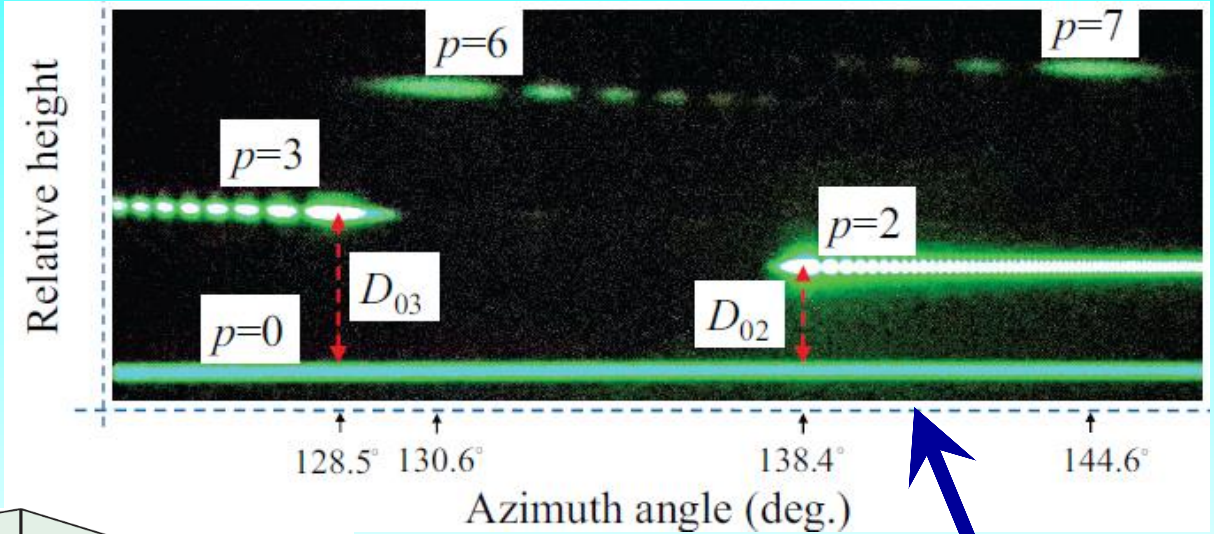
< 10 nm

Rainbow – metrology

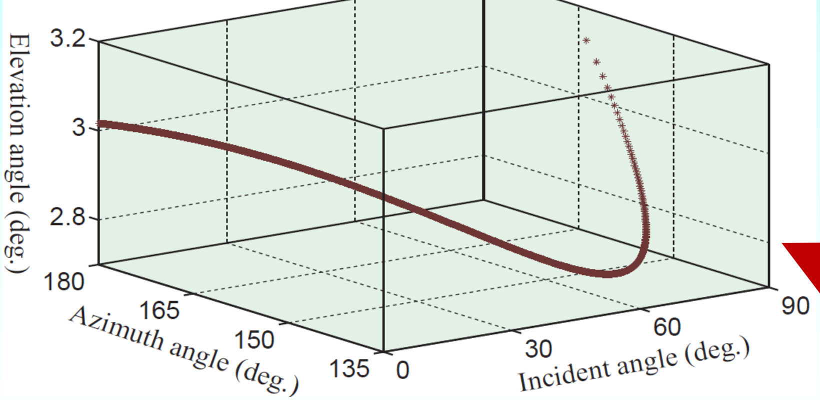
Rainbow of a real liquid jet



Profile of liquid jet



Experimental scattering patterns.



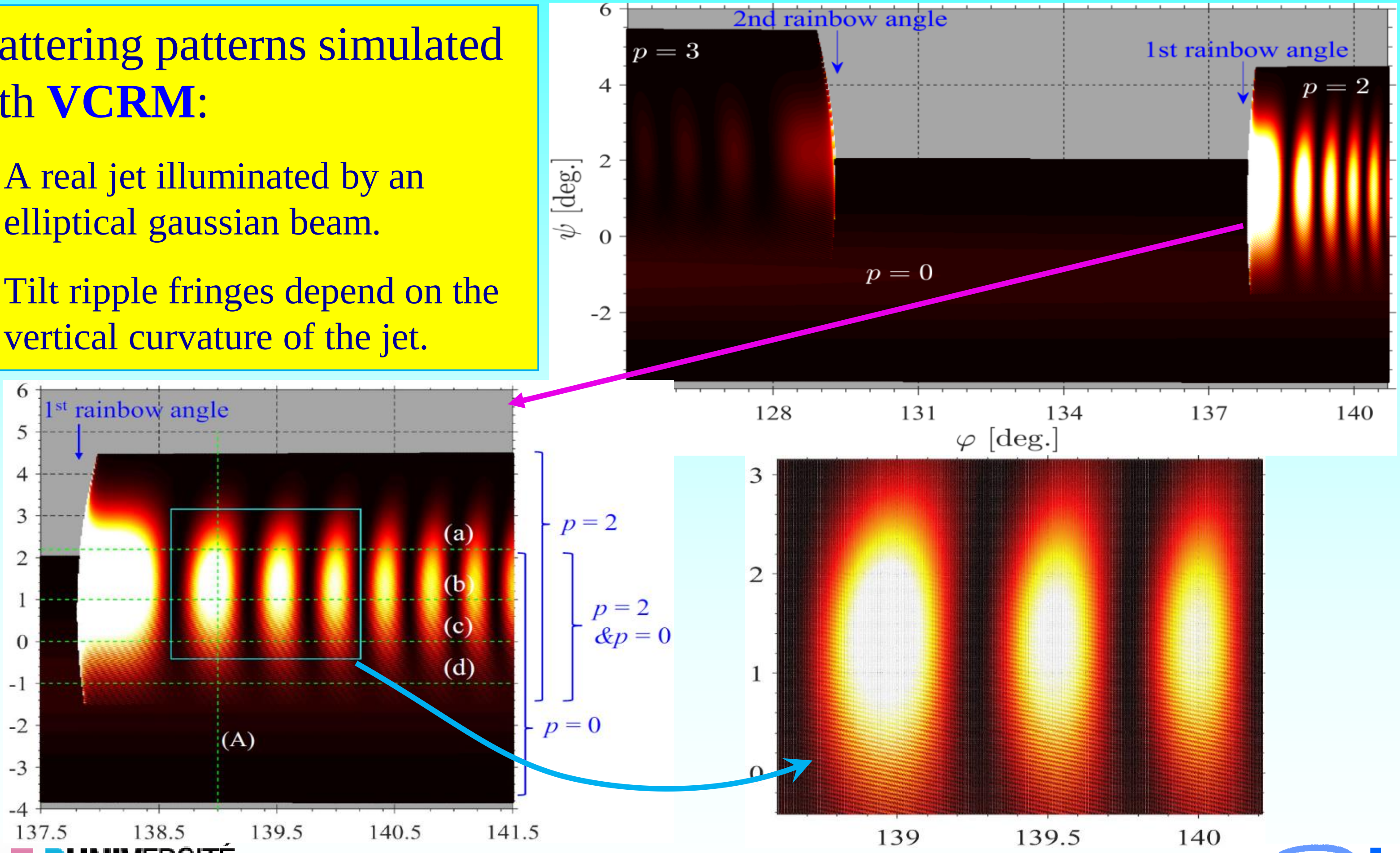
Simulated rainbow positions.

Rainbow – metrology

Rainbow of a real liquid jet

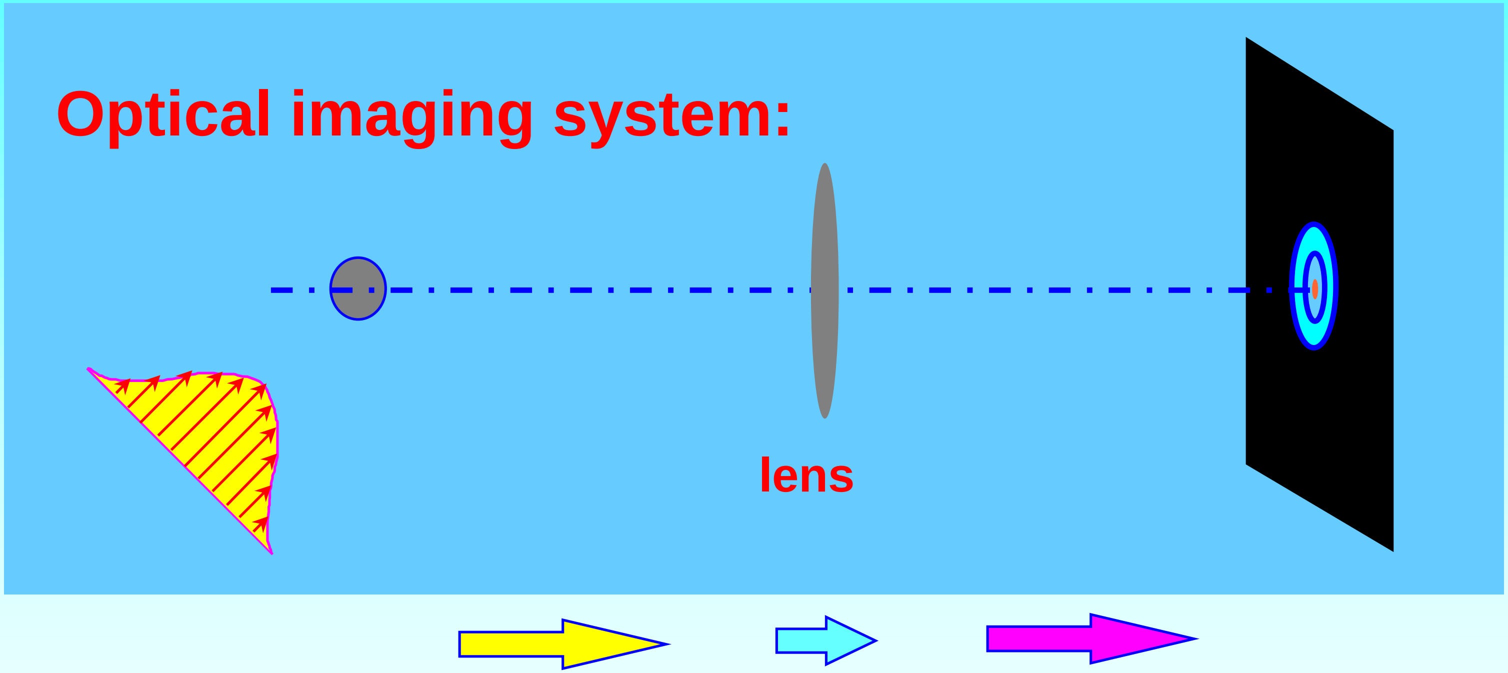
Scattering patterns simulated with **VCRM**:

- A real jet illuminated by an elliptical gaussian beam.
- Tilt ripple fringes depend on the vertical curvature of the jet.



Imaging technique – principle

Theoretical principle



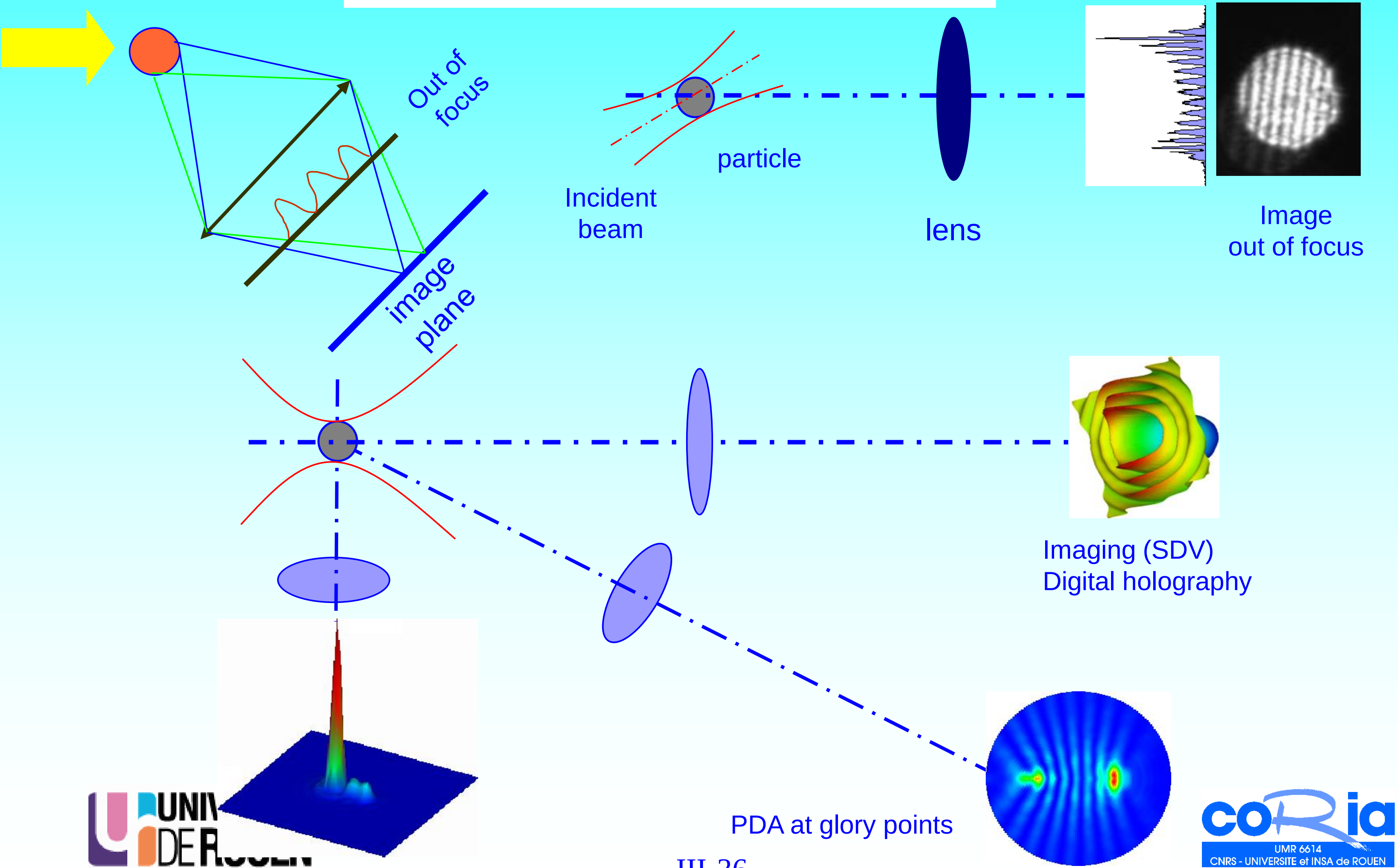
TLMG/TLM

Phase function of the lens

Huygens-Fresnel integration

Imaging technique – principle

Imaging in different stages



Out of focus

image plane

Incident beam

particle

lens

Image out of focus

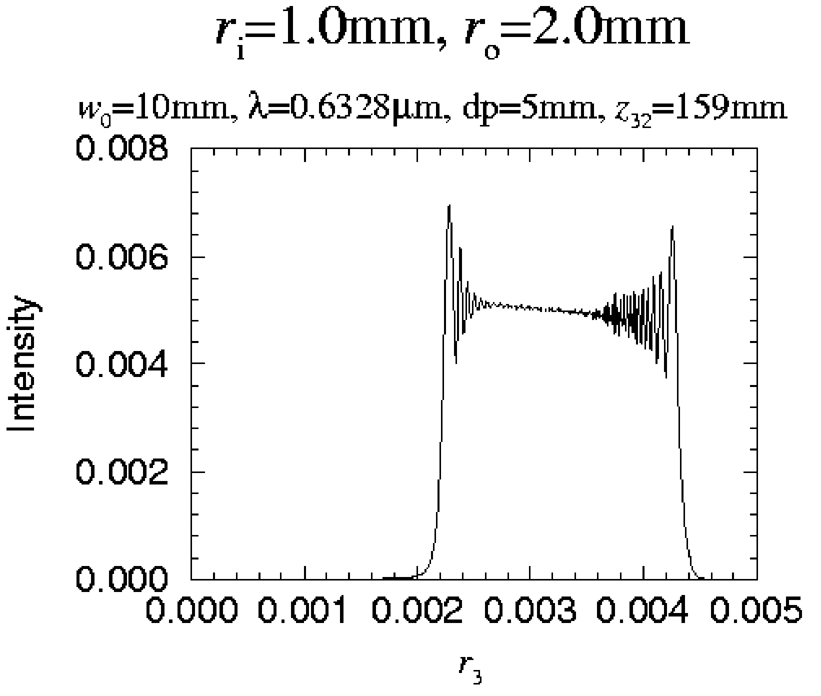
Imaging (SDV)
Digital holography

PDA at glory points

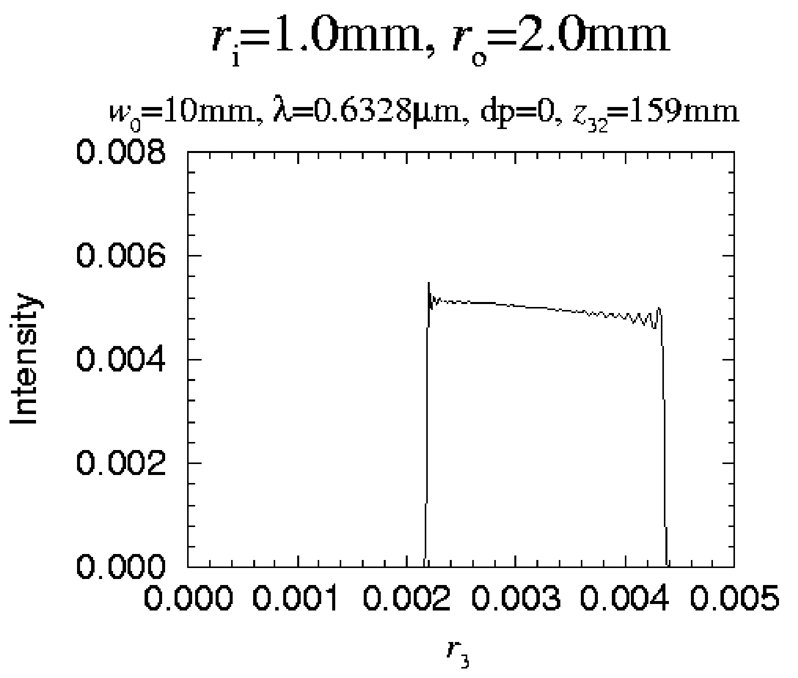
Imaging technique

Effect of diffraction

Image of a hole by TLMG:


r_i=1.0mm, r_o=2.0mm

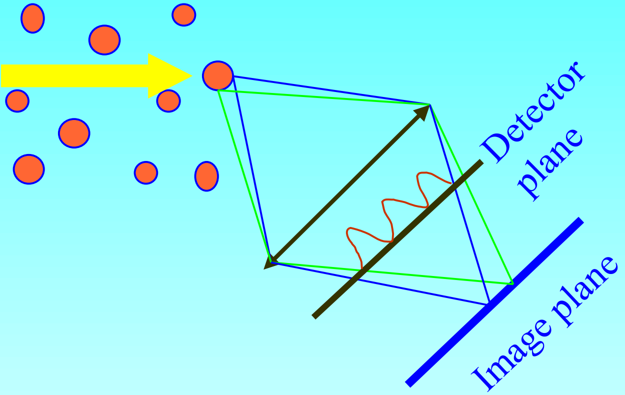
w_0=10mm, λ=0.6328µm, dp=5mm, z_{32}=159mm

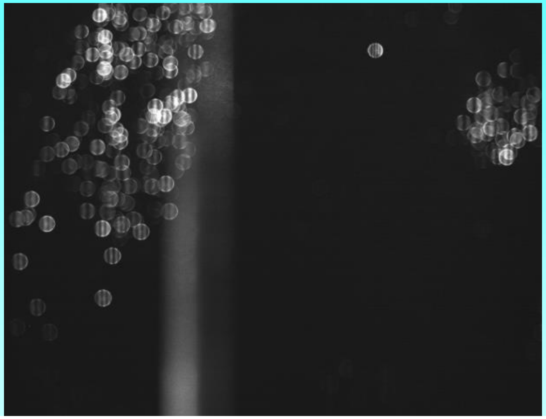

r_i=1.0mm, r_o=2.0mm

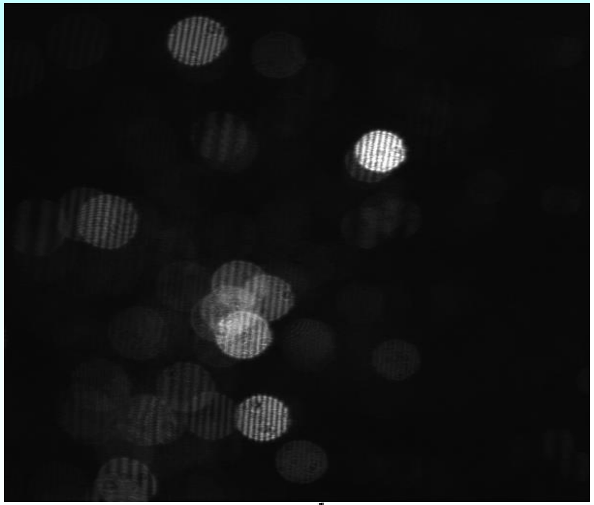
w_0=10mm, λ=0.6328µm, dp=0, z_{32}=159mm

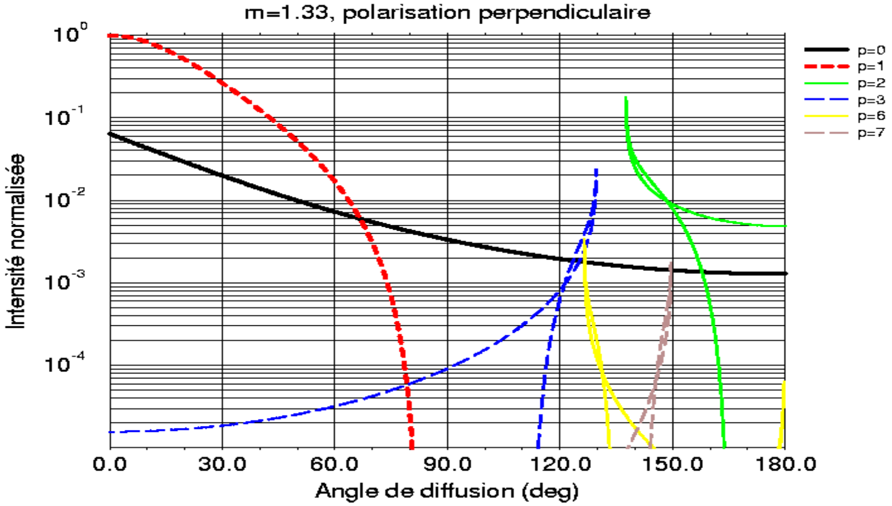
Imaging technique

Out of focus Imaging







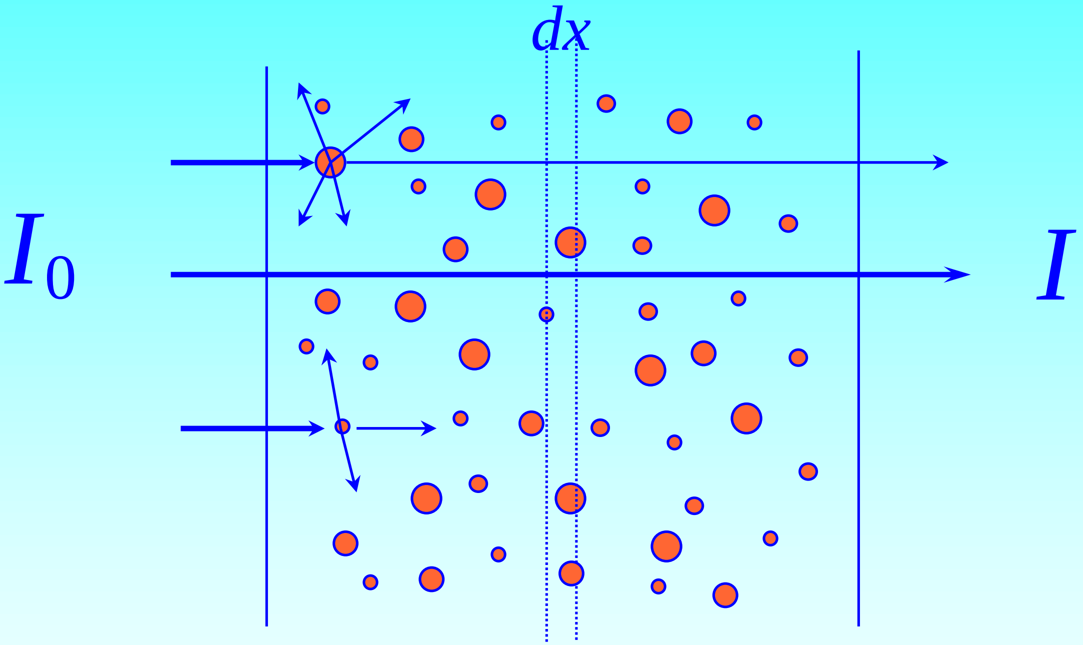


Measurement techniques - 2

- **Measurement of individual particles**
  - LDV and PDA
  - Rainbow
  - Imaging technique
- **Measurement of cloud particles**
  - Extinction spectrometry
  - Refractometer (Malvern)
  - Global rainbow

Extinction spectroscopy - Principle

Beer-Lambert law



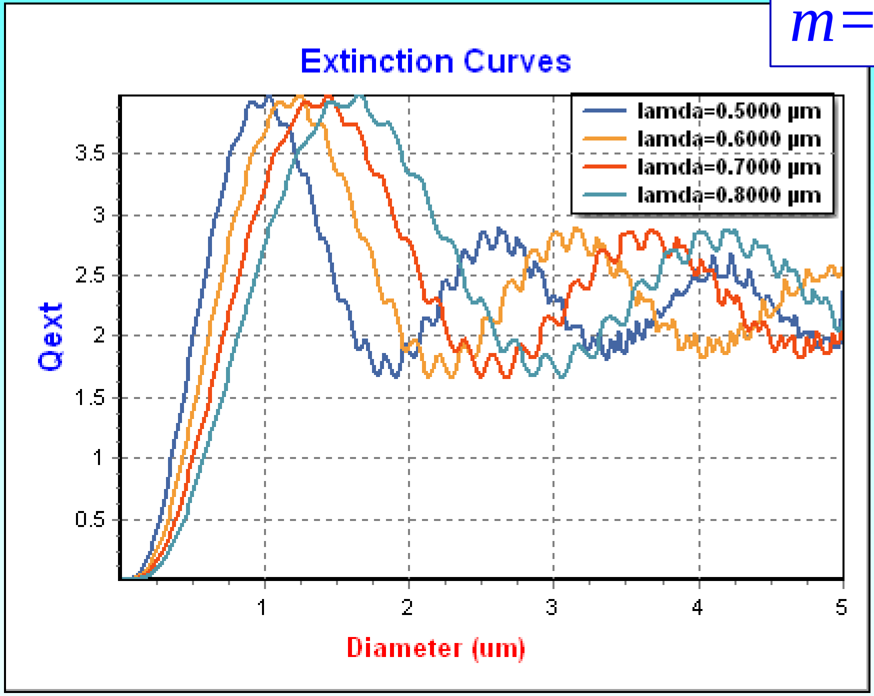
Balance of energy: $dI = -I \; n(r) \; C_{ext}(r,\lambda) dx$

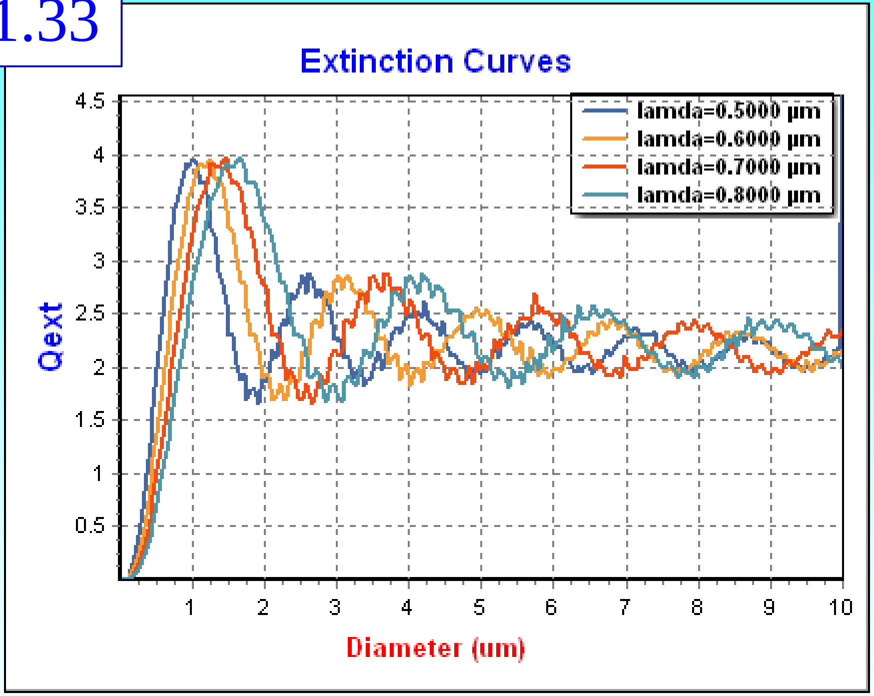
Beer-Lambert law: $I = I_0 \exp\left[-L \int_0^\infty C_{ext}(r,\lambda) n(r) dr \right]$

Extinction spectroscopy

Extinction factor behaviors

$m=1.33$





$d \ll \lambda,$

$$Q_{ext} = \frac{8}{3}\left(\frac{\pi d}{\lambda}\right)^4 \mathrm{Re}\left(\frac{m^2-1}{m^2+2}\right)$$

Extinction spectroscopy

Inversion problem

Mono-dispersed particles :

$$I = I_0 \exp(-L C_{ext} N)$$

$$L C_{ext} N = \ln(I_0/I)$$

$$N = \frac{\ln(I_0/I)}{L C_{ext}}$$

Poly-dispersed particles :

$$\sum_i C_{ext}(r_i, \lambda_j) N(r_i) = \frac{1}{L} \ln\left(\frac{I_0}{I(\lambda j)}\right)$$

Inversion problem :

$$\int_0^\infty A(r,\lambda) n(r) dr = s(\lambda)$$

Extinction section

concentration

Measured spectrum

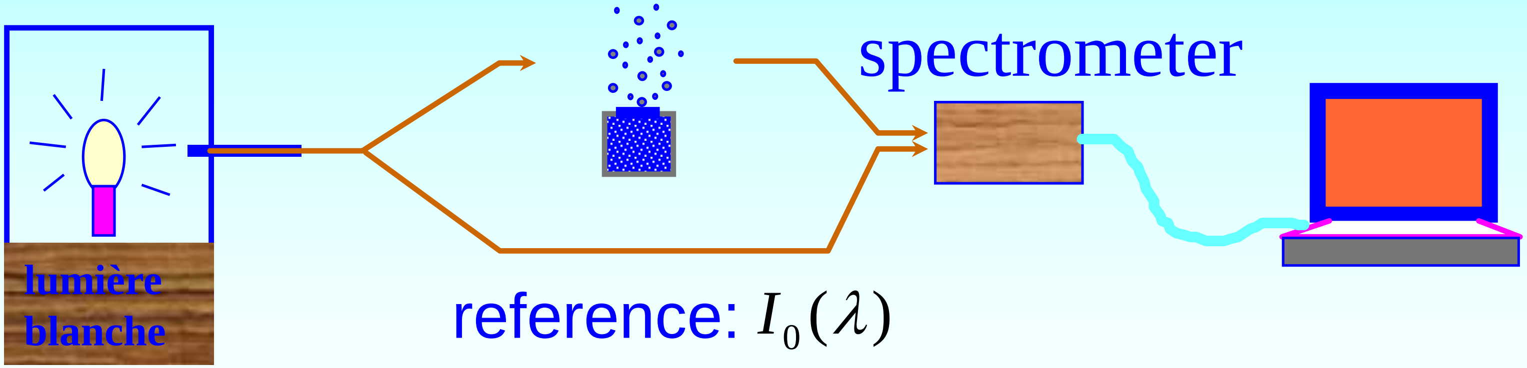
Extinction spectroscopy

Inversion problem

$$I = I_0 \exp\left[-L\int_0^{\infty} C_{ext}(r,\lambda)N(r)dr\right]$$

RINVERSE: N(r)

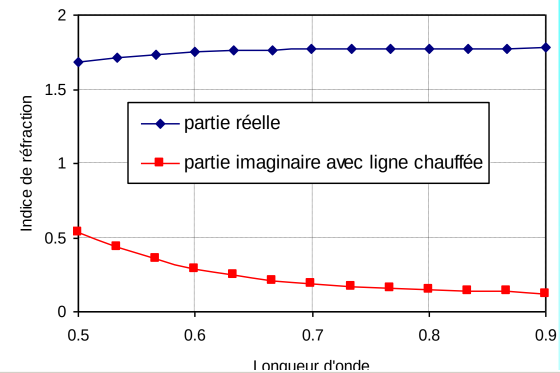
measured: $I(\lambda)$

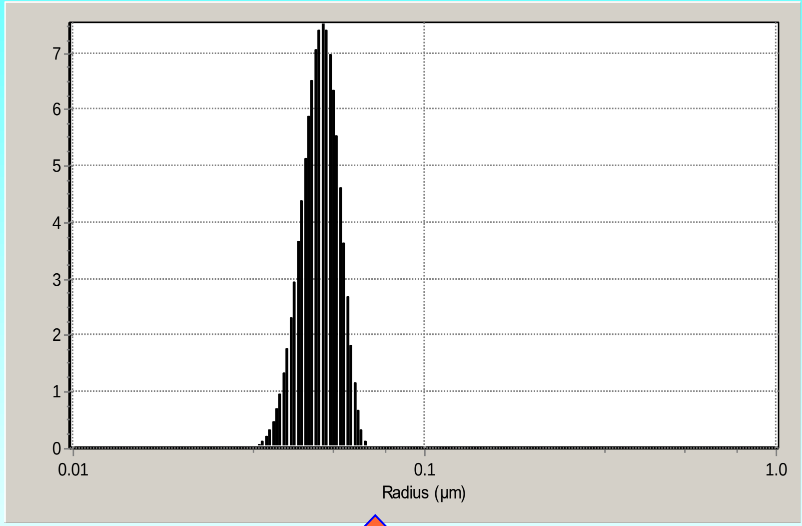


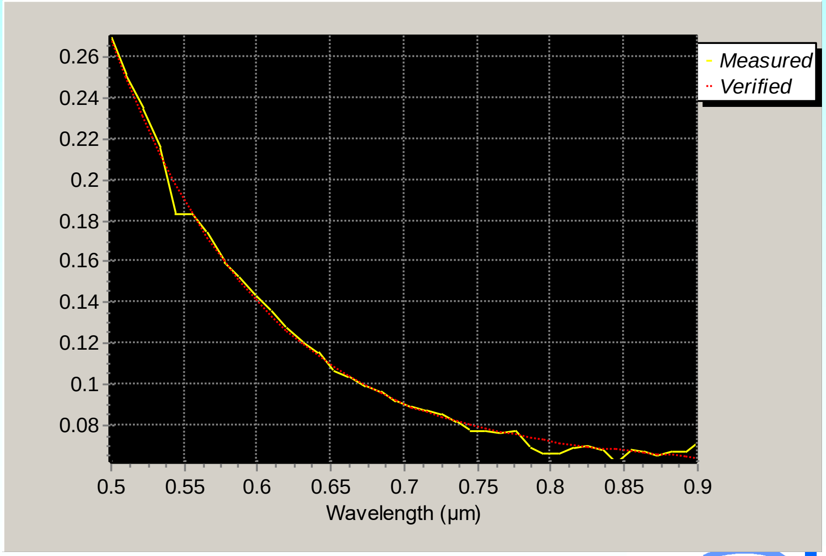
reference: $I_0(\lambda)$

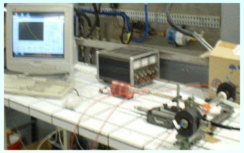
Extinction spectroscopy

Measurement of soot emitted by a motor (CERTAM)



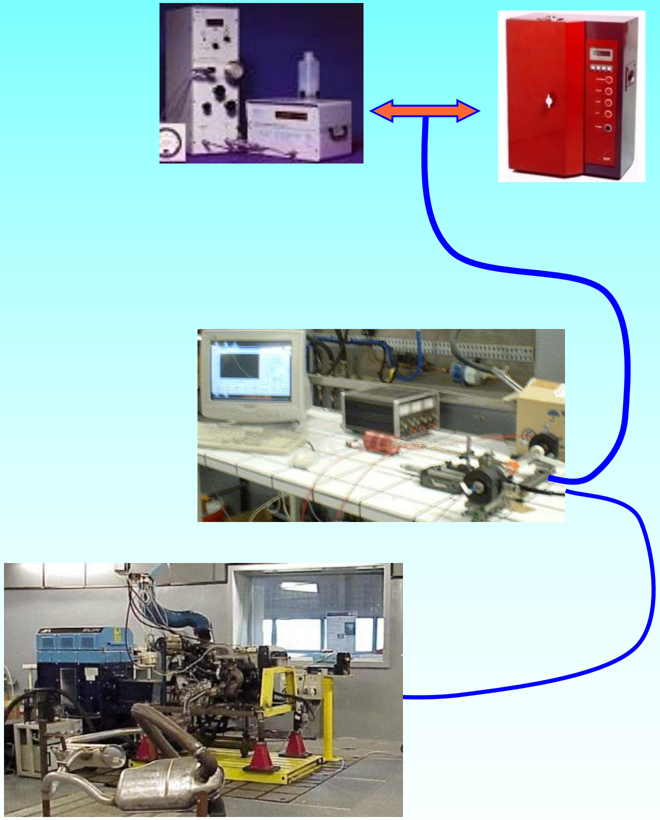


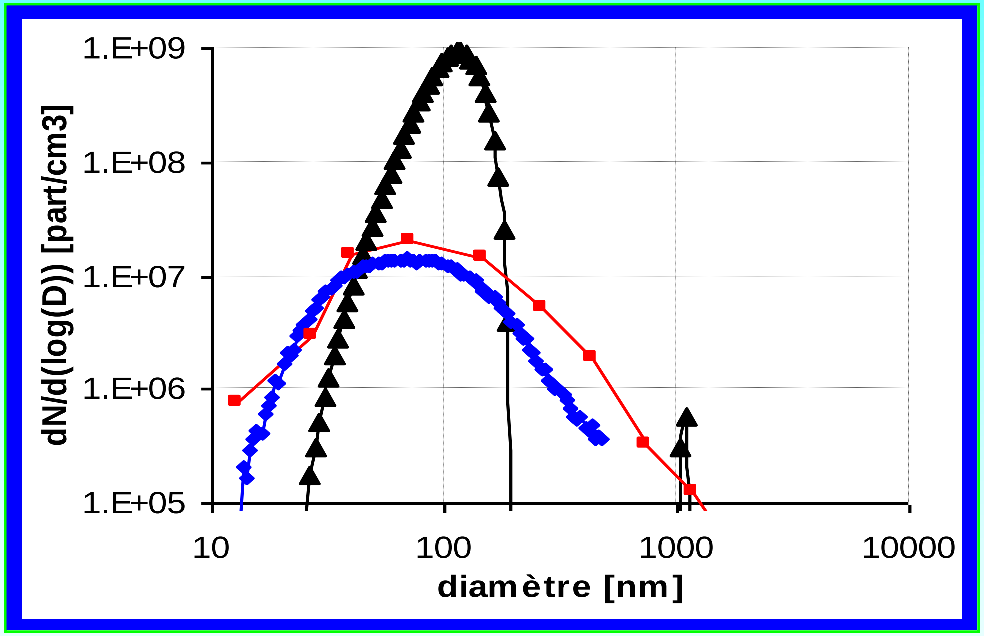




Extinction spectroscopy

Measurement of soot emitted by a motor (CERTAM)





	SMPS	ELPI	Exinction spect.
D moyen (nm)	86	111	101
D modal (nm)	68	71	101
f_v	1.07E-8	1.62E-7	6.07E-8
C_n (part/m^3)	1.01E13	1.22E13	7.85E13

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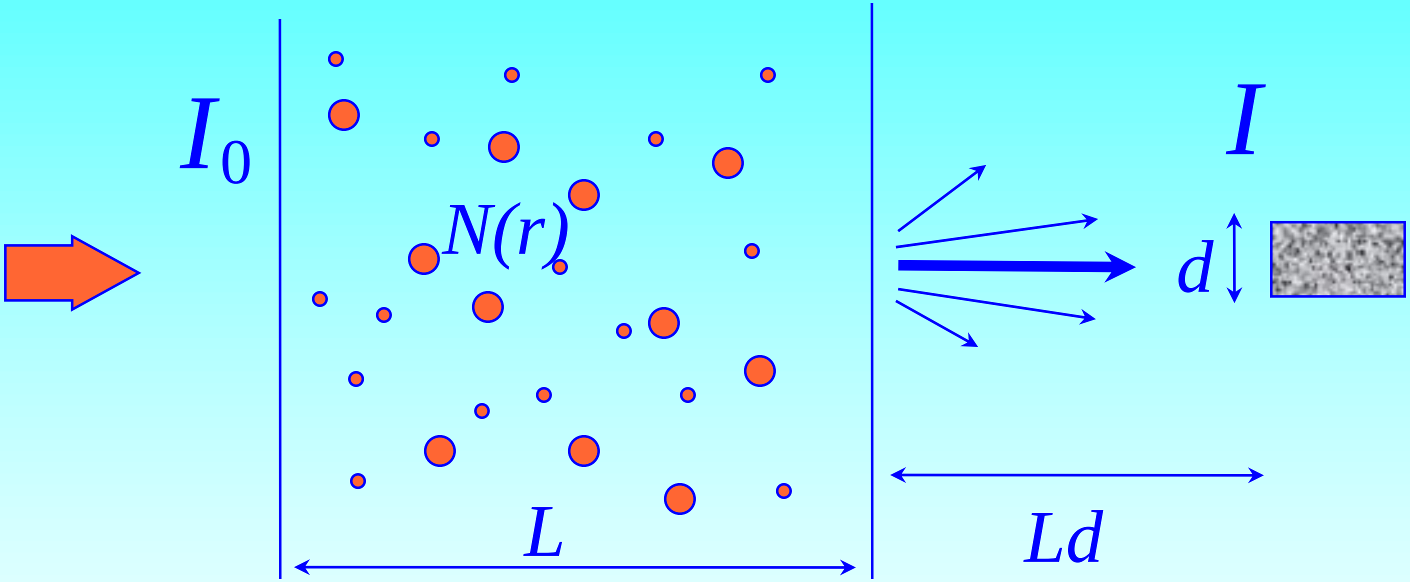
Extinction spectroscopy

Caution for a dense medium



- $N(r)L$ must not be too small (measurement noise) neither too big (multiple broadcast),
- d/Ld must be small enough.

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Refractometry

General inversion problem

Mono-dispersed particles :

$$I(\theta)=I_0\left(\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right)^2$$

Poly-dispersed particles :

$$I(\theta)=I_0\sum_i N(r_i)\left(\frac{J_1(kr_i\sin\theta)}{kr_i\sin\theta}\right)^2$$

Inversion problem :

$$\int_0^\infty A(r,\theta)n(r)dr=I(\theta)$$

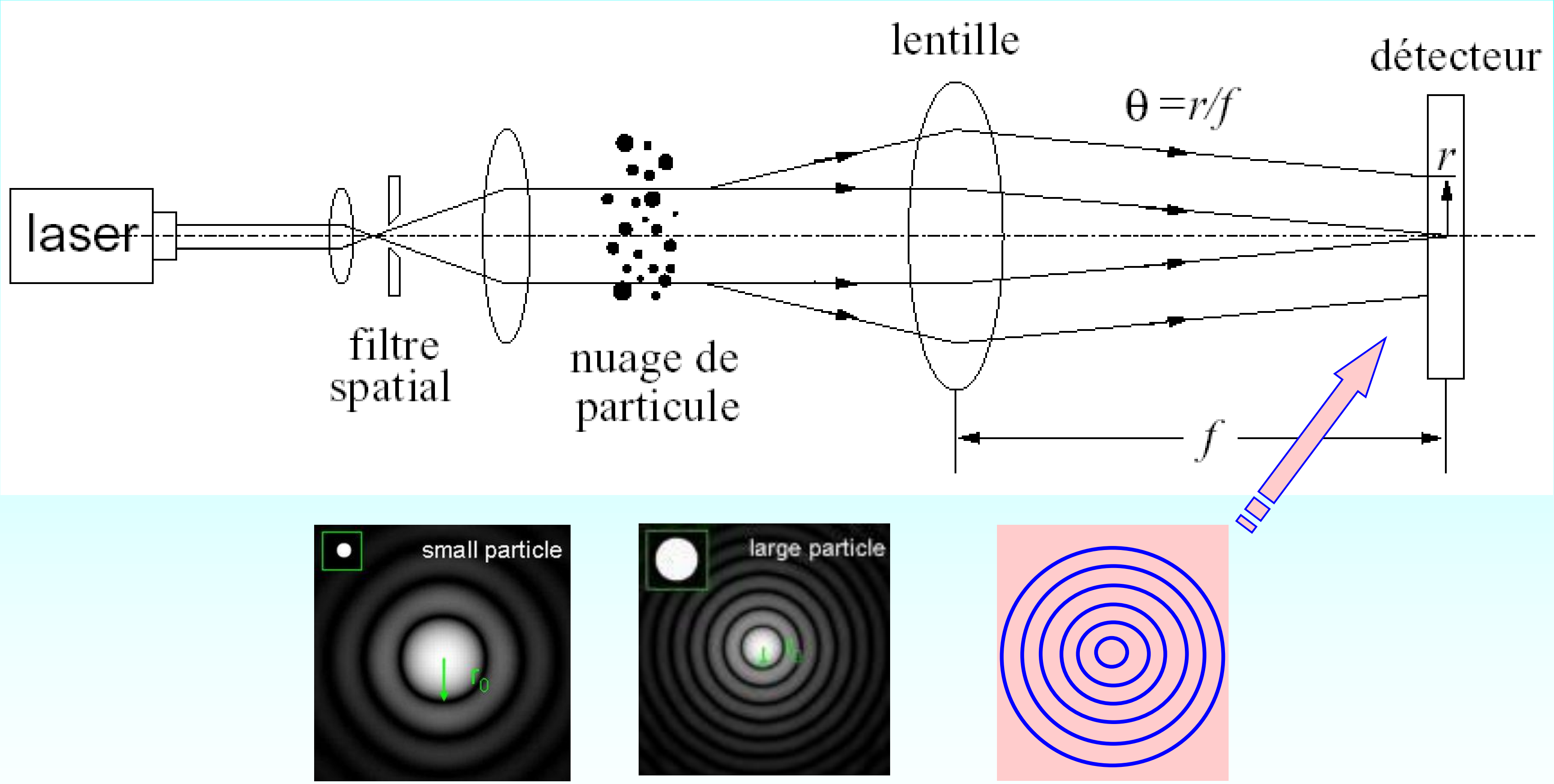
Diagramme de diffusion

concentration

Intensité angulaire

Refractometry

Principle of refractometer (Malvern)



Global rainbow

Principle of global rainbow

Mono-dispersed Particles :

$$\boxed{I(m,r,\theta)} \qquad f_\theta \sim r,\ \Delta\phi \sim \Delta r,\ \theta_{airy} \sim m$$

Poly-dispersed Particles

$$I_t(\theta)=I_0\sum_i N(r_i)I(r_i,\theta)$$

Inversion problem:

$$\int_0^\infty I(r,\theta)n(r)dr=I_t(\theta)$$

Scattering diagram

concentration

Angular Intensity

Inversion problem

Algorithm of inversion problem

Fredholm equation of the first kind

$$\int_0^\infty K(x,y)f(y)dy = g(x)$$

Discretization:

$$g_i + \varepsilon_i = \sum_i w_j K_{i,j} f_i$$

ε_i – measurement error, w_i – discretization weight.

Usually this is an ill conditioned equation.

Regularization:

$$M = \|Af - g\|^2 + B$$

Inversion problem

Algorithm of inversion problem

Phillips-Twomey :

$$(A^T A+\gamma H)f = A^T g$$

Physical constraints:

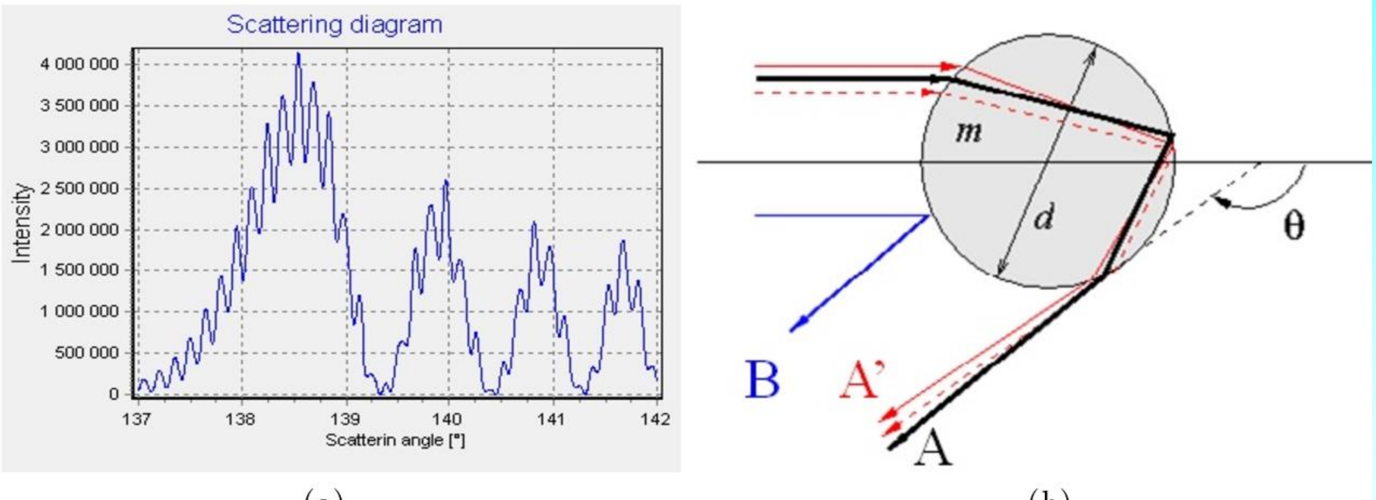
$$\begin{gathered} n(r)>0 \\ f(r=0)=0 \\ f(r\to\infty)=0 \\ f'(r=0)=0 \end{gathered}$$

Algorithm NNLS (Lawson, SVD, …)

Rainbow

Fig (a) presents the scattering diagram of a water droplet illuminated by a plane wave and Fig (b) a schema of rays around the rainbow angles.

1. Explain the formation of the ripple structure (high frequency) and the Airy structure in Fig. (a) with help of the Fig. (b).
2. Estimate the radius of the particle from the Fig. (a).



(a) (b)

1. The ripple structure (high frequency) is formed by the interference of rays external reflected rays B and the internal reflected rays and A'. The Airy structure is formed by the rays around A, i. e. rays of types represented by solid and dashed red lines.
2. We have 35 ripple peaks over 5°, i.e. the peak number over 180° is N=35*180/5=1260 ~ α. So the radius a ~ αλ/2π=130 μm. The real size is 200 μm.