Lecture at Xidain University
On frontiers of modern optics

Scattering of shaped beam by particles and its applications

IV. Ray theory of wave and its applications

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西安电子科技大学
现代光学前沿专题

波束散射理论和应用

第四讲：波的射线理论及其应用

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Plan of lecture

I. Fundamentals of geometrical optics

II. Geometrical optics of a sphere particle

III. Extension of the geometrical optics

IV. VCRM - Vectorial complex ray Model

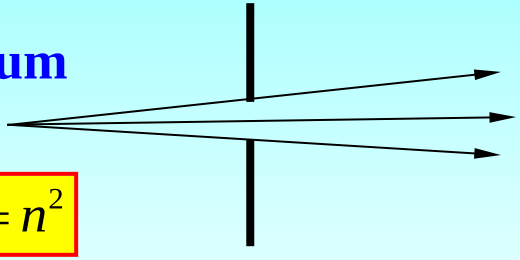
Fundamentals of geometrical optics

Condition of application

$$\lambda << l$$

Wavelength λ much smaller the dimension of the object l.

1. **Straight line in homogeneous medium**



2. **Eikonal eq. (程函方程):** $(\nabla S)^2 = n^2$

3. **Eq. of ray:** $\nabla n = \frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right)$

The rays are perpendicular to the wave front.

Fundamentals of geometrical optics

Basic laws

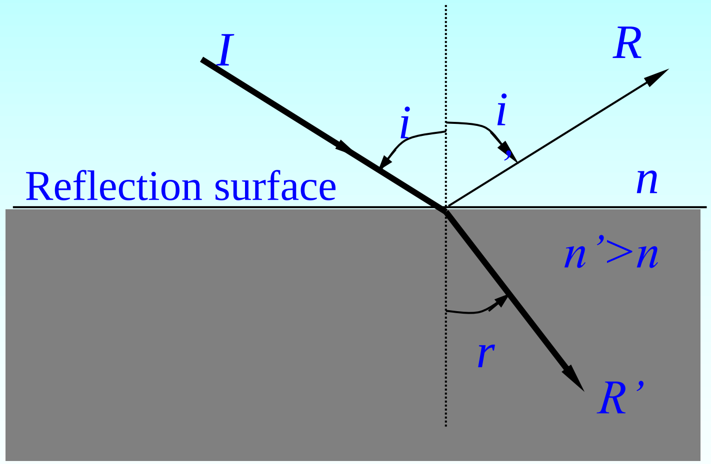
1. Snell-Descartes law:

Reflection: $i = i'$

Refraction: $n \sin i = n' \sin r$

The tangent component of wave vector is continuous

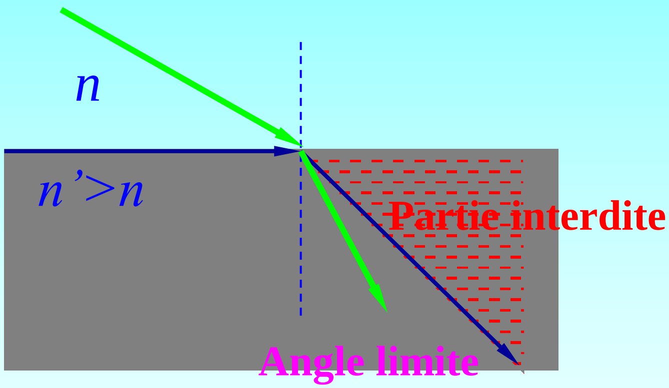
$$k_\tau = k'_\tau$$

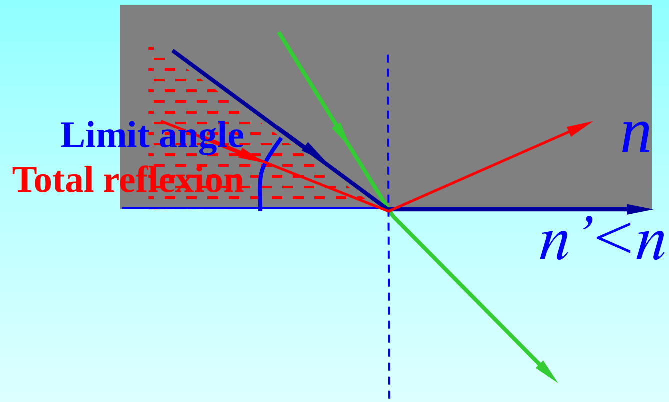


Fundamentals of geometrical optics

Total reflection :

$$i_l = \arcsin\left(\frac{n'}{n}\right)$$





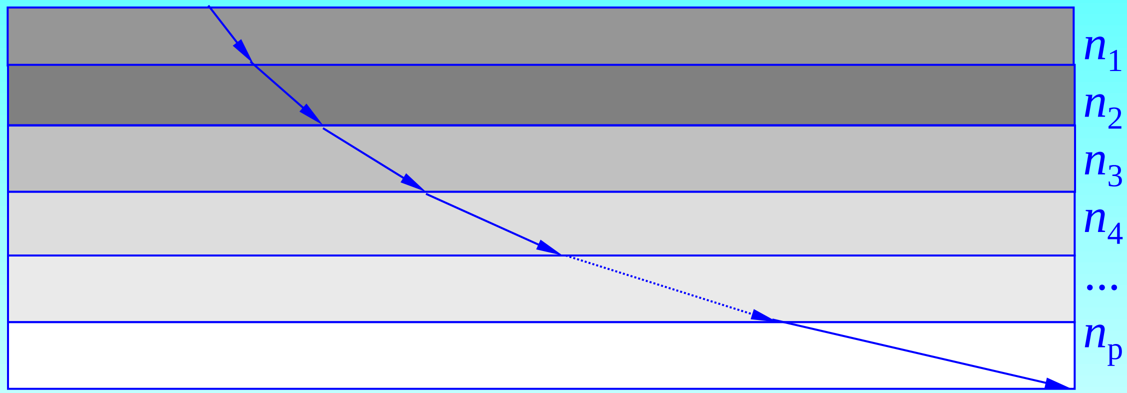
$$n'/n = 1.333 \quad i_l = 48.6°$$
$$n'/n = 1.500 \quad i_l = 41.8°$$

Fundamentals of geometrical optics

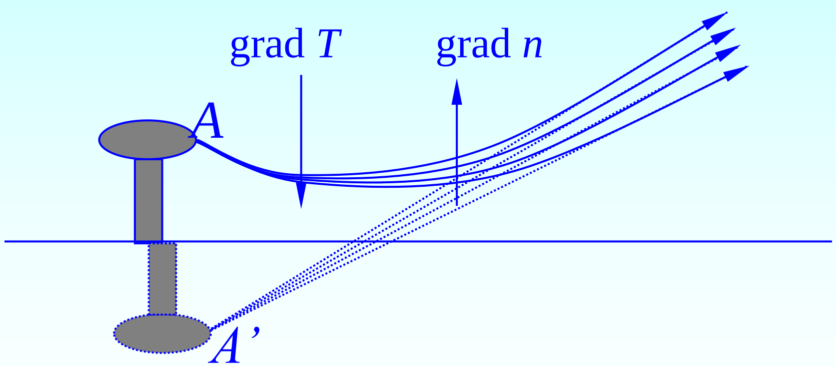
Example of application:

Propagation d 'un rayon lumineux dans une milieu stratifié:

$n_1 > n_2 > n_3 > \ldots > n_p$



Mirage 海市蜃楼



Mirages on the road

California (USA) 2014.10.19 by Ren

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Fundamentals of geometrical optics

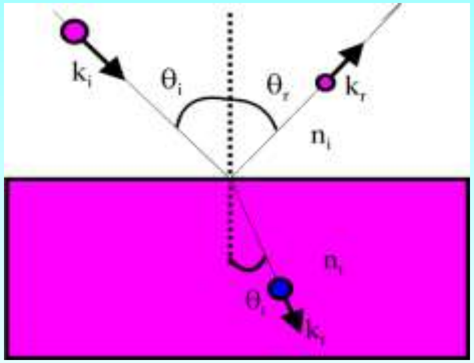
2. Fresnel's Equations :

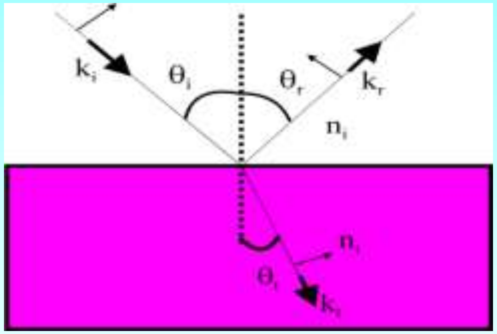
$$r_{//} \equiv \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$r_{\perp} \equiv \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$t_{//} \equiv \frac{2 n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{2\sin\theta_i \cos\theta_t}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

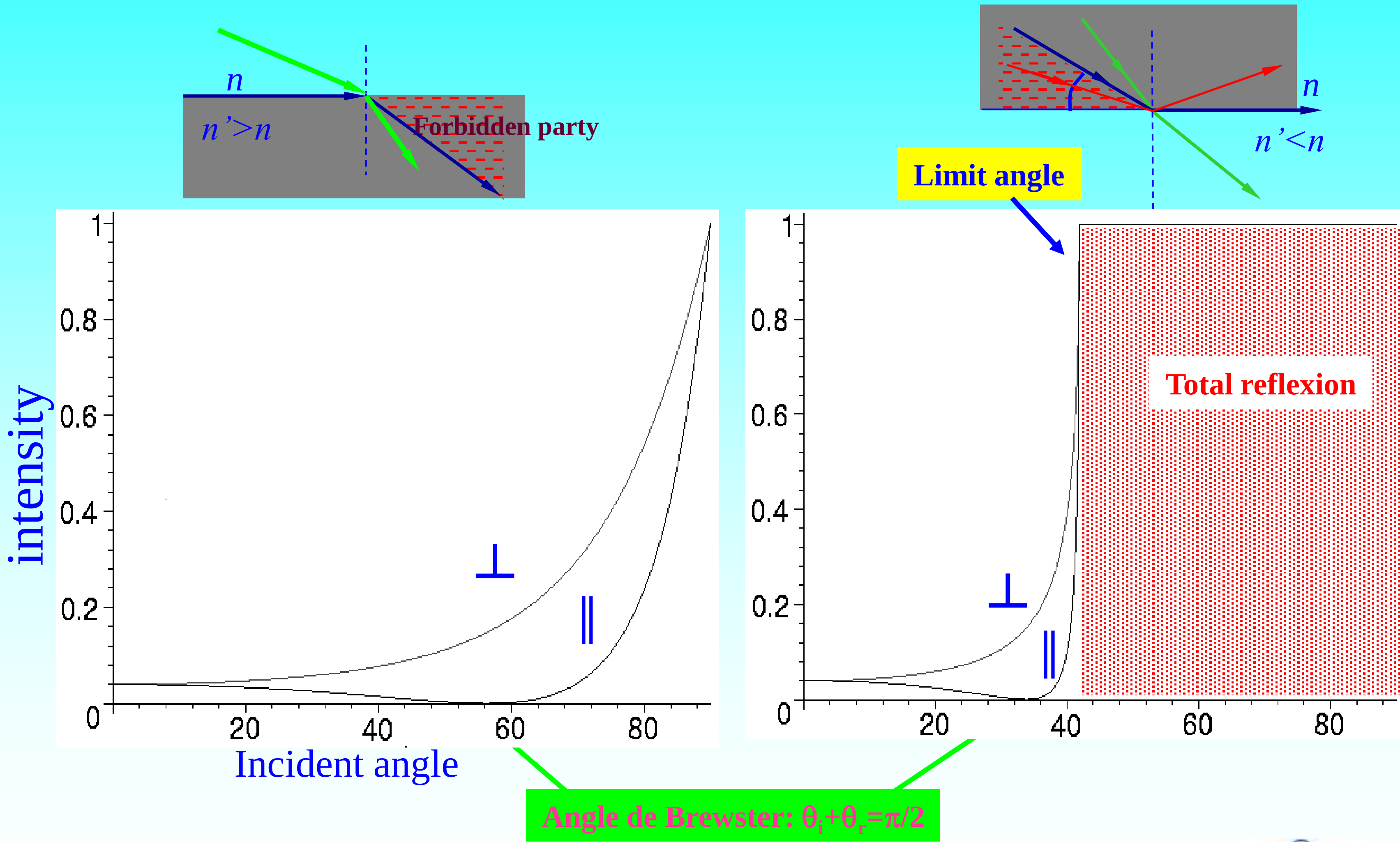
$$t_{\perp} \equiv \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{2\sin\theta_i \cos\theta_t}{\sin(\theta_i + \theta_t)}$$





- Phase due to reflection
- complex refractive index

Fundamentals of geometrical optics



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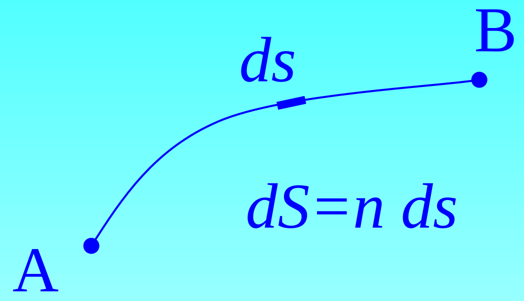
Fundamentals of geometrical optics

3. Optical path:

$$S(AB) = \int_A^B n(s)\,ds$$



- **Complex refractive index:** $m = m_r - i m_i$
- **Phase difference :** $\Delta\phi(AB) = k m_r \Delta s$
- **Absorption:** $I(B) = I(A)\exp(-k m_i \Delta S)$
- **if *m* is constant:**

$$\Delta\phi = k m_r \Delta s$$

$$E = E_0 \exp(-k m_i \Delta s)$$

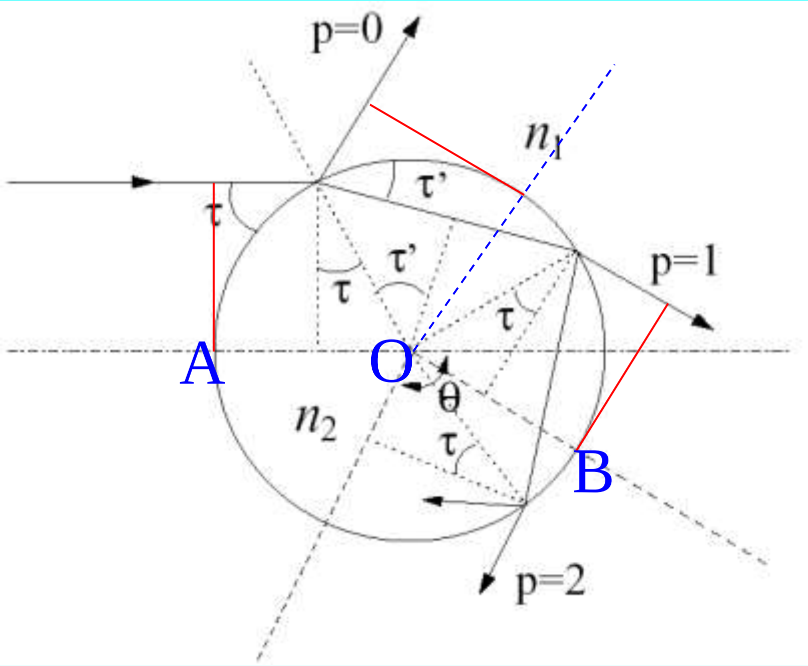
Geometrical optics - sphere

Application of GO to scattering

1. Scattered Intensity

$$I(\theta_p) = I_0 \varepsilon_X^2 D$$

ε_X: Fresnel coefficient
D: divergence factor



- **Deviation angle :**

$$\theta = 2\tau - 2p\tau'$$

- **Scattering angle:**

$$\theta_p = q_p(2p\tau' - 2\tau - 2k_p\pi)$$

- **Phase:**

$$\Delta\Phi = \frac{2\pi d}{\lambda}(\sin\tau - pm\sin\tau')$$

Geometrical optics - sphere

- **Fresnel factors** (Reflection and refraction) **:**

$$\varepsilon_X^2 = r_X^2 \qquad p = 0$$

$$\varepsilon_X^2 = r_X^{2(p-1)}(1-r_X^2)^2 \qquad p \geq 1$$

p : order of ray,

***X* :** polarization ($\perp$ or $\parallel$),

r_X : Fresnel coefficients.

- **Divergence factor for a sphere:**

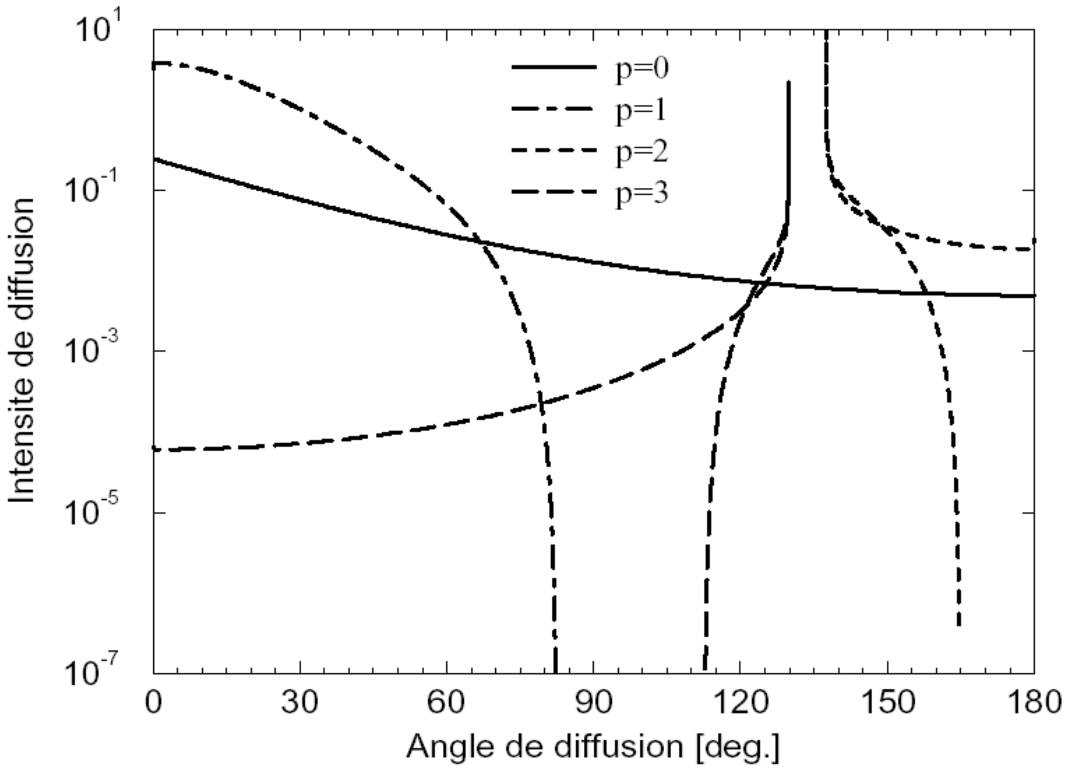
$$I_p = \varepsilon_X^2 \frac{I_0 \sin\tau dA}{r^2 d\Omega} = \varepsilon_X^2 \frac{I_0 a^2 \cos\tau \sin\tau d\tau d\varphi}{r^2 \sin\theta d\theta d\varphi} = \frac{a^2}{r^2}\varepsilon_X^2 I_0 D$$

$$D = \frac{\sin(2\tau)}{4\sin\theta\left(p\dfrac{\sin\tau}{\sqrt{m^2-\cos^2\tau}}-1\right)}$$

Geometrical optics - sphere

Scattering diagram according to GO:

Without interference between different modes



- **Water droplet**
- **Polarization ⊥**
- **Intensity → ∝ near rainbow angles.**

Extension of geometrical optics

Extension of Geometrical optics

To be considered:

- Interference between all the rays.
- Diffraction.
- Extended to an arbitrary shaped beam.

So to be calculated:

- Direction of propagation.
- Divergence/convergence.
- Phase (path, focal lines, phase of the incident wave).
- Amplitude (Fresnel coefficient, absorption).
- Summation of the *complex amplitudes of all modes* at given point (direction).

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Extension of geometrical optics

Application to a homogeneous sphere:

To take into account the interference, we calculate the amplitude of each ray and count carefully the phase.

Amplitude :

We do not calculate the intensity !

$$A = A_0 \varepsilon_X \sqrt{D} \varepsilon_a$$

D: divergence factor.

ε_X: Fresnel coefficient.

ε_a: Absorption coefficient:

L real distance,

k_n: normal component of ***k***.

$$\varepsilon_a = e^{-k_n m_i L}$$

Extension of geometrical optics

Application to a homogeneous sphere:

- **Phases:**

$$\phi_p = (\pi) + \phi_{p,PH} + \phi_{p,FL}$$

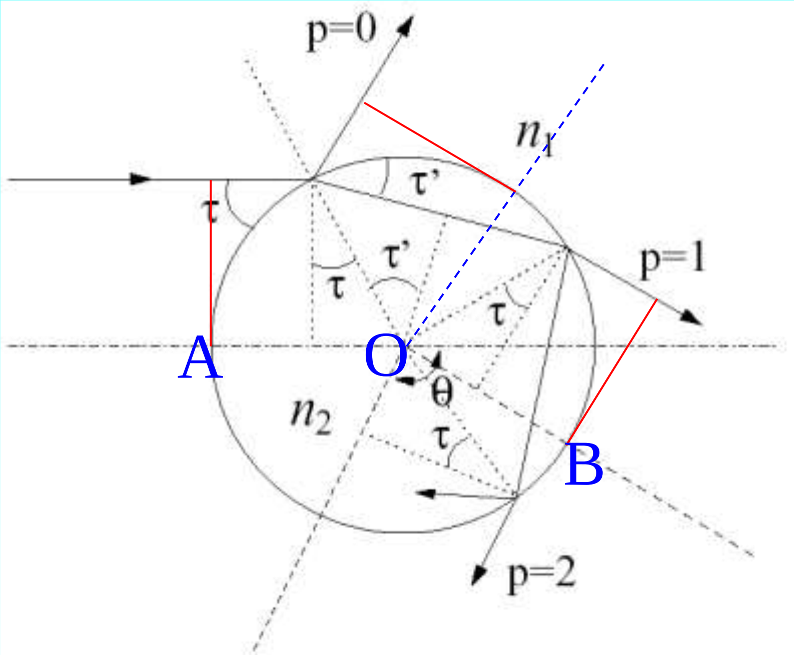
- **Phase difference due to the path difference :**

$$\phi_{p,PH} = \frac{2\pi d}{\lambda}(\sin\tau - pm\sin\tau')$$

- **Phase due to the focal lines:**

$$\phi_{p,FL} = \frac{\pi}{2}(p - 2k_p + \frac{1}{2}s - \frac{1}{2}q_p)$$

s the sign of δτ'/δτ



Extension of geometrical optics

Application to a homogeneous sphere:

- **Due to focal lines:**

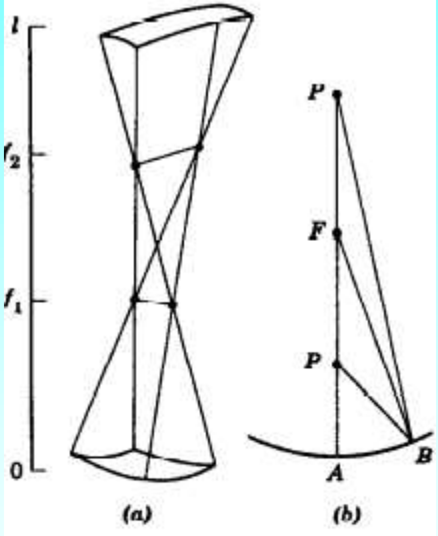
$$\phi_{p,FL} = \frac{\pi}{2}\left(p - 2k_p + \frac{1}{2}s - \frac{1}{2}q_p\right)$$

Amplitude:

$$u_P = u_A \left(1 - \frac{l}{f_1}\right)^{-1/2} \left(1 - \frac{l}{f_2}\right)^{-1/2}$$

When $f - l$ is negative -> phase factor $e^{-i\pi/2}$.

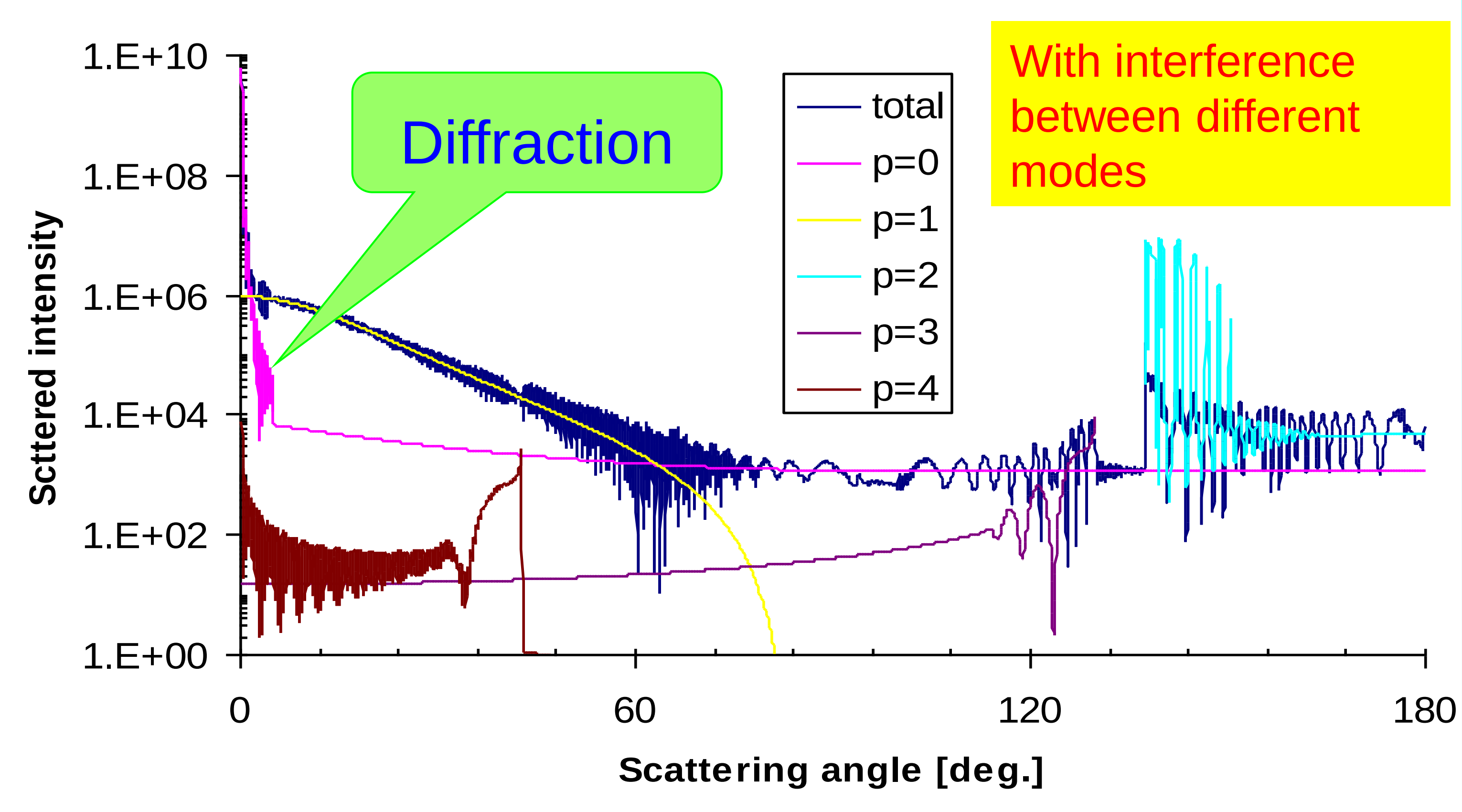
Rule: The phase advances by π/2 at the passage of a focal line.



See Van de Hulst, sec. 3.21, 12.1 and 12.22.

Extension of geometrical optics

Scattering diagram according to GO:



Extension of geometrical optics

Shaped beam:

Ray direction - Application to the Gaussian beam (on axis)

- **Phase function:**

$$\phi_i(x,y,z) = -k_1\left[\left(z-z_0\right)+\frac{x^2+y^2}{2R}\right]+\tan^{-1}\left(\frac{z-z_0}{z_R}\right)$$

$$F_y = \frac{\partial \phi_i}{\partial y} = -\frac{2z_R y(z-z_0)}{w_0^2[z_R^2+(z-z_0)^2]},$$

$$F_z = \frac{\partial \phi}{\partial z} = \frac{z_R}{z_R^2+\left(z-z_0\right)^2} - \frac{z_R y^2\left[z_R^2-(z-z_0)^2\right]}{w_0^2\left\{z_R^2+\left[z_R^2-(z-z_0)^2\right]^2\right\}^2} - k_1$$

- **Incident angle on a sphere:**

$$\cos\theta_i = \frac{\left|yF_y + zF_z\right|}{\sqrt{(y^2+z^2)(F_y^2+F_z^2)}}$$

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Extension of geometrical optics

Shaped beam:

Diffraction of a Gaussian beam:

$$S_d = \frac{\alpha^2}{2\pi}\left(\frac{w_0}{w}\right)\int_0^{2\pi} d\varphi' \int_0^1 \exp(-At^2)\exp[i(Bt + Ct^2)]tdt$$

$$\text{where } A = \left(\frac{r}{w}\right)^2,\ B = -\alpha \tan\theta \cos(\varphi - \varphi') \text{ and } C = \frac{\alpha^2}{2k_1 R}.$$

When A is small, i.e., the radius of the particle is smaller than the local beam waist radius ($r \leq w$), the integral equation can be evaluated analytically through using Legendre polynomial approximation (Chevaillier et al., 1986)

Extension of geometrical optics

Shaped beam:

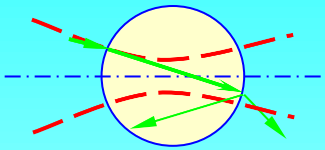
Total field

- **Amplitude:**

$$S_j = S_{dif} + \sum_{p=0}^{\infty} S_{j,p}$$

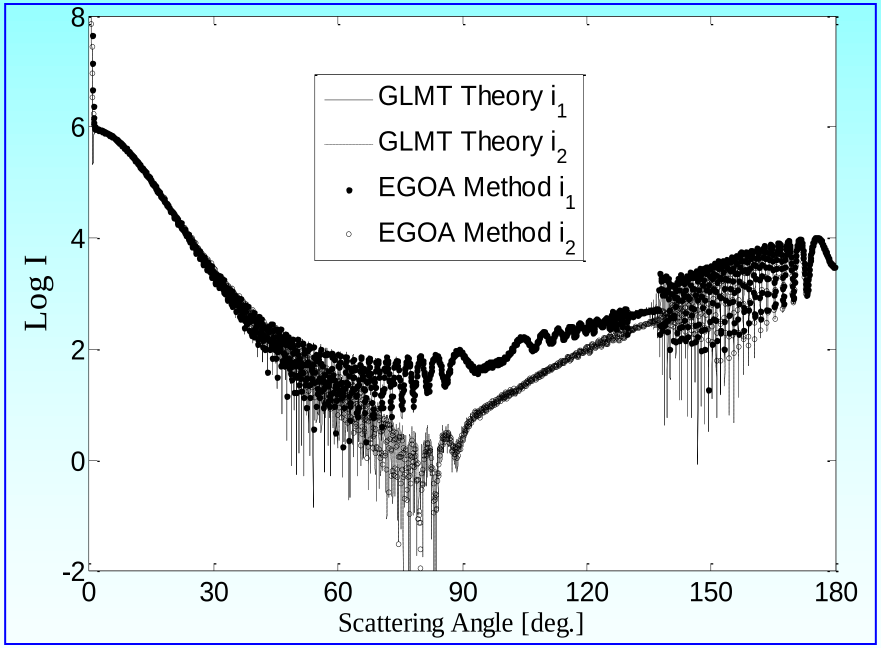
- **Intensity :**

$$I = \frac{I_0}{(kr)^2} |S_j(\theta)|^2$$



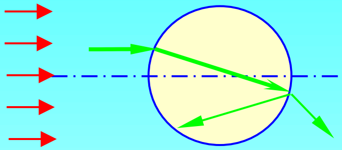
Sphere a=100 µm, Gaussian beam **w_0=50 µm**

The agreement is very good.



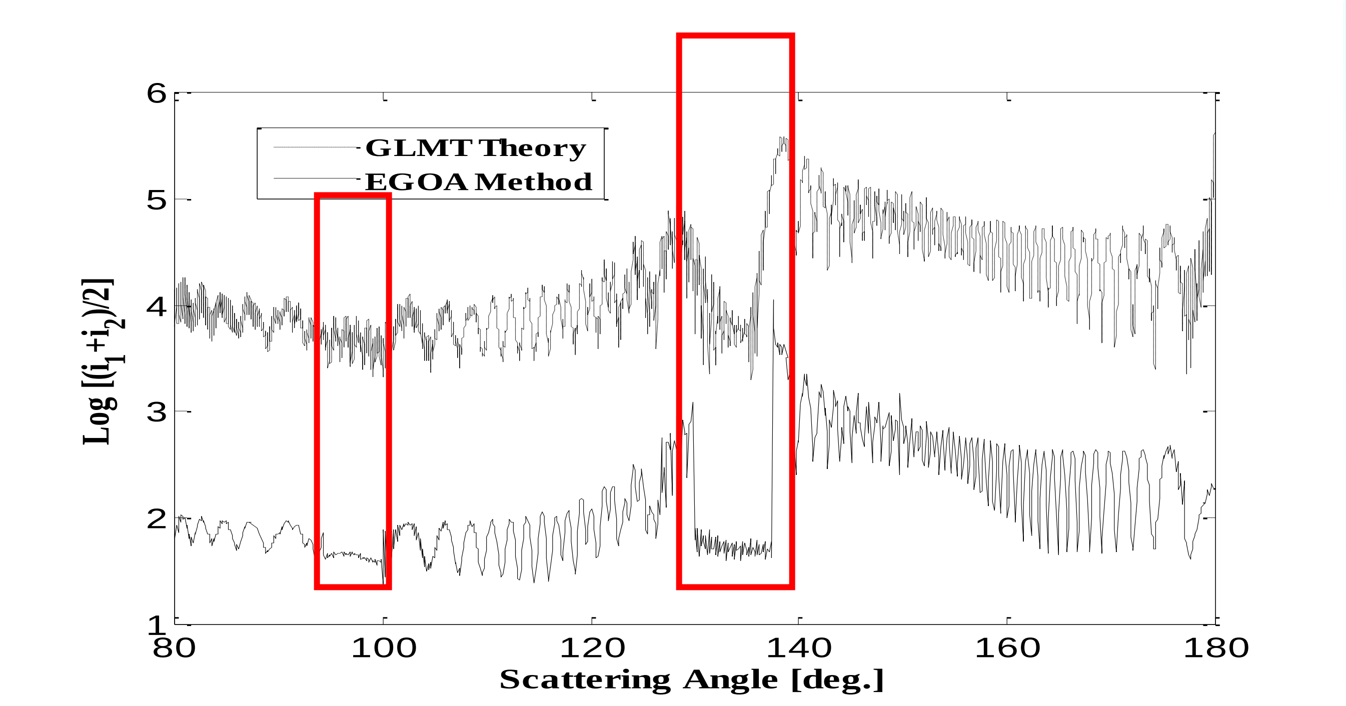
Extension of geometrical optics

Shaped beam:



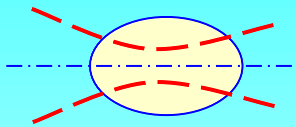
Sphere a=100 µm **plane wave**

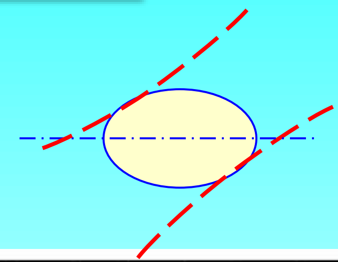
Discrepancy found near rainbow angles

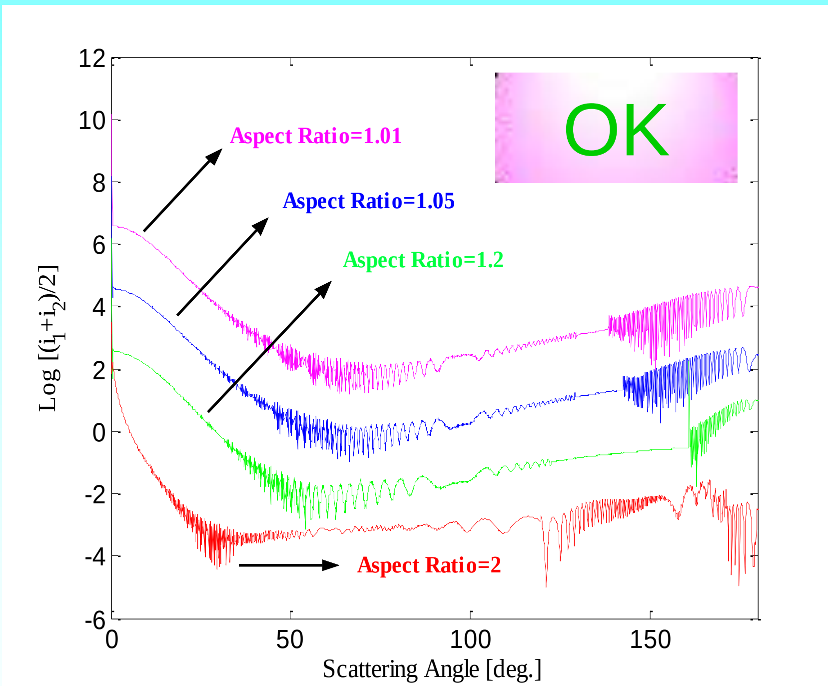


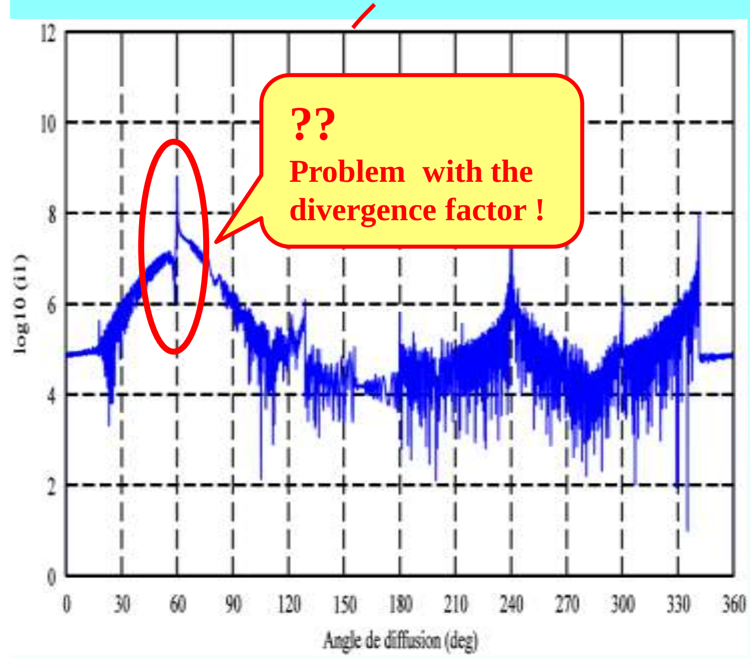
Extension of geometrical optics

Extension to a spheroid









a=100 µm, w_0=50 µm, m=1.33, x_0=y_0=z_0=0

a = 200μm, b = 100μm, m = 1.33, λ = 632.8nm, α_0 = 60°

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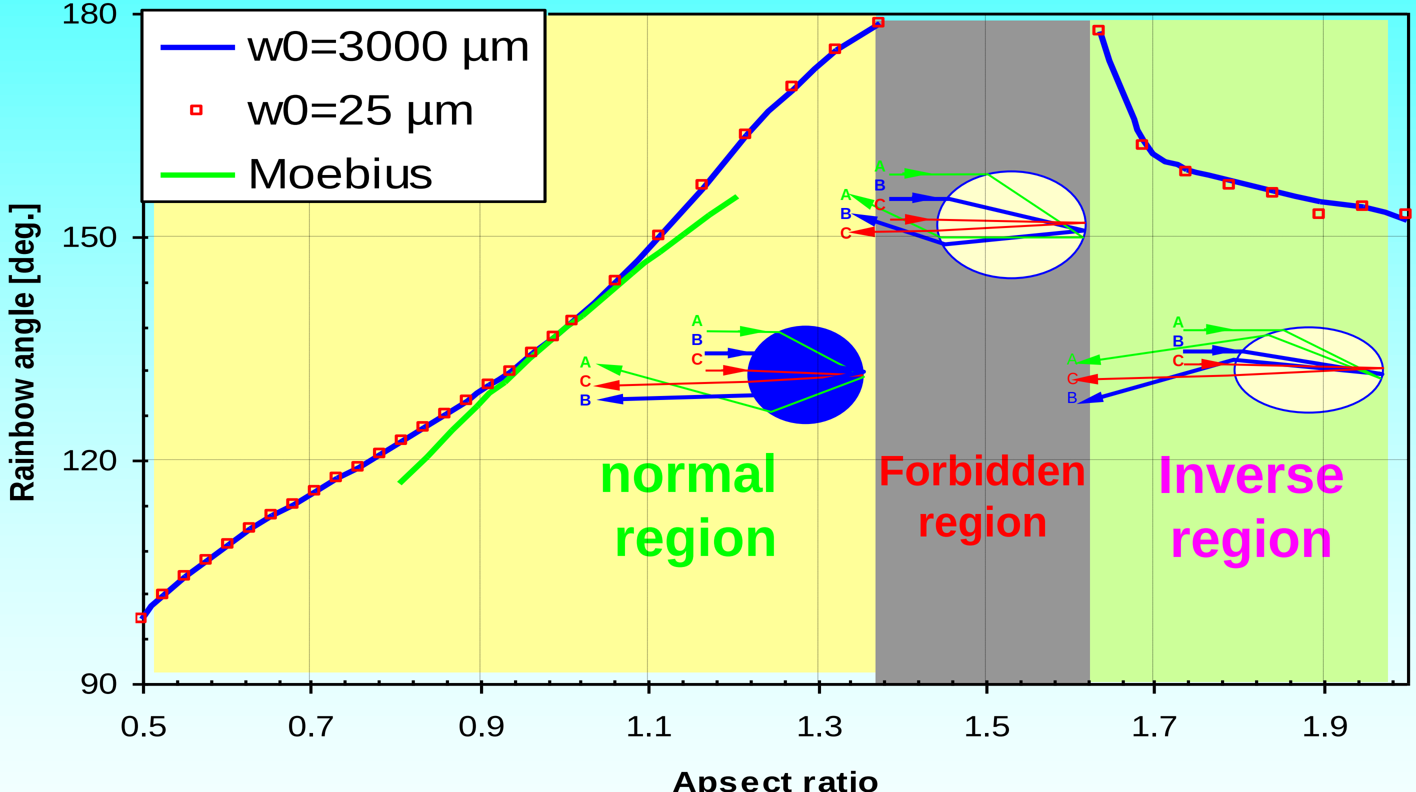
Extension of geometrical optics

First order rainbow position of a spheroid

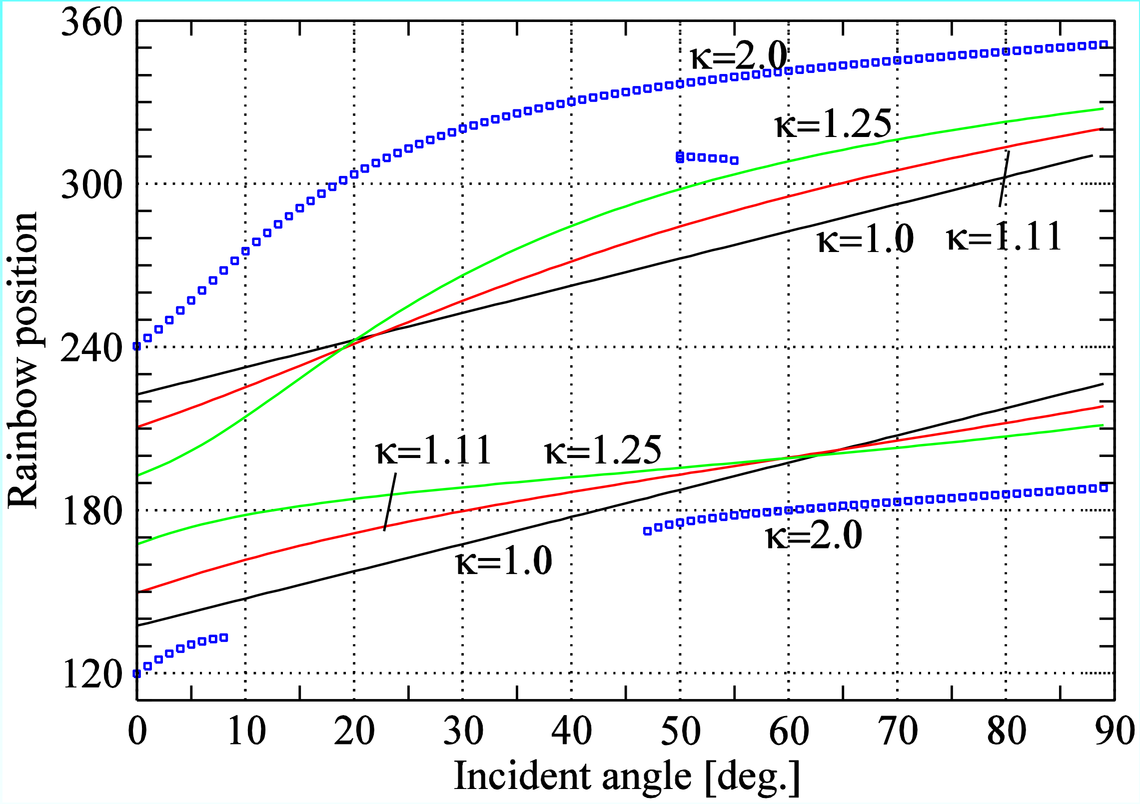


Rainbow position of a spheroid with arbitrary ellipticity
The limit of Moebius model is surpassed

Extension of geometrical optics

Rainbow of an elliptical cylinder

K. L. Jiang et al,
Appl. Opt.. 2012
To be published.



Relation of incident angle and the rainbow position with the different aspect ratio

Extension of geometrical optics

Preliminary conclusions

- By taking into account correctly the interferences, the Ray model can predict the scattering diagram in **ALL directions**.
- It can be applied to the scattering of **any shaped beam**.
- It works also for a circular infinite **cylinder**.

Limitations

NOT appropriate for a spheroid or an ellipsoid particle, as well as any irregular shaped particles.

The key problem is **the divergence factor.**

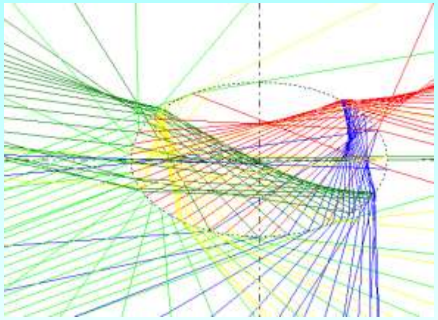
Vectorial complex ray model

Key point: Wave front curvature in the ray model

Vectorial Complex Ray Model (VCRM)

Ren et al, *Opt. Lett*. 36(3): 370-372, 2011

- **Principle:**
  - **Wave front and dioptric surface** described by their local curvatures,
  - Divergence and focal lines calculated by the curvatures,
  - Rays as vectors – easier to implement,
  - Interference of all rays.



- **Advantages:**
  - Any form of incident wave,
  - Object of any shape with smooth surface,
  - Sufficiently precise – scattering in all directions,
  - Prediction of all scattering properties of the objet.

Vectorial complex ray model

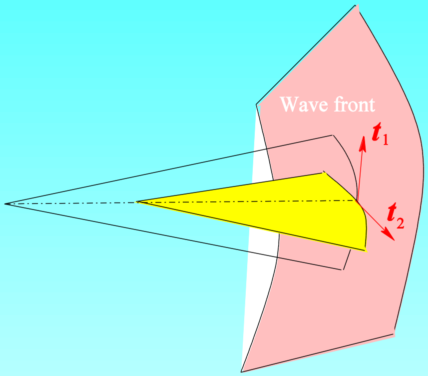
Description of the model

- Vectorial complex rays :

$$\vec{S}_i = A_i e^{-i\Phi_i}\hat{k}_i$$

- Wave front - Curvature matrix :

$$Q = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$$

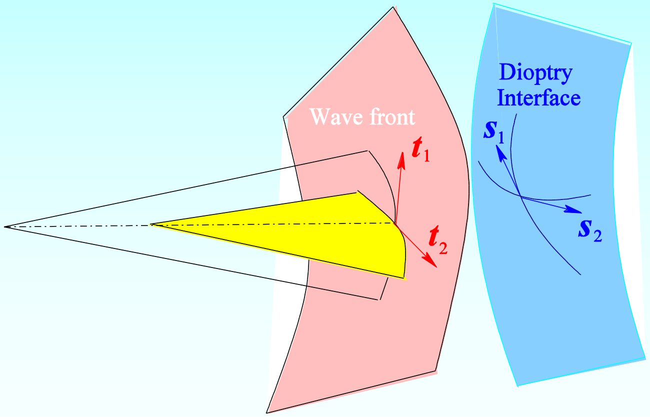


- Dioptric surface :

$$C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

- Projections matrix:

$$\Theta = \begin{pmatrix} \hat{t}_1 \cdot \hat{s}_1 & \hat{t}_1 \cdot \hat{s}_2 \\ \hat{t}_2 \cdot \hat{s}_1 & \hat{t}_2 \cdot \hat{s}_2 \end{pmatrix}$$



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Vectorial complex ray model

- **Fundamental laws**

1. Ray direction – Law of Snell Descartes :

The tangent component of wave vector is continuous:

$$k_\tau = k_\tau^{'}$$

The normal component :

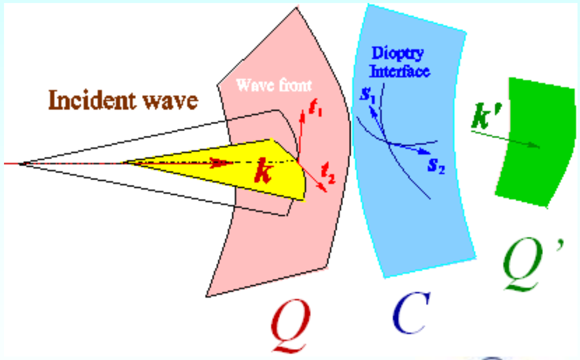
$$k_n = \sqrt{k^2 - k_\tau^2}$$

The wave vector for a arbitrary shaped beam:

$$\vec{k} = k \frac{\vec{\nabla} s}{|\vec{\nabla} s|}$$

2. Wave front equation:

$$\left(k_n^i - k_n^t\right) C = k' \Theta'^T Q' \Theta' - k \Theta^T Q \Theta$$



Vectorial complex ray model

Special case of the wave front equation:

When the rays remain in the same plane – a main direction of the wave front and particle surface:

- Spherical particle
- Infinite cylinder at normal incidence
- Ellipsoidal particle in the symmetric plane.

➢ Curvature matrix:

$$C = \begin{pmatrix} \frac{1}{\rho_1} & 0 \\ 0 & \frac{1}{\rho_2} \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{1}{R_1} & 0 \\ 0 & \frac{1}{R_2} \end{pmatrix}$$

$$Q' = \begin{pmatrix} \frac{1}{R'_1} & 0 \\ 0 & \frac{1}{R'_2} \end{pmatrix}$$

➢ Wave front equation:

$$\frac{{k'_n}^2}{k'R'_1} = \frac{k_n^2}{kR_1} + \frac{k'_n - k_n}{\rho_1}$$

$$\frac{k'}{R'_2} = \frac{k}{R_2} + \frac{k'_n - k_n}{\rho_2}$$

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Vectorial complex ray model

- **Amplitude:** $A_i = \sqrt{D}\varepsilon_X \varepsilon_a$
- **Phase :** $\Phi = \Phi_{inc} + \Phi_{fl} + \Phi_{path} + (\Phi_{\lambda/2})$
- **Total field:** $E = S_{diff} + \sum_{i=1}^{N} S_i$

Divergence factor:

$$D = \frac{R_{01}R_{02}}{R_{01}^{'}R_{02}^{'}}\frac{R_{11}R_{12}}{R_{11}^{'}R_{12}^{'}}\cdots\frac{R_{p1}R_{p2}}{(d-R_{p1}^{'})(d-R_{p2}^{'})}$$

Fresnel coefficients: ε :

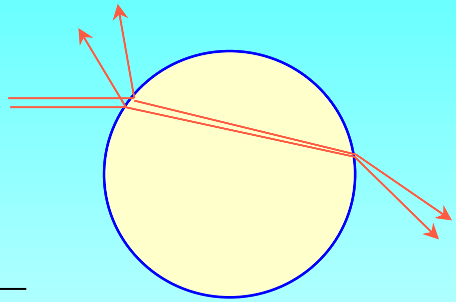
$$\tilde{r}_{\perp} = \frac{k_n - \tilde{k}_n^{'}}{k_n + \tilde{k}_n^{'}} \qquad \tilde{r}_{\parallel} = \frac{\tilde{m}^2 k_n - \tilde{k}_n^{'}}{\tilde{m}^2 k_n + \tilde{k}_n^{'}}$$

All expressed in wave vector components.

Vectorial complex ray model

Applications to a sphere and a cylinder

- **Sphere**



  - Reflection: $R_1 = -\frac{a\cos\alpha}{2} \quad R_2 = -\frac{2}{a\cos\alpha} \rightarrow D = \frac{1}{4}$
  - Refraction p =1:

After 1st refraction: $R'_{11} = -\frac{am\cos^2\beta}{m\cos\beta - \cos\alpha} \quad R'_{12} = -\frac{am}{m\cos\beta - \cos\alpha}$

After 2nd refraction:

$$R'_{21} = \frac{m\cos\beta - 2\cos\alpha}{2(m\cos\beta - \cos\alpha)}\cos\alpha \quad R'_{22} = \frac{2a\cos\beta(m\cos\beta - \cos\alpha) - m}{2(m\cos\beta - \cos\alpha)(\sin\alpha\sin\beta - \sin\alpha\sin\beta)}$$

Divergence factor: $D = \frac{m\sin(2\alpha)\cos\beta}{4\sin[2(\beta-\alpha)](\cos\alpha - m\cos\beta)}$

Identical to the classical one.

- **Cylinder: $R_2 = \infty$**
  - Reflection : $D = \frac{a\cos\alpha}{2}$
  - Refraction p=1: $D = \frac{m\cos\alpha\cos\beta}{2(\cos\alpha - m\cos\beta)}$

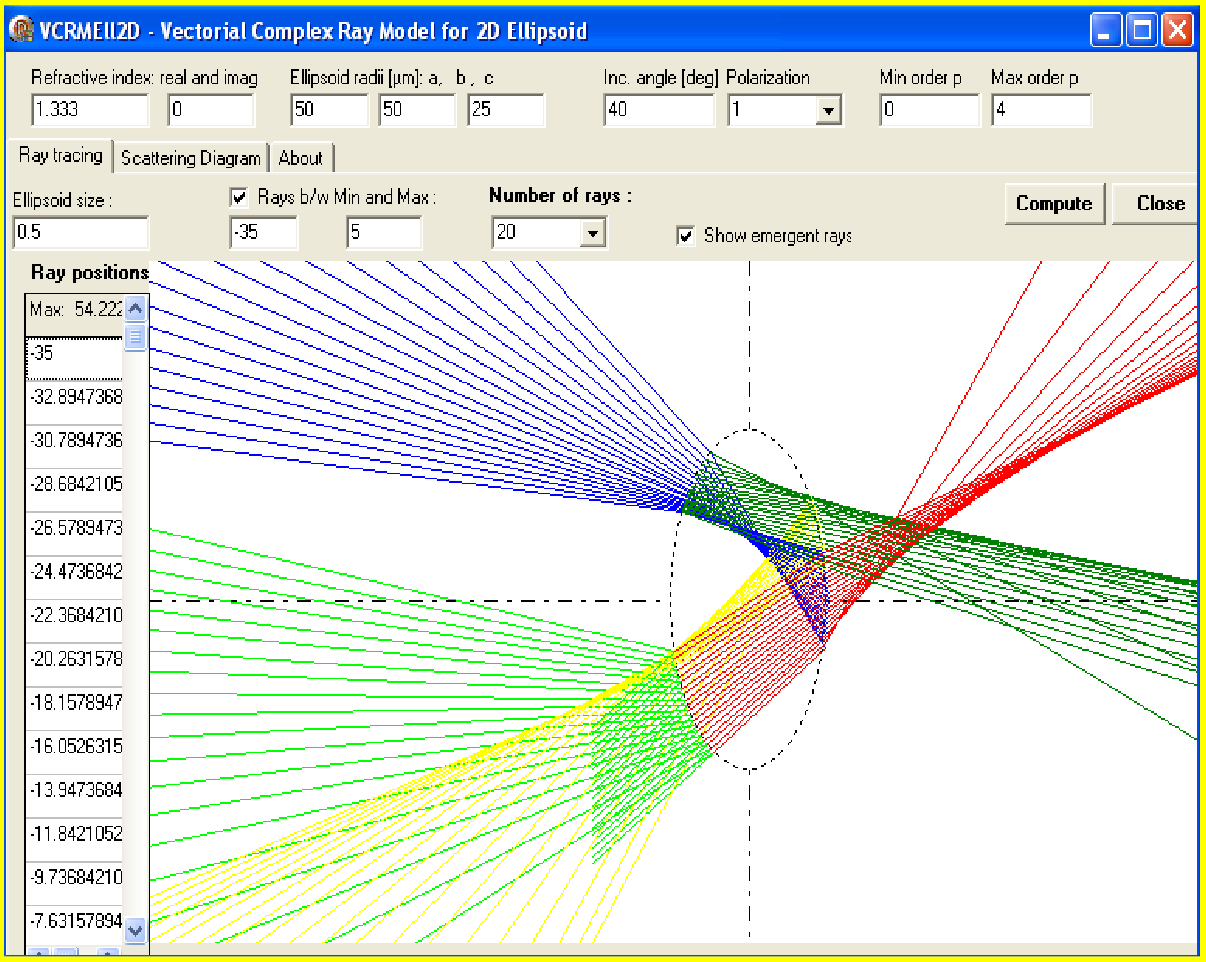
IV-32

Vectorial complex ray model

Software for an ellipsoid

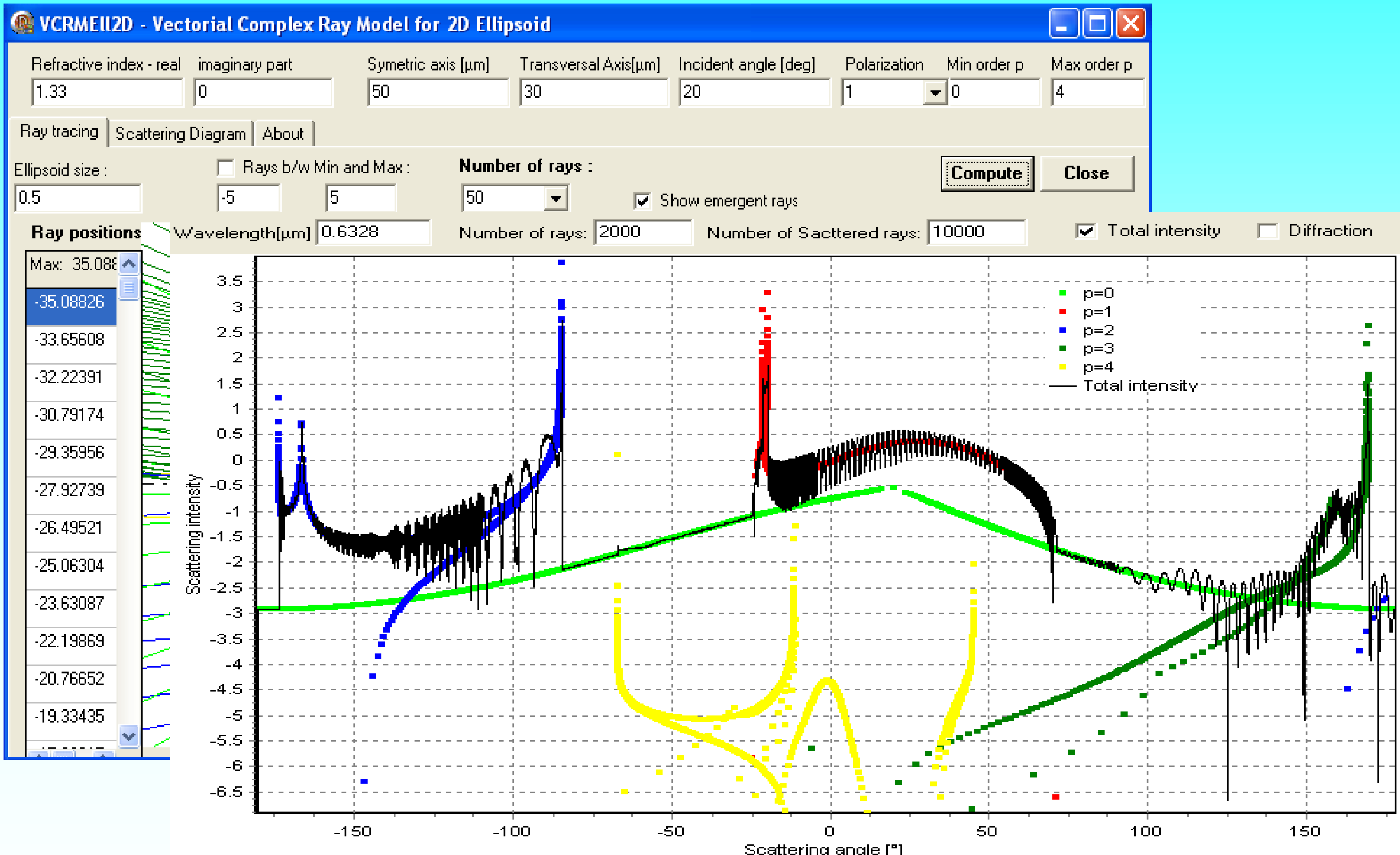
Ray tracing Module

- Tracing of all the rays or a part of them
- Any number of rays
- Any position of rays



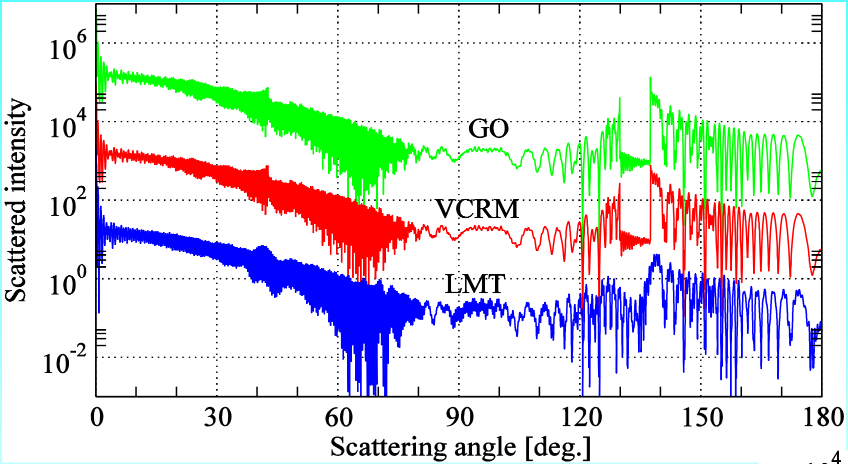
Vectorial complex ray model

Software for spheroid



Vectorial complex ray model

Comparison with exact numerical method

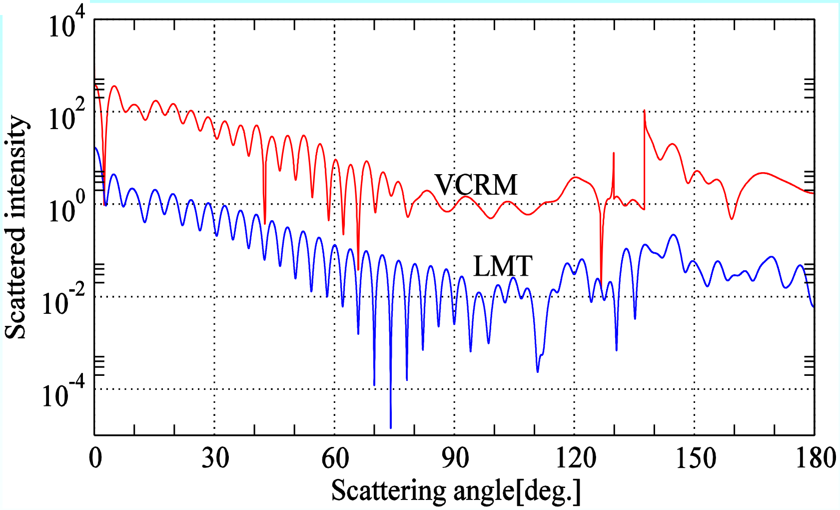


← Scattering diagrams by LMT, GO and VCRM for an infinite circular cylinder :

- refractive index: m = 1.33,
- **radius a = 50 μm**
- wavelength λ = 0.6328 μm.

→ Scattering diagrams by LMT, GO and VCRM for an infinite circular cylinder :

- refractive index: m = 1.33,
- **radius a = 5 μm**
- wavelength λ = 0.6328 μm.



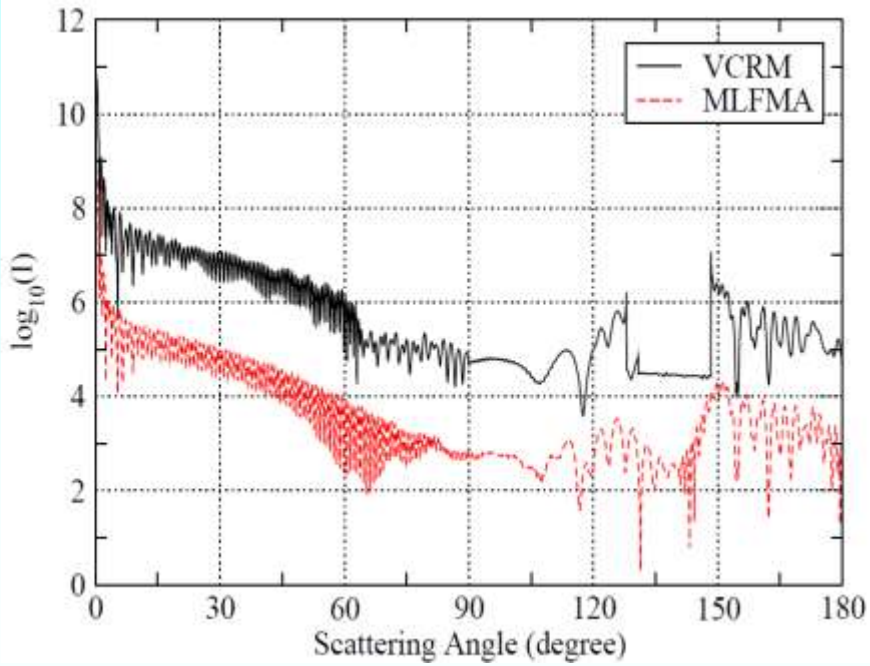
Vectorial complex ray model

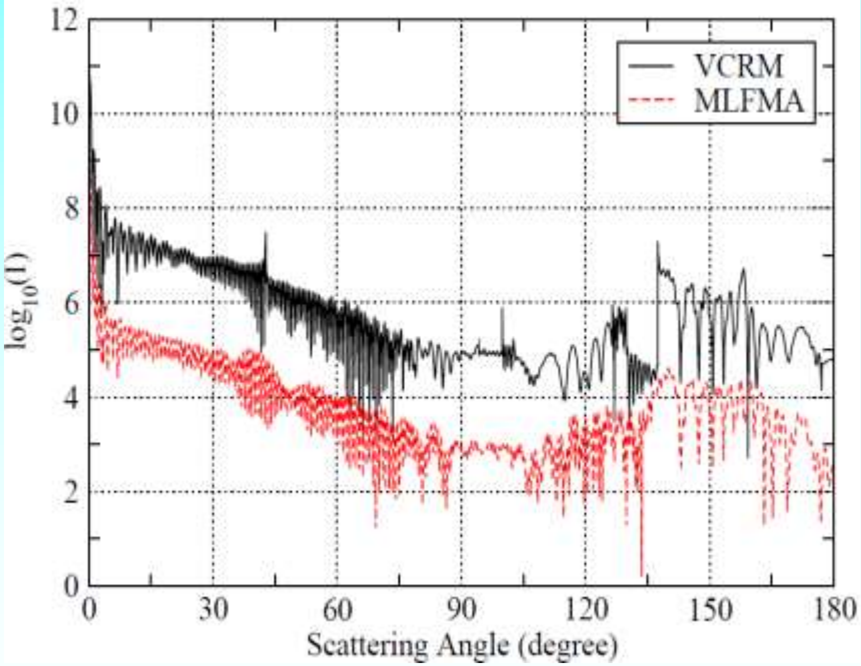
Comparison with exact numerical method MLFMA

A spheroidal water droplet (m=1.33) a = **30 μm** and a plane wave of wavelength λ=0.785 μm.

prolate: $a = b$, $c = 1.1a$

oblate: $a = 0.9b$, $c = a$





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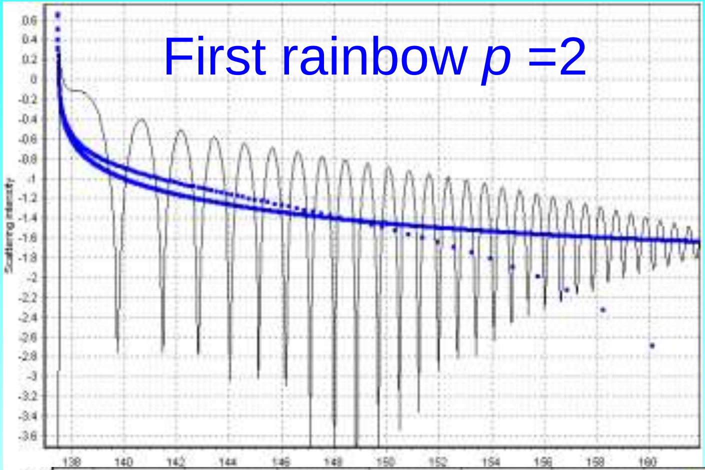
Vectorial complex ray model

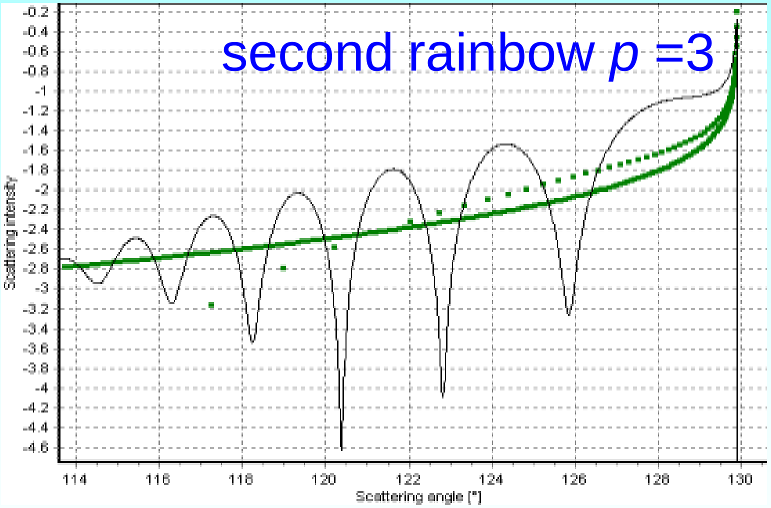
Interference near rainbow angles

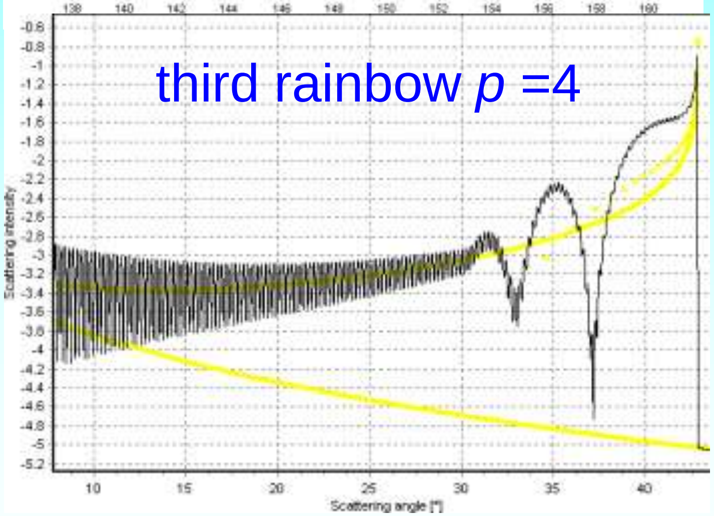
Plane wave incident on a sphere:

λ=0.6328 μm,

a = 100 *μ*m, *m* = 1.33

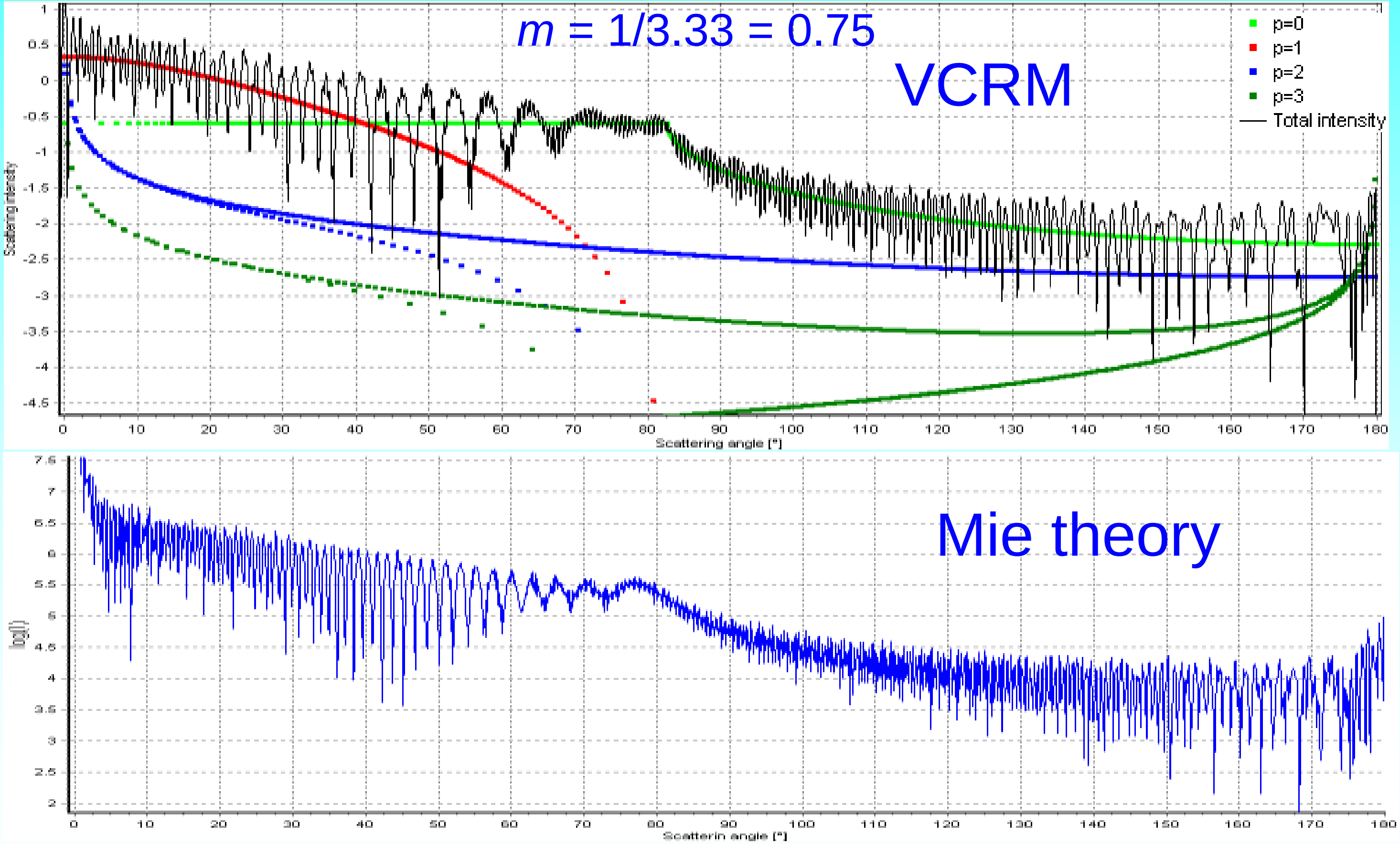






Vectorial complex ray model

Comparison with Mie theory for a bubble



Vectorial complex ray model

Transversal convergence of an ellipsoid

Ren et al, *JQSRT.* 2012

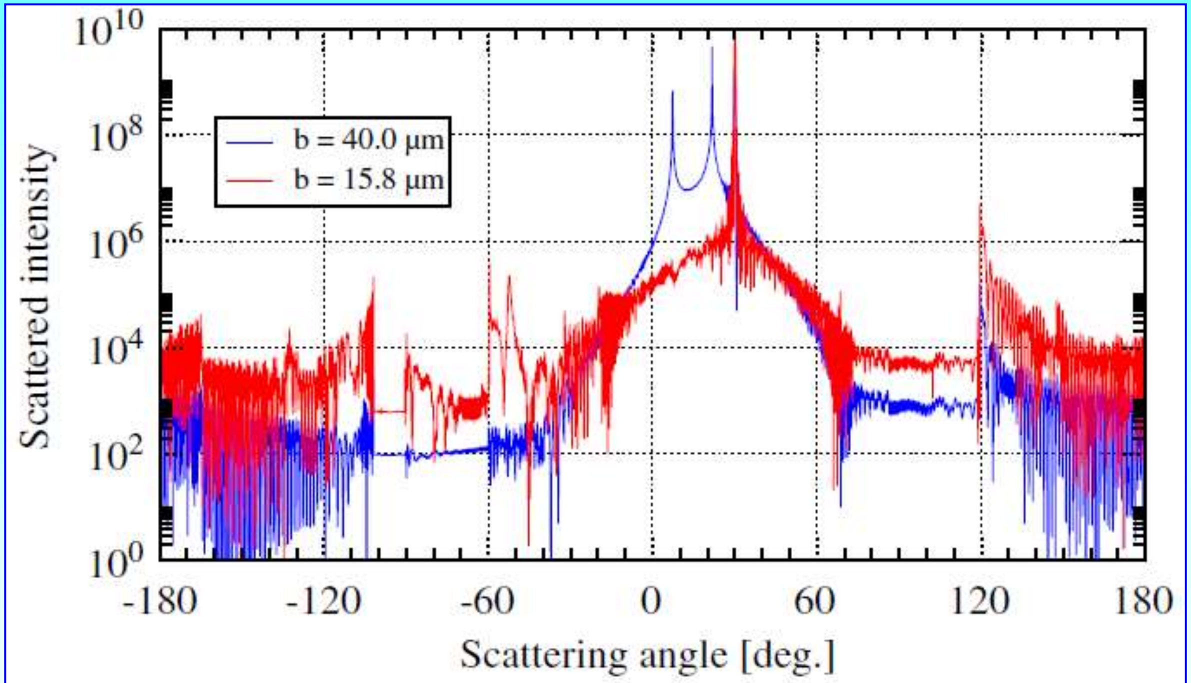
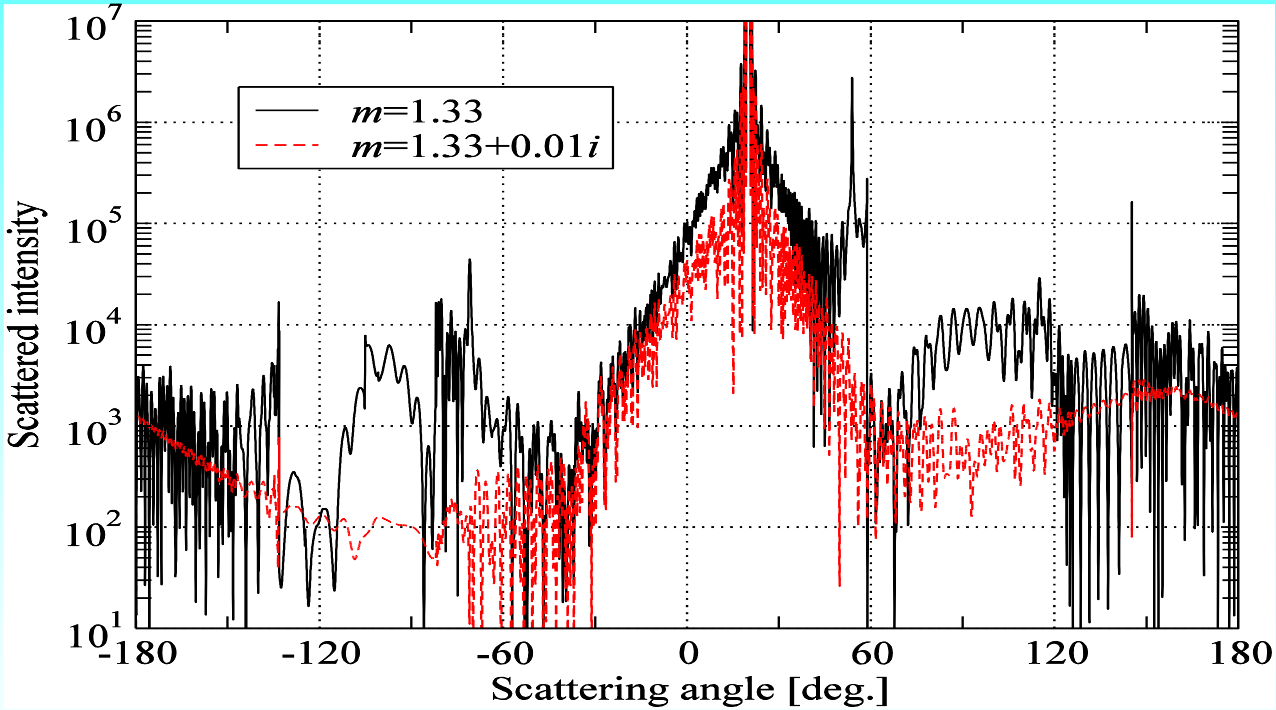


Fig. 6. Scattering diagrams of an ellipsoid of water $a = 50\ \mu m$, $c = 30\ \mu m$ illuminated by a plane wave of wavelength $\lambda = 0.6328\ \mu m$ with an incident angle $\theta_0 = 30^\circ$ and b as parameter.

Vectorial complex ray model

Scattering diagram of an absorbing ellipsoid

Ren et al, *ISAPE*. 2012



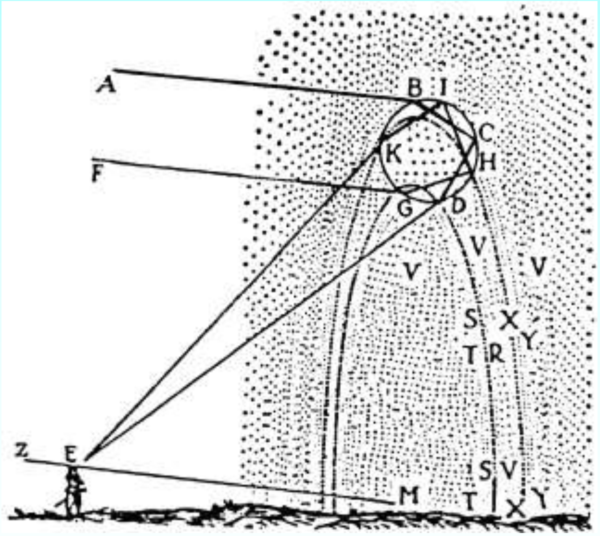
Scattering diagrams of an ellipsoid of water of semi-axes a=30 μm, b=20 μm, c=10 μm illuminated by a plane wave of wavelength λ = 0.6328 μm and incident angle θ_0 = 20° with refractive index as parameter.

Ray Theory of Wave

The rainbow is a marvel of nature.

L'arc-en-ciel est une merveille de la Nature si remarquable, et sa cause a été de tout temps si curieusement recherchée par les bons esprits, et si peu connue, que je ne saurais choisir de matière plus propre à faire voir comment, par la méthode dont je me sers, on peut venir à des connaissances que ceux dont nous avons les écrits n'ont point eues.

彩虹乃自然之奇观
自古引圣贤索其源



The Colors of bows are due to the refractive index.

But the intensity tends to infinity !!

Ray Theory of Wave

Airy theory (1838)

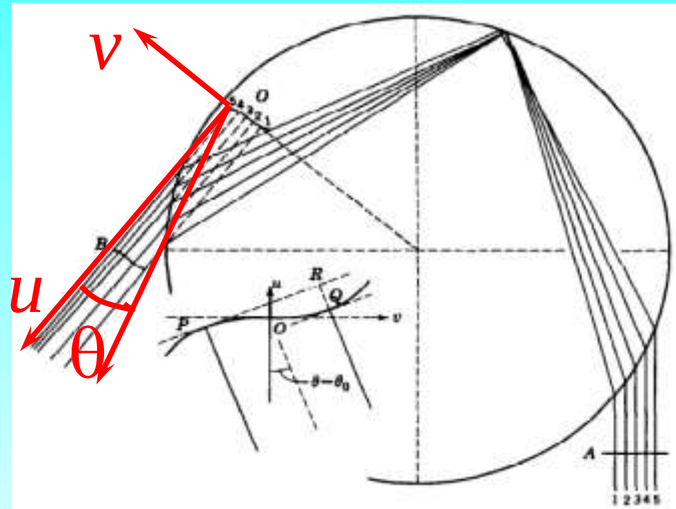
Sir George Airy
(1801-1892)

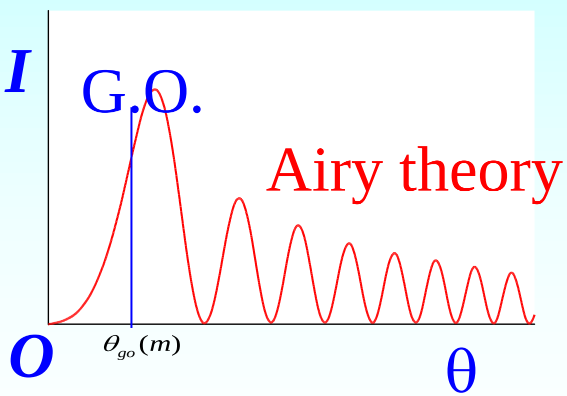
- Phase difference is:

$$\Delta\Psi = khv^3/3a^2$$

- Amplitude is constant for all emergent rays
- Amplitude of scattered field in θ direction:

$$\int_{-\infty}^{\infty} e^{-ikv(\theta-\theta_0)+ikhv^3/3a^2}\,dv$$





Airy theory predict a good profile,

Questions :

- The phase function found in v -> 0 but integration to infinity ?
- The amplitude is not constant near rainbow angles!
- It is only valid for a sphere.

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Ray Theory of Wave

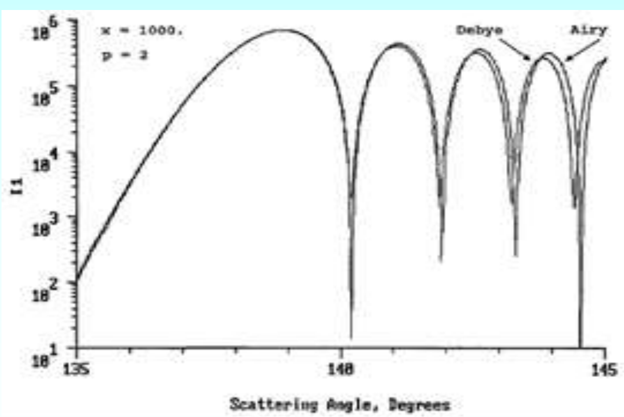
Airy theory in 1990s'

R. Wang and van de Hulst
Appl. Opt. 30(1):106, 1991

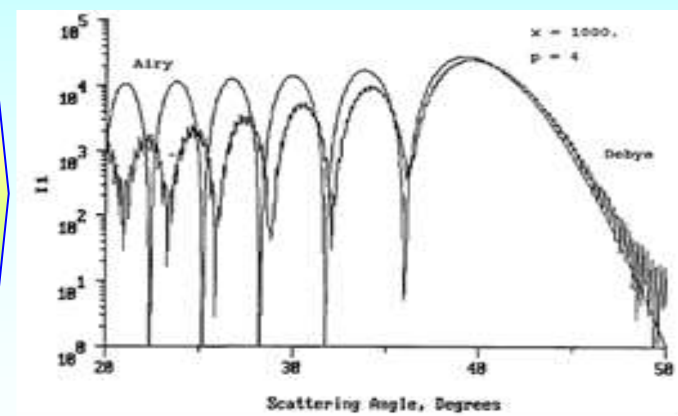
Can anything new be said about the rainbow? Yes. The insight that this phenomenon arises from the play of light in a single spherical drop is 7 centuries old, the full geometrical optics theory of Descartes 3-1/2 centuries, and its modification by Airy to take account of diffraction just 1-1/2 centuries. Exactly a century ago

- Wang, van de Hulst et Lock: same principle but with amplitude correction:

$$E^{p}_{Airy}(\theta) = x\left(\frac{2\pi\sin\theta_i^R}{\sin\theta^R}\right)^{1/2}\frac{x^{1/6}}{h^{1/3}}T^{21}(\theta_i^R)\times\left[R^{11}(\theta_i^R)\right]^{p-1}T^{12}(\theta_i^R)\times Ai\left(\frac{-x^{2/3}\Delta}{h^{1/3}}\right)\exp(2\pi i L^R/\lambda)\exp\left[(ix\Delta)\left(\frac{p^2-n^2}{p^2-1}\right)^{1/2}\right]$$



Airy theory compared to Debye theory (rigorous)



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Ray Theory of Wave

Airy theory in 1990s'

- **Revisit of Airy theory** to:
  1. understand the method,
  2. Extend to **non-spherical particle**.
- **Theoretical demonstration for a sphere**
  - We know for each emergent ray:

    phase: $\Phi = 2ka(\sin\tau - pm\sin\tau')$

    deviation angle: $\theta = 2\tau - 2p\tau'$
  - The derivatives: ($v \sim 0$):

$$\left.\frac{du}{dv}\right|_{\tau_0} = 0 \qquad \left.\frac{d^2u}{dv^2}\right|_{\tau_0} = 0 \qquad \frac{d^3\Delta\Phi}{d\tau^3} = -2kah\sin^3\tau_0 = -\frac{2ka(p^2-1)}{p^2}\cos\tau_0$$

so

$$h = \frac{p^2-1}{p^2\sin^2\tau_0\tan\tau_0} = \frac{(p^2-1)^2\sqrt{p^2-m^2}}{p^2(m^2-1)^{3/2}}$$

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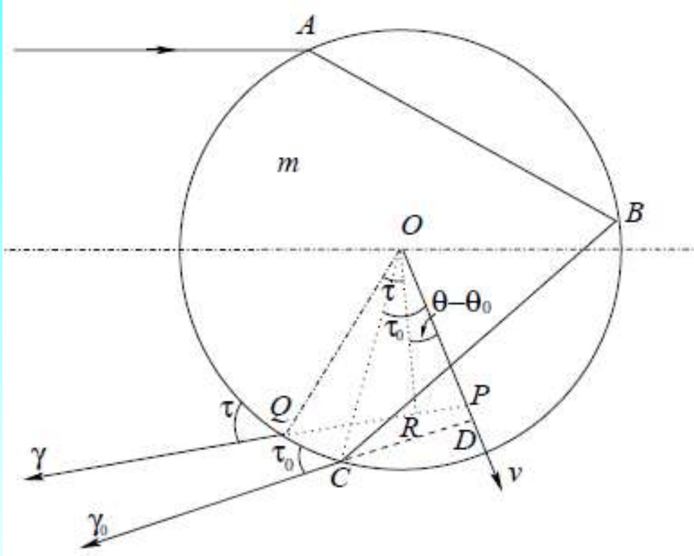
Ray Theory of Wave

Airy theory in 21st century

- **Phase:** $\Delta\Phi = \Phi(k) - \Phi(k_r) - k\overline{PR}$

  with

$$
\begin{aligned}
\overline{OD} &= \boldsymbol{r}_0 \cdot \hat{\boldsymbol{k}}_{r\perp} \\
\overline{OR} &= \boldsymbol{r} \cdot \hat{\boldsymbol{k}}_{\perp} \\
\overline{OP} &= \frac{\overline{OR}}{\hat{\boldsymbol{k}}_r \cdot \hat{\boldsymbol{k}}} = \frac{\overline{OR}}{\hat{\boldsymbol{k}}_{r\perp} \cdot \hat{\boldsymbol{k}}_{\perp}} \\
\overline{PR} &= \overline{OP}(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_{r\perp}) \\
v &= \overline{OP} - \overline{OD}
\end{aligned}
$$



- **Amplitude**

  The amplitude of each ray is naturally calculated in VCRM.

- All values are calculated **numerically**,
- **Without any hypothesis/condition.**
- The model is directly applicable to **any shaped particle.**

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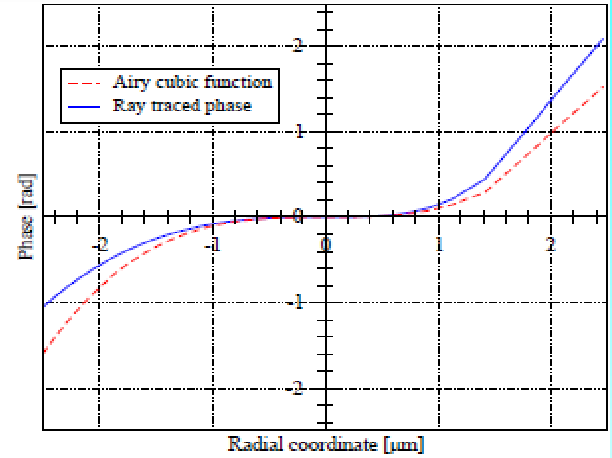
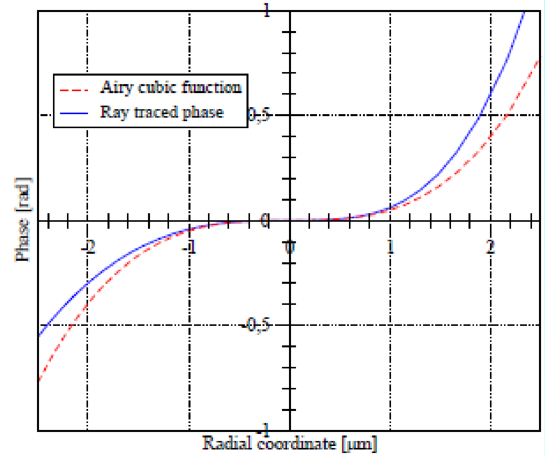
Ray Theory of Wave

Airy theory in 21st century

Phases calculated by Airy and VCRM

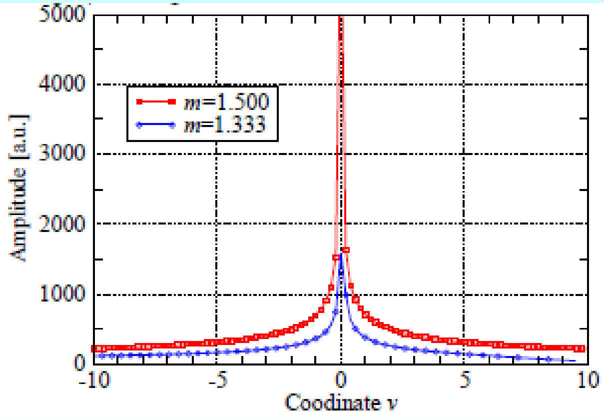


(a). $m = 1.333$



(b). $m = 1.5$

Amplitudes calculated by Airy and VCRM



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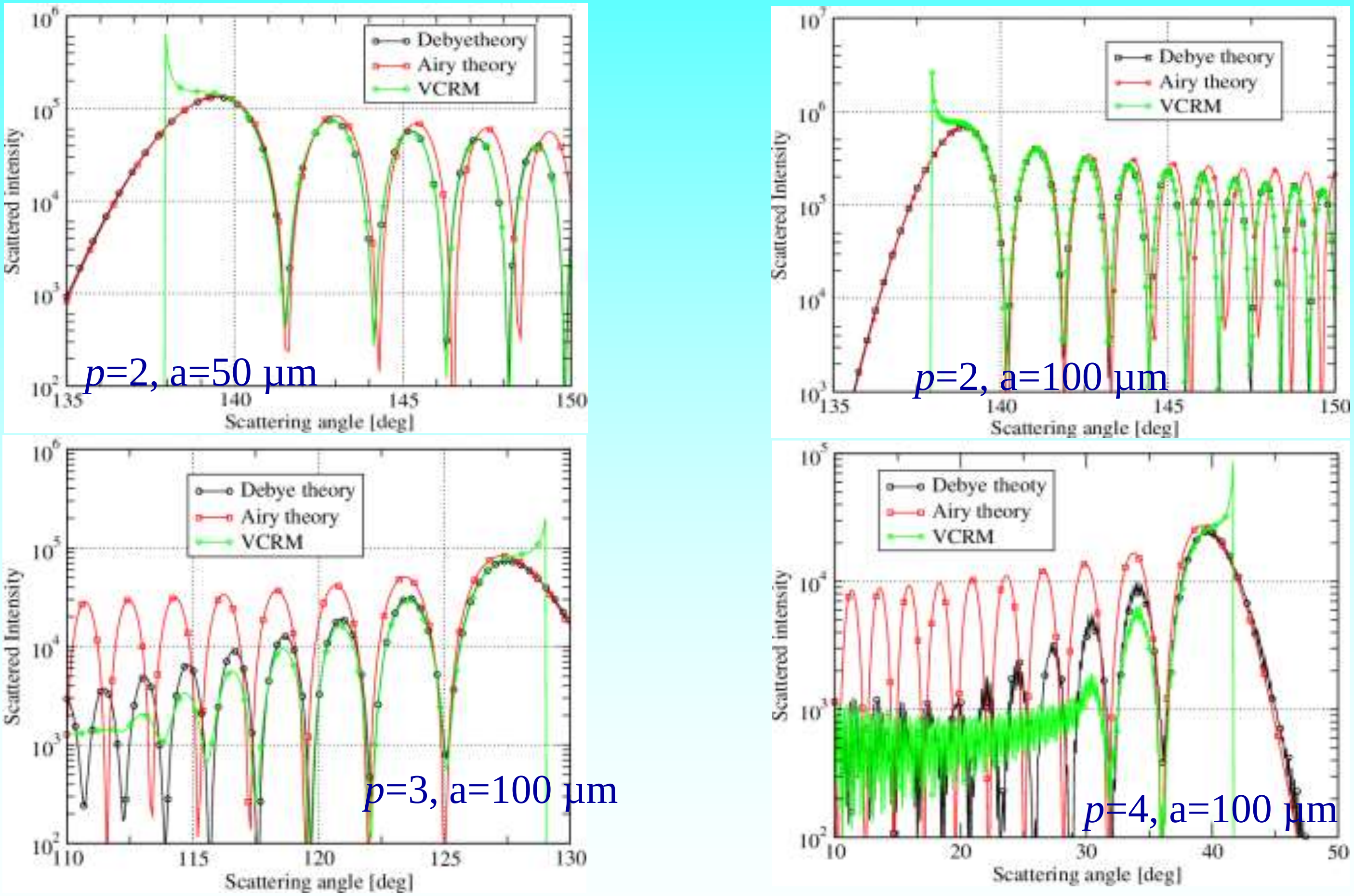
Ray Theory of Wave

Comparison of the three methods



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Conclusions

- **Conclusions**
  - The Vectorial Complex Ray model (VCRM) is established.
  - VCRM has been validated :
    - theoretically for the spherical and cylindrical particle.
    - Numerically 2D scattering of a ellipsoid.
  - A software for 2D ellipsoid is realized.
  - VCRM has been applied to the elliptical cylinder and Gaussian beam,
  - First step to the Ray Theory of Wave achieved.
- **Application fields:**
  - Metrology of non-spherical particles in multiphase flow,
  - Prediction of radiation force, torque, and stress,
  - Application to freeform optical systems,
  - Diagnostics for micro-fluidics.
  - …

Thank you for your attention