Airy theory revisited with VCRM and physical optics

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Abstract

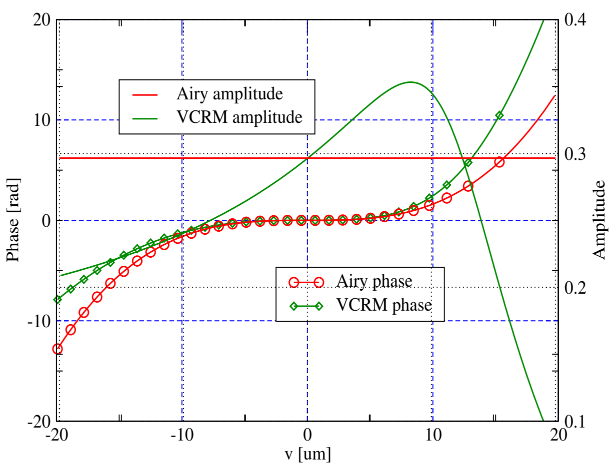
Airy published a paper in 1838 to remedy the problem of the infinite intensity in the rainbow angles of a spherical droplet predicted by the geometrical optics (GO) [1]. His theory has been studied by mathematicians and physicists since then from different points of view [2]. Airy theory predicts the main and secondary peak positions and amplitudes as function of the refractive index and the particle size. Therefore, it is still largely used in the optical particle measurement though it is known that its precision is limited compared to the rigorous theories [3].

In this communication, the Airy theory is revisited with the Vectorial Complex Ray model (VCRM) [4] and the physical optics (PO). Two key points are examined: The first is that the cubic phase function in Airy theory is obtained from the relation in the vicinity of the rainbow angles, but this is extended to infinity in the integration. The second is the assumption of constant amplitude of the emergent rays, which is not true because the amplitude varies due to the convergence and divergence of the wave on the curved surface of the particle.

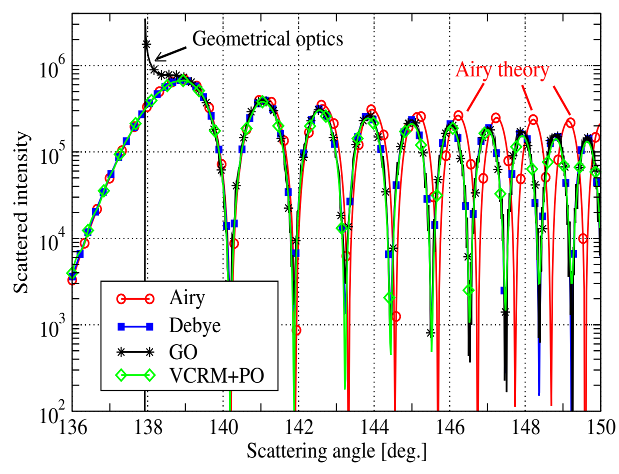
We follow the idea of Airy, but the phase and the amplitude of the emergent rays are calculated in the framework of VCRM [4] with rigorous geometry. Figure 1 (a) shows that the phase on the virtual line v given by the cubic function of Airy [2] (red line with circles) is in good agreement with that calculated by VCRM (green line with diamants) around $v = 0$ (corresponding to the geometrical rainbow angle) but differs more and more when the point is far from 0. The variation of the amplitude is also significant, from about 0.1 to 0.35 for v in the range [-20,20].

The physical optics is then applied to calculate the scattered intensity according to the phase and the amplitude calculated by VCRM. Figure 1 (b) shows that the intensity predicted by our method (VCRM+PO) is in very good agreement with Debye theory (rigorous) while Airy peaks differ from Debye theory more and more with increase of the scattering angle and GO predict an infinite intensity in the rainbow angle ($\sim 138°$).

Furthermore, our method can be applied directly to particles of arbitrary shape with smooth surface.



(a) Phase and amplitude on the virtual line.



(b) Scattering diagrams near the rainbow angle.

Figure 1: A plane wave of wavelength $\lambda = 0.6328$ µm is scattered by a spherical droplet of radius $a = 100$ µm and refractive index $m = 1.333$ for the order of ray $p = 2$. The incident wave is polarized perpendicularly to scattering plane.

References

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