Optical diagnostics in fluid mechanics

Metrology of particles

Part 2: Theories

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Theories of light scattering

- **Approximate models and theories**
  - **Geometric optics → VCRM**
  - **Diffraction**
  - **Ray theory of waves**
  - **Rayleigh theory**
  - **… …**
- **Rigorous theories**
  - **Lorenz-Mie theory**
  - **Generalized Lorenz-Mie theory**
- **Numerical methods**
  - **T-matrix, DDA**
  - **MoM, FTDT, FEM, …**

Geometric optics

What is it used for ?

- **Imaging in daily life:**
  - Image by reflection: mirror in car, water surface, summer route …
  - Image by refraction: fish, stick in water, mirage in the nature …
- **Optical instruments:**
  - Camera, telescope, microscope, ..
  - Industrial measurement systems,
  - Optical fiber for telecommunication
- **Scientific research:**
  - Fluid mechanics: PIV, holography, LDV, PDA,
  - Measurement of temperature, size distribution, …
  - Biological imaging, …

Advantages:

- Simple
- Object of arbitrary shape

Geometric optics

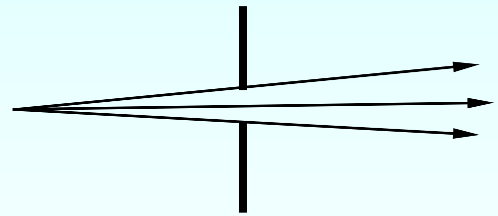
Condition and simple applications

Conditions :

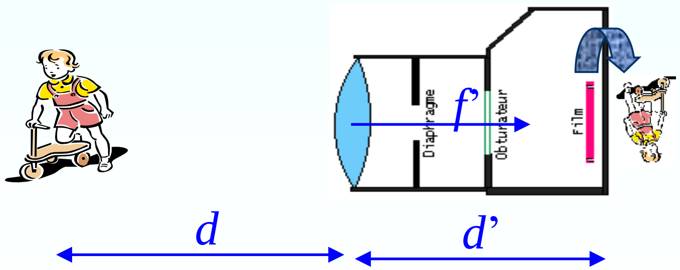
$$\lambda \ll l$$

The wave length* λ *is much longer than the dimension of the object* l*.

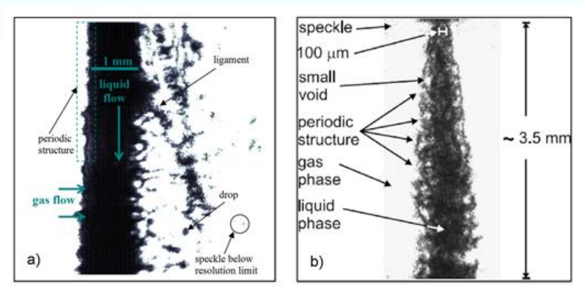
1. **Straight propagation in a homogeneous medium**



Photography



Imaging



Geometric optics – reflection and refraction

Reflection and refraction laws

Reflection and refraction on a surface between two media

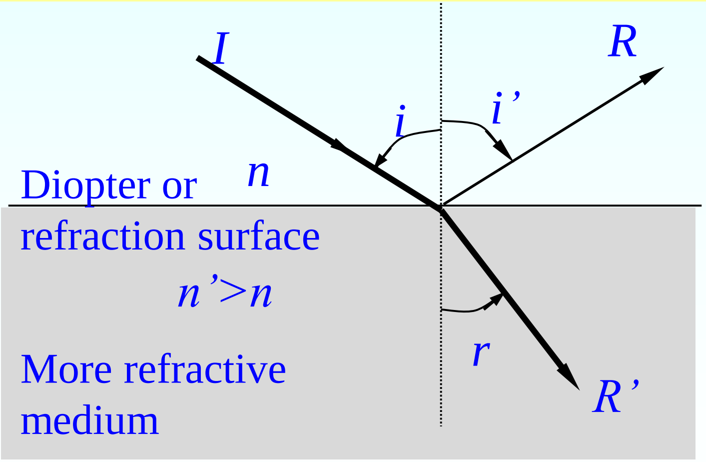
Law of Snell-Descartes

Reflection: $\boxed{i = i'}$

Refraction: $\boxed{n \sin i = n' \sin r}$

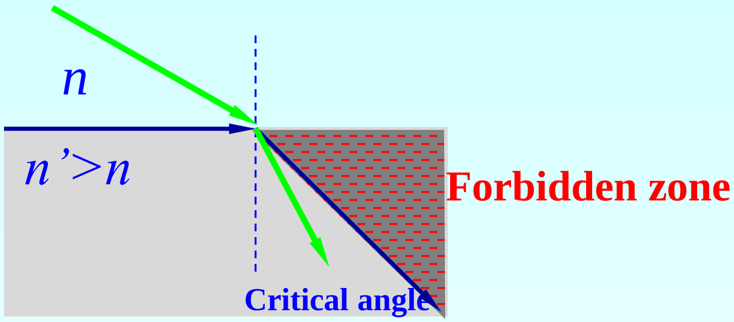
The velocity of light v depends on the refractive index n of the medium:

$$\boxed{n = \frac{c}{v}}$$



Geometric optics – reflection and refraction

Total reflection

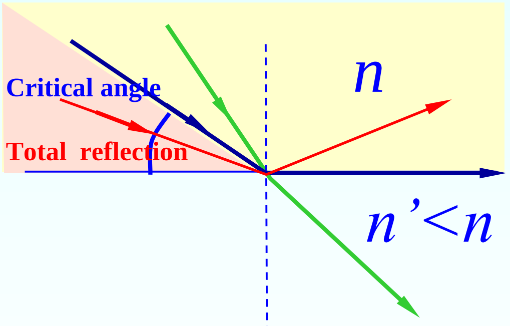


Critical angle:

$$i_l = \arcsin\left(\frac{n'}{n}\right)$$

Total reflection :

$$i \geq i_l$$



Examples :

$n_2 / n_1 = 1/1.333 \quad i_l = 48.6°$

$n_2 / n_1 = 1/1.500 \quad i_l = 41.8°$

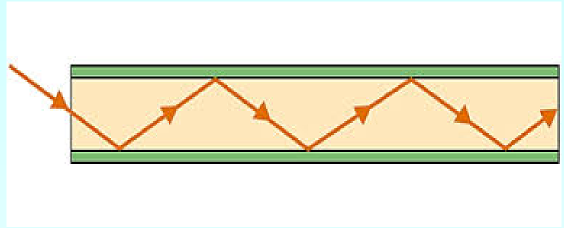
Applications

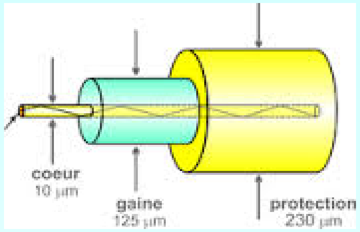
- Optical fiber
  - Telecommunication
  - Transport de laser beam
- Critical angle for measuring bubbles
- Optical gauge
- Natural mirage

Geometric optics – reflection and refraction

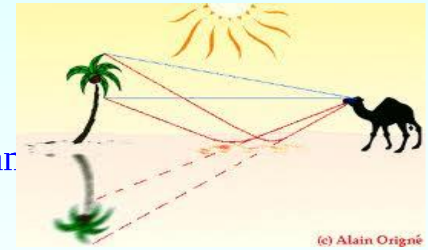
Total reflection - applications

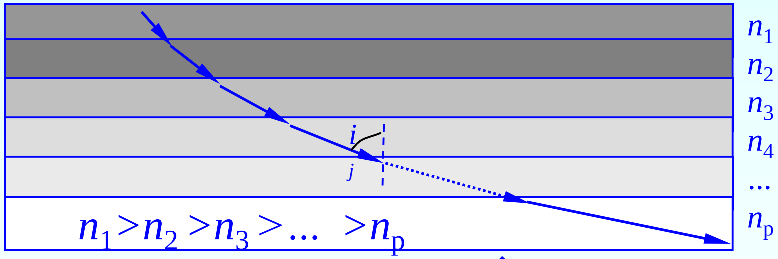
Application 1:
Optical fiber





Application 2:
Effect of mirage

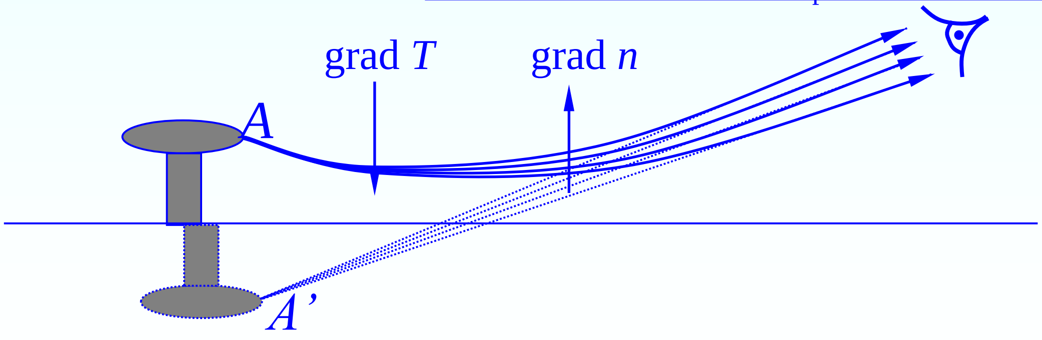




Propagation of a light bear
in a parallel stratified
environment:

$$n_j \sin i_j = C$$

$n_i \downarrow$ until i_j=90°,
i.e. total reflection.



Geometric optics – reflection and refraction

Relation of amplitudes and intensities

Fresnel formulas – relation between the amplitudes of the reflected/refracted and incident waves

$$r_X = \frac{E_X^r}{E_X^i}, \qquad t_X = \frac{E_X^t}{E_X^i}$$

$X = \perp$ ou $\parallel$ polarisation state

E_X^i: amplitude of incidente wave

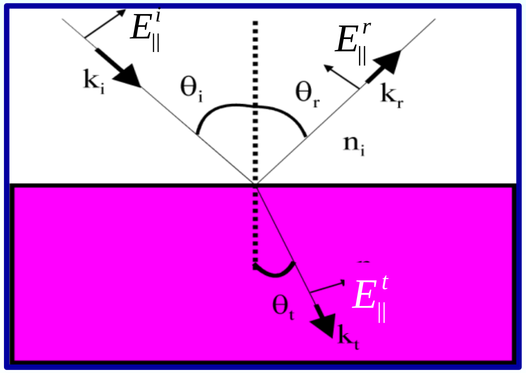
E_X^r, E_X^r amplitudes of reflected/refracted waves

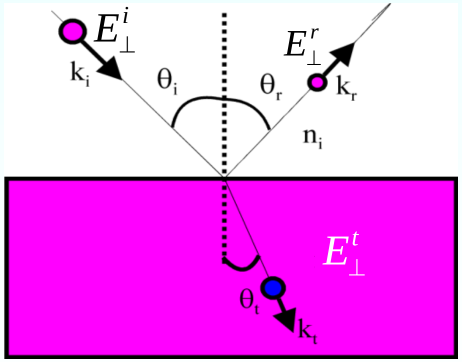
$$r_{//} \equiv \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$r_{\perp} \equiv \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$t_{//} \equiv \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

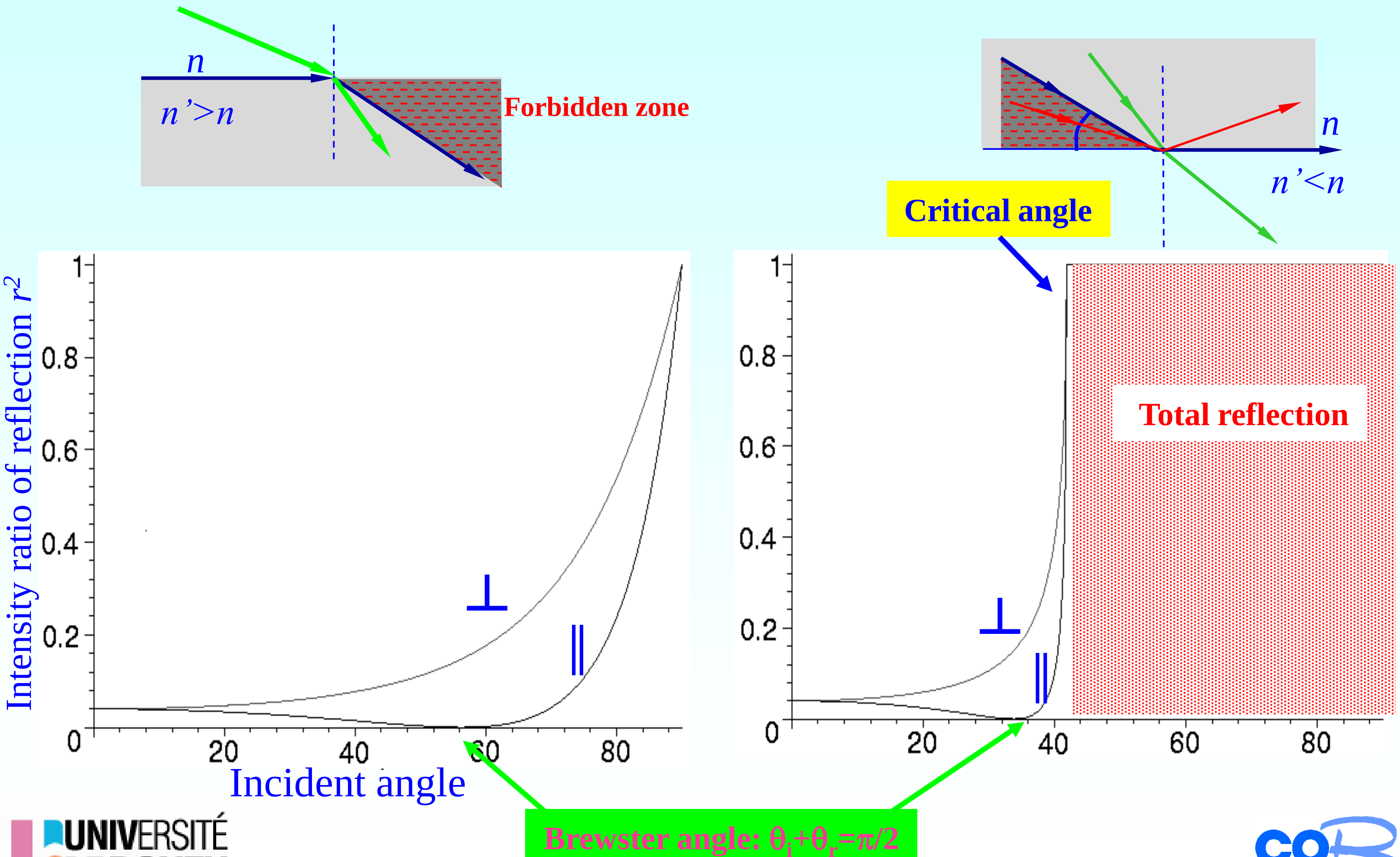
$$t_{\perp} \equiv \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$





Geometric optics – reflection and refraction

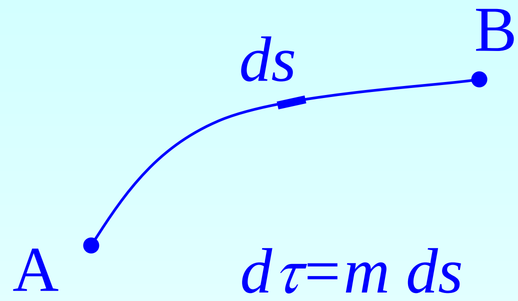
Relation of amplitudes and intensities



Geometric optics – propagation of light in a medium

optical path, phase and intensity

Propagation of light in a medium:



Optical path:

$$\tau(AB) = \int_A^B m(s)ds$$

Complex refractive index:

$$\tilde{m} = m_r - im_i$$

- **Phase difference :** $\Delta\varphi(AB) = k\tau_r = km_r\Delta S$
- **Absorption:** $I(B) = I(A)\exp(-2k\tau_i) = I(A)\exp(-2km_i\Delta S)$
- **If m is constant:** $\boxed{\Delta\phi = km_r\Delta S}$

$$\boxed{I = I_0\exp(-2km_i\Delta S)}$$

Geometric optics – application to light scattering

Can GO be applied to light scattering ?

YES to the simple particles:

- Homogeneous sphere,
- Homogeneous circular cylinder

Possible for objects of complex shape:

- Pure ray model, precision is very limited
- Ray model + electromagnetic integration, much better
- Ray model + wave properties → **Vectorial Complex Ray Model**

→ **New concept, very precise and easy to use.**

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Geometric optics – application to light scattering

Reflection and refraction in a particle

Light scattering by a sphere:

important

- **Intensity :**

$$\epsilon_X = r_X^2 \qquad p = 0$$
$$\epsilon_X = r_X^{2(p-1)}(1 - r_X^2)^2 \qquad p \geq 1$$

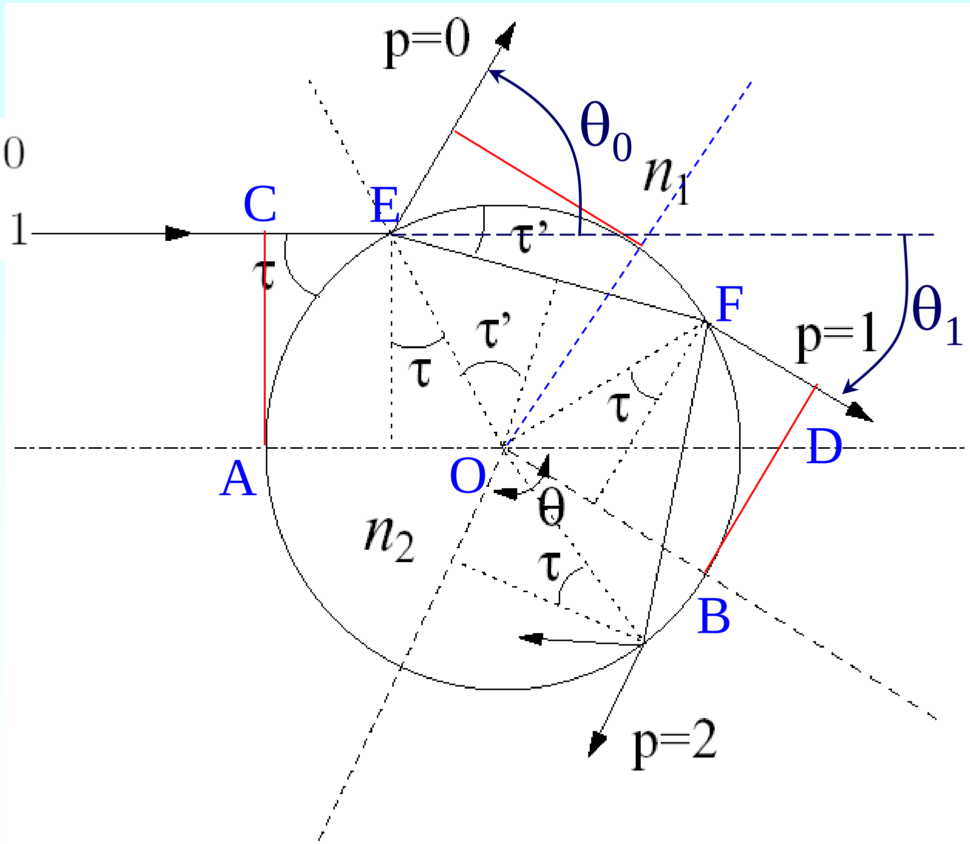
where p is the scattering order,
X represents the polarization sate ($\perp$ or $\parallel$), r_X Fresnel coefficients.

- **Length of the path in the particle:**

$$\Delta_p = 2pa\sin\tau'$$

- **Deviation angle de :**

$$\theta_p = 2\tau - 2p\tau'$$



Geometric optics – application to light scattering

Reflection and refraction in a particle

important

➢Phases:

- Phase difference due to the difference of the optical paths :

$$\Delta\phi = \frac{2\pi d}{\lambda}\left|(\sin\tau - pm\sin\tau')\right)$$

- Jump of phase due to reflection on the surface and the focal lines.

1) Reflection: phase of the complex number: $r_{X,}$
2) Focal lines: phase jump π/2 at each focal line.

➢ Divergence factor :

$$D_p = \frac{\cos\theta_i \sin\tau}{\sin\theta_p \left|\frac{d\theta_p}{d\tau}\right|}$$

Summary
These equations are enough to calculate the intensity of each order and the total field.

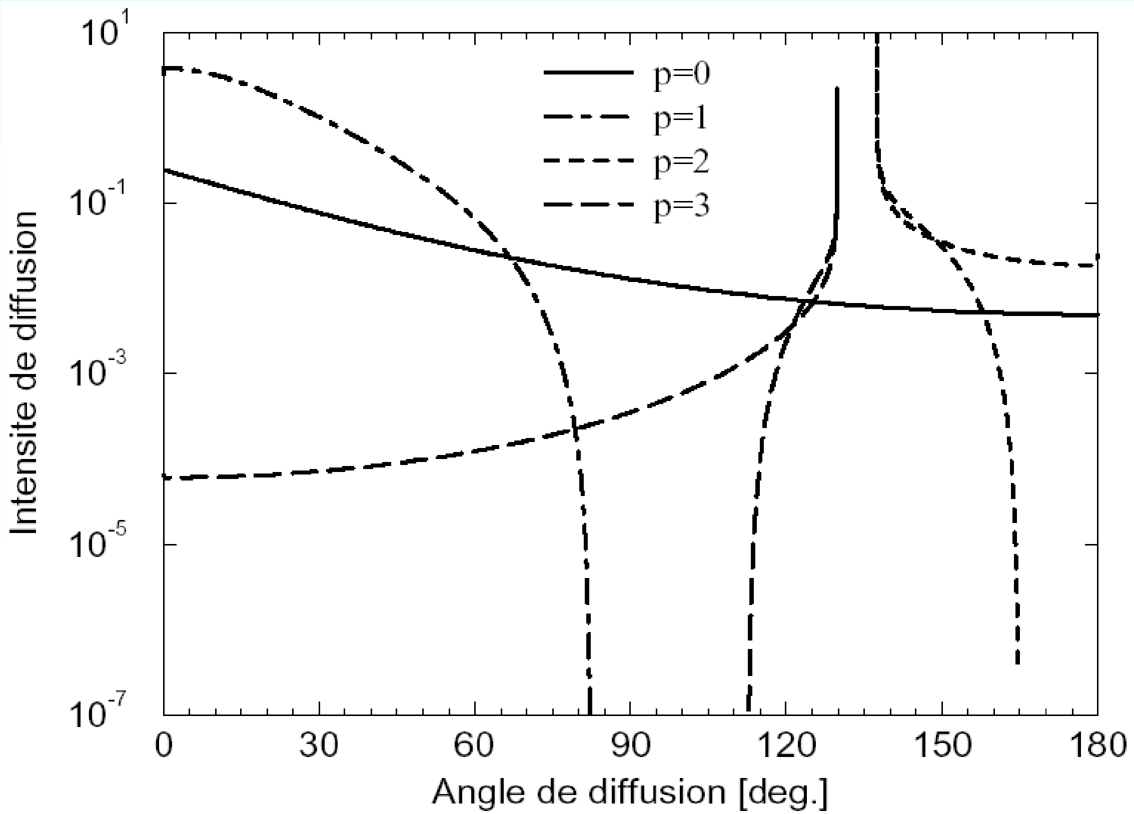
$$I_{s,p} = \frac{I_0 \varepsilon_X dS_i}{dS_s} = \frac{I_0 \varepsilon_X a^2 \cos\tau \sin\tau d\tau d\varphi}{r^2 \sin\theta_p d\theta_p d\varphi} = \frac{a^2}{r^2} I_0 \varepsilon_X D_p$$

$$\tilde{A}_{s,t} = \sum_{p=0}^{N} \sqrt{I_{s,p}}\, e^{-i\Delta\phi_P}$$

Geometric optics – application to light scattering

Scattering diagram according to geometrical optics

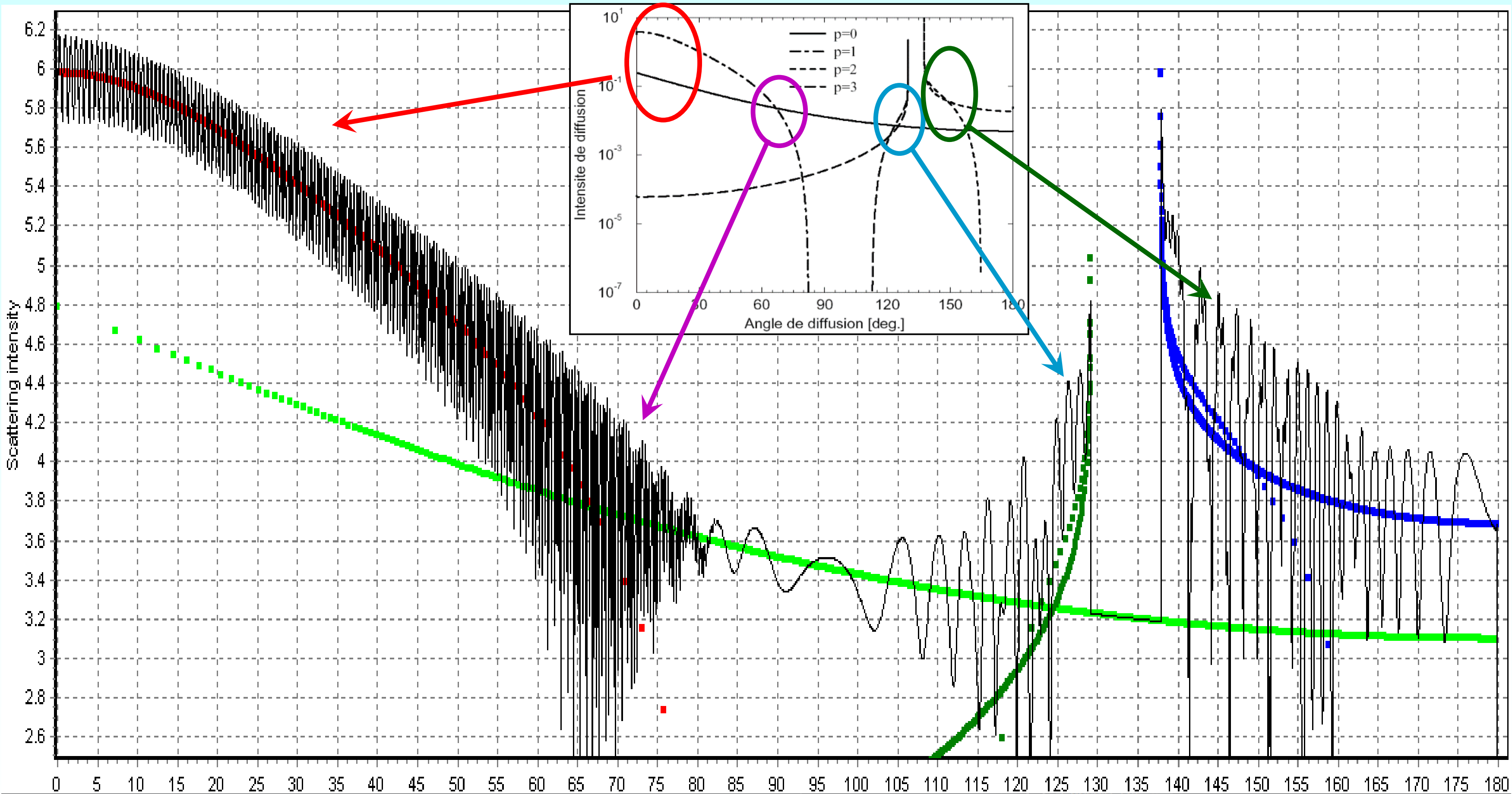
Without interference between different modes



- **Sphere of water**
- **Polarization $\perp$**
- **Intensity $\rightarrow \infty$ at rainbow angle**

Geometric optics – application to light scattering

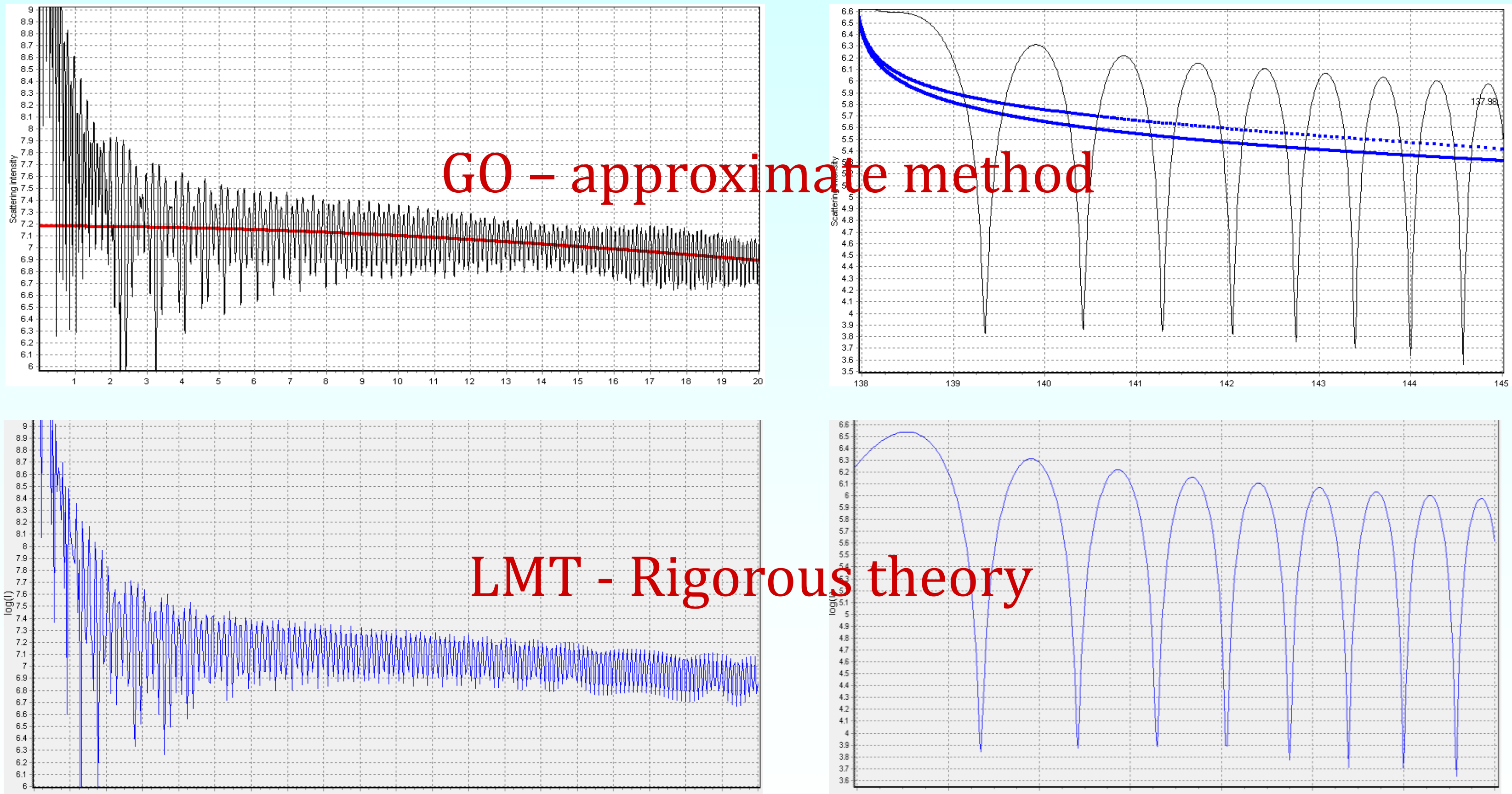
Scattering diagram of a water droplet: a=50µm, λ=0.6328, m=1.333



Geometric optics – application to light scattering

Scattering diagram of a water droplet: a=200µm, λ=0.6328, m=1.333

Forward direction　　　　First rainbow



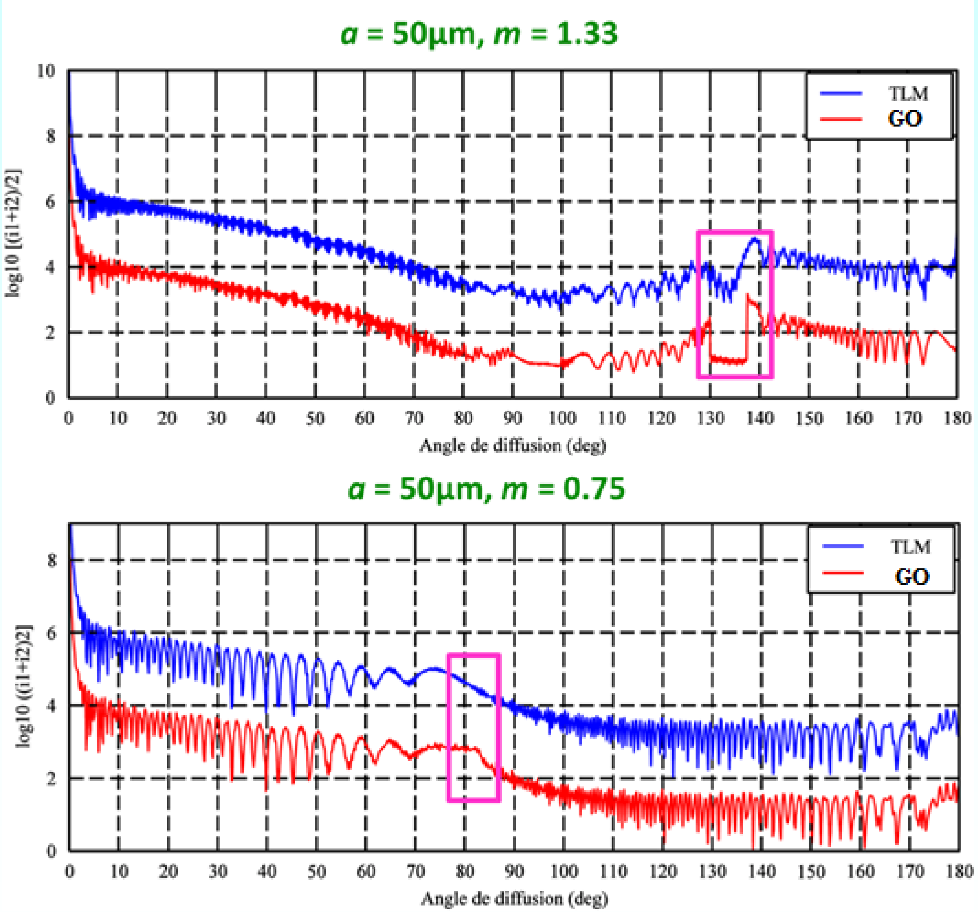
Geometric optics – application to light scattering

Comparison with rigorous theory

A homogeneous sphere

Total intensity with interference:

A sphere of water or a air bubble in the water illuminated by a plane wave of wavelength of 0.6328 µm.



Geometric optics – application to light scattering

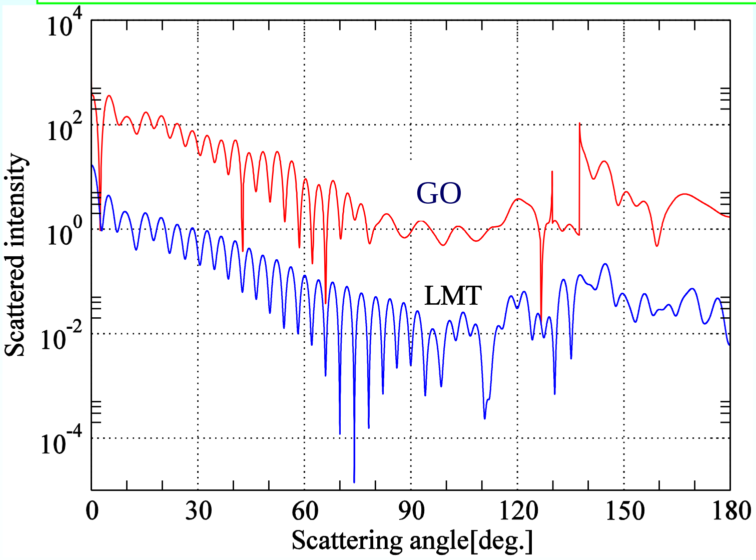
Comparison with rigorous theory

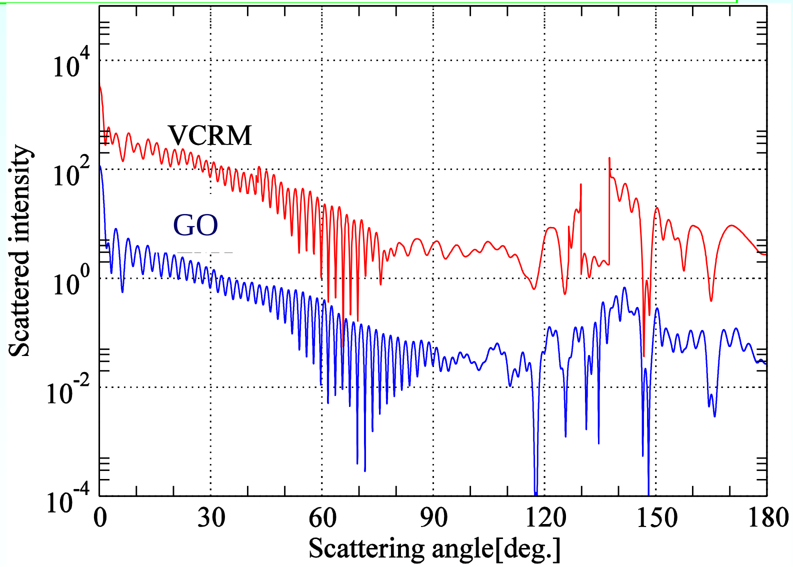
A homogeneous infinite cylinder

Total intensity with interference

A circular cylinder of water with a radius of 5 µm (left) or 10 µm (right) illuminated by a plane wave with a wavelength of 0.6328 µm.

Geometric optics still work very well for *a* ~ 10λ.

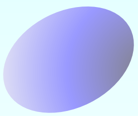




Geometric optics – application to light scattering

Preliminary conclusions

- By taking into account correctly the interferences, the Ray model can predict the scattering diagram in **ALL directions**.
- It can be applied to the scattering of **any shaped beam**.
- It works also for a circular infinite **cylinder**.



Limitations

NOT appropriate for a spheroid or an ellipsoid
NO for any irregular shaped particles.

The key problem is **the divergence factor.**

How to improve the model ?

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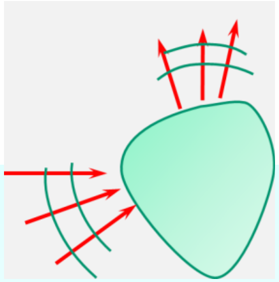
Geometric optics → VCRM

Possible candidate?

- ✗ ~~**Rigorous theories:**~~ the particle shape must correspond to a coordinates system
- ✗ ~~**Numerical methods:**~~ size limited, very time communing
- ✓ **Ray models: precision to be improved**

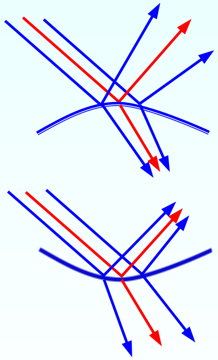


Key problem: lack of wave properties

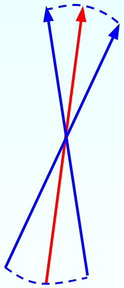
Our strategy: Extension of ray model

- **Inclusion of wave front curvature**
- Interference between all the rays.
- Diffraction.

Vectorial Complex Ray Model



Divergence/ Convergence



Phase in focal lines

Vectorial Complex Ray Model

Geometrical optics + wave form

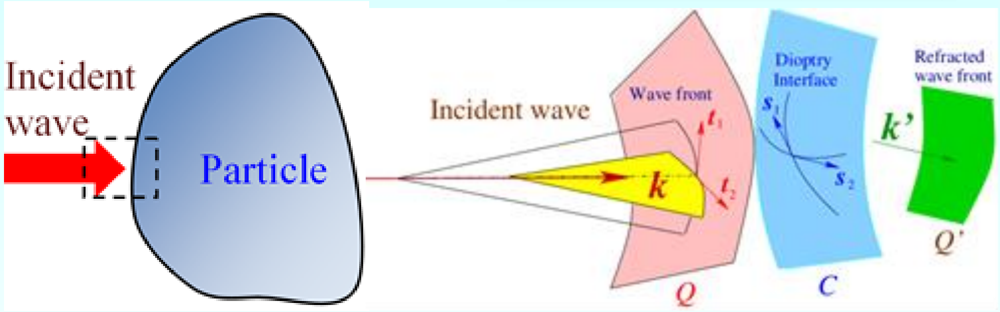
- **Vectorial Complex Ray Model – new**

For details:
- Ren et al, *Opt. Lett.* 36(3), 2011
- http://www.amocops.eu

important

✓ **5 properties of a ray:**

Classical optics
1. direction,
2. amplitude,
3. phase,
4. polarization



New **5. Wave front curvature**

✓ **Advantages:**
- Objects of any shape with smooth surface,
- Incident wave of any form,
- Sufficiently precise – scattering in all directions,
- All scattering properties of the objet.

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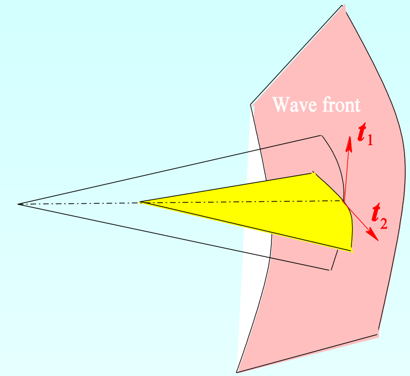
Vectorial Complex Ray Model

Essential of VCRM

- **Vectorial complex Ray :**

$$\boxed{\vec{S}(\hat{\boldsymbol{k}})_s = \sum_{p=0}^{N}\left(A_p^{\perp} e^{-i\Phi_p^{\perp}}\hat{\boldsymbol{e}}_{\perp} + A_p^{\parallel} e^{-i\Phi_p^{\parallel}}\hat{\boldsymbol{e}}_{\parallel}\right)}$$



- **Wave front – curvature matrix:**

$$Q = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$$

Examples :

Sphere : $C = \begin{pmatrix} \frac{1}{r} & 0 \\ 0 & \frac{1}{r} \end{pmatrix}$

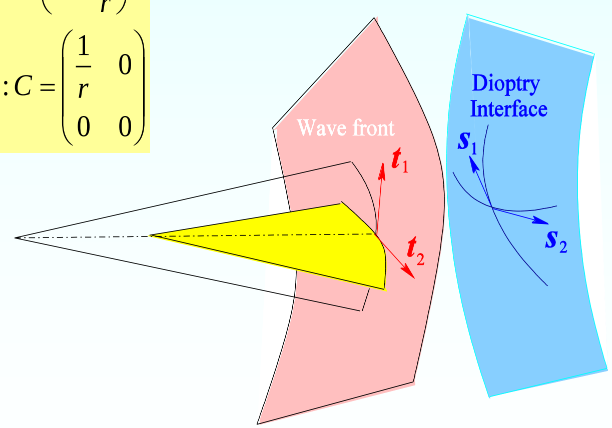
Cylinder : $C = \begin{pmatrix} \frac{1}{r} & 0 \\ 0 & 0 \end{pmatrix}$

- **Surface of dioptry:**

$$C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

- **Projection matrix:**

$$\Theta = \begin{pmatrix} \hat{t}_1 \cdot \hat{s}_1 & \hat{t}_1 \cdot \hat{s}_2 \\ \hat{t}_2 \cdot \hat{s}_1 & \hat{t}_2 \cdot \hat{s}_2 \end{pmatrix}$$



Vectorial Complex Ray Model

Special case of the wave front equation

The rays remain in the same plane – a main direction of the wave front and the particle surface:

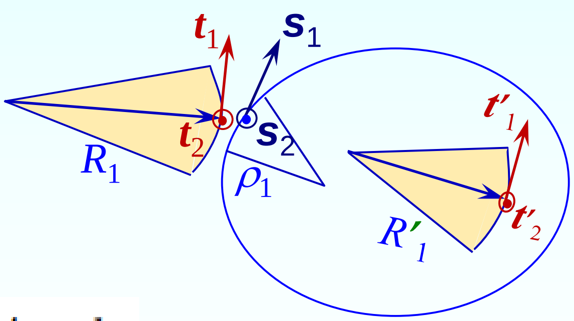
- Spherical particle
- Infinite cylinder at normal incidence
- Ellipsoidal particle in the symmetric plane.

➢ **Curvature matrix:**

$$C = \begin{pmatrix} \frac{1}{\rho_1} & 0 \\ 0 & \frac{1}{\rho_2} \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{1}{R_1} & 0 \\ 0 & \frac{1}{R_2} \end{pmatrix}$$

$$Q' = \begin{pmatrix} \frac{1}{R'_1} & 0 \\ 0 & \frac{1}{R'_2} \end{pmatrix}$$



➢ **Wave front equation:**

$$\frac{{k'_n}^2}{k' R'_1} = \frac{k_n^2}{k R_1} + \frac{k'_n - k_n}{\rho_1}$$

$$\frac{k'}{R'_2} = \frac{k}{R_2} + \frac{k'_n - k_n}{\rho_2}$$

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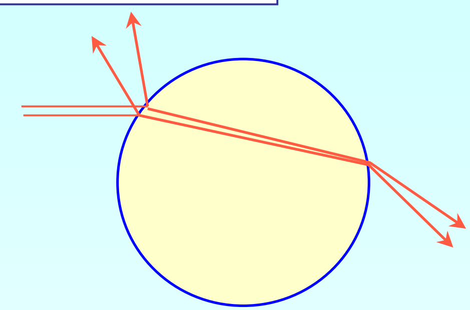
Vectorial Complex Ray Model

Theoretical Validation – divergence factor



- **Sphere**
  - Reflection: $R_1 = -\dfrac{a\cos\alpha}{2} \quad R_2 = -\dfrac{2}{a\cos\alpha} \rightarrow D = \dfrac{1}{4}$
  - Refraction p =1:

After 1[st] refraction: $R'_{11} = -\dfrac{am\cos^2\beta}{m\cos\beta - \cos\alpha} \quad R'_{12} = -\dfrac{am}{m\cos\beta - \cos\alpha}$

After 2[nd] refraction: $R'_{21} = \dfrac{m\cos\beta - 2\cos\alpha}{2(m\cos\beta - \cos\alpha)}\cos\alpha \quad R'_{22} = \dfrac{2a\cos\beta(m\cos\beta - \cos\alpha) - m}{2(m\cos\beta - \cos\alpha)(\sin\alpha\sin\beta - \sin\alpha\sin\beta)}$

Divergence factor: $D = \dfrac{m\sin(2\alpha)\cos\beta}{4\sin[2(\beta-\alpha)](\cos\alpha - m\cos\beta)}$

- **Cylinder:** $R_2 = \infty$
  - Reflection : $D = \dfrac{a\cos\alpha}{2}$
  - Refraction p=1: $D = \dfrac{m\cos\alpha\cos\beta}{2(\cos\alpha - m\cos\beta)}$

Identical to the classical formula (page II-13).

Vectorial Complex Ray Model

Basic laws of geometrical optics

1. Snell-Descartes law:

Reflection: $\boxed{i = i'}$

Refraction: $\boxed{n \sin i = n' \sin r}$

2. Fresnel's Equations :

$$r_{//} \equiv \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$t_{//} \equiv \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{2\sin\theta_i \cos\theta_t}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

$$r_{\perp} \equiv \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$t_{\perp} \equiv \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{2\sin\theta_i \cos\theta_t}{\sin(\theta_i + \theta_t)}$$

VCRM

The tangent component of wave vector is continuous

$$\boxed{k_\tau = k'_\tau}$$

$$r_\perp = \frac{k_{in} - k_{rn}}{k_{in} + k_{rn}}$$

$$t_\perp = \frac{2k_{in}}{k_{in} + k_{rn}}$$

$$r_\parallel = \frac{m^2 k_{in} - k_{rn}}{m^2 k_{in} + k_{rn}}$$

$$t_\parallel = \frac{2mk_{in}}{m^2 k_{in} + k_{rn}}$$

Vectorial Complex Ray Model

Summary of VCRM

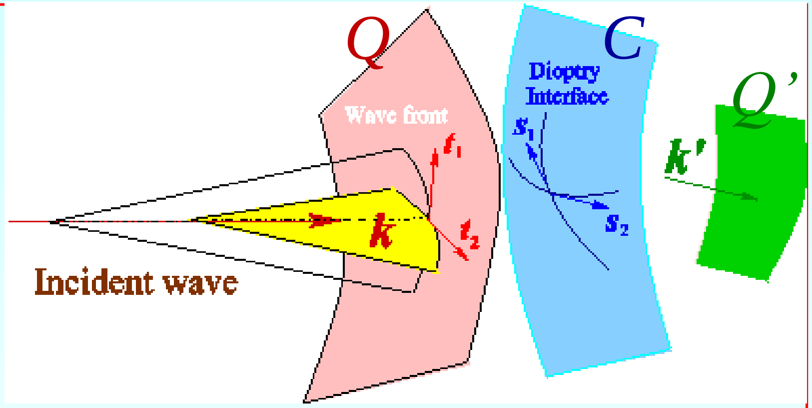
- **Fundamental laws**

1. Wave front equation:

$$\left(k_n^i - k_n^t\right) C = k' \Theta'^T Q' \Theta' - k \Theta^T Q \Theta$$

2. Law of Snell-Descartes **in vectors**:

$$k_\tau^i = k_\tau^t$$



- Amplitude:

$$A = \sqrt{D}\varepsilon$$

- Phase:

$$\Phi = \Phi_{inc} + \Phi_{fl} + \Phi_{path} + \Phi_\varepsilon$$

- Total field:

$$E = S_{diff} + \sum_{i=1}^{N} S_i$$

Divergence factor:

$$D = R'_{11} R'_{21} \frac{R'_{12} R'_{22}}{R_{12} R_{22}} \cdots \frac{R'_{1q} R'_{2q}}{R_{1q} R_{2q}}$$

Fresnel coefficients: ε

$$\tilde{r}_\perp = \frac{k_n - \tilde{k}'_n}{k_n + \tilde{k}'_n} \qquad \tilde{r}_\parallel = \frac{\tilde{m}^2 k_n - \tilde{k}'_n}{\tilde{m}^2 k_n + \tilde{k}'_n}$$

All expressed in wave vector components.

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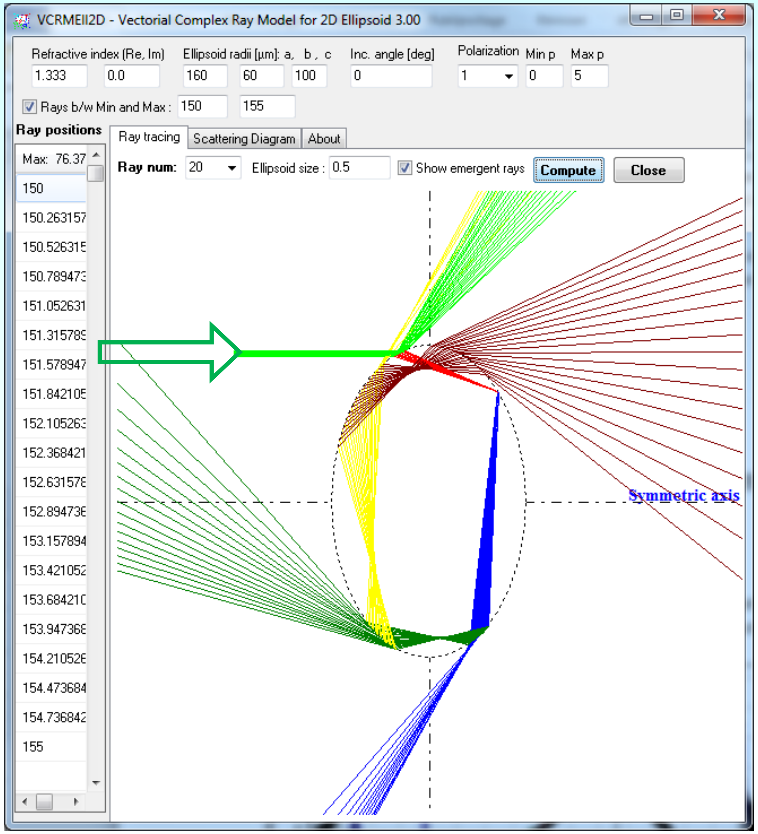
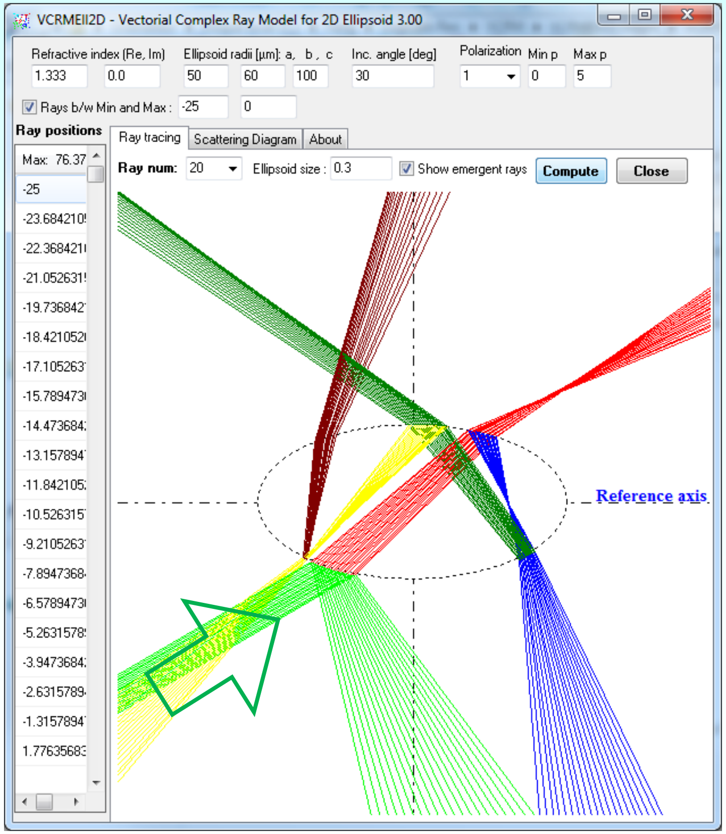
Vectorial Complex Ray Model

Available at **www.amocops.eu**

VCRMEll2D : Vectorial Complex Ray Model (VCRM) for Scattering of plane wave by an Elliptical particle in the 2D plane

1. Module for ray tracing





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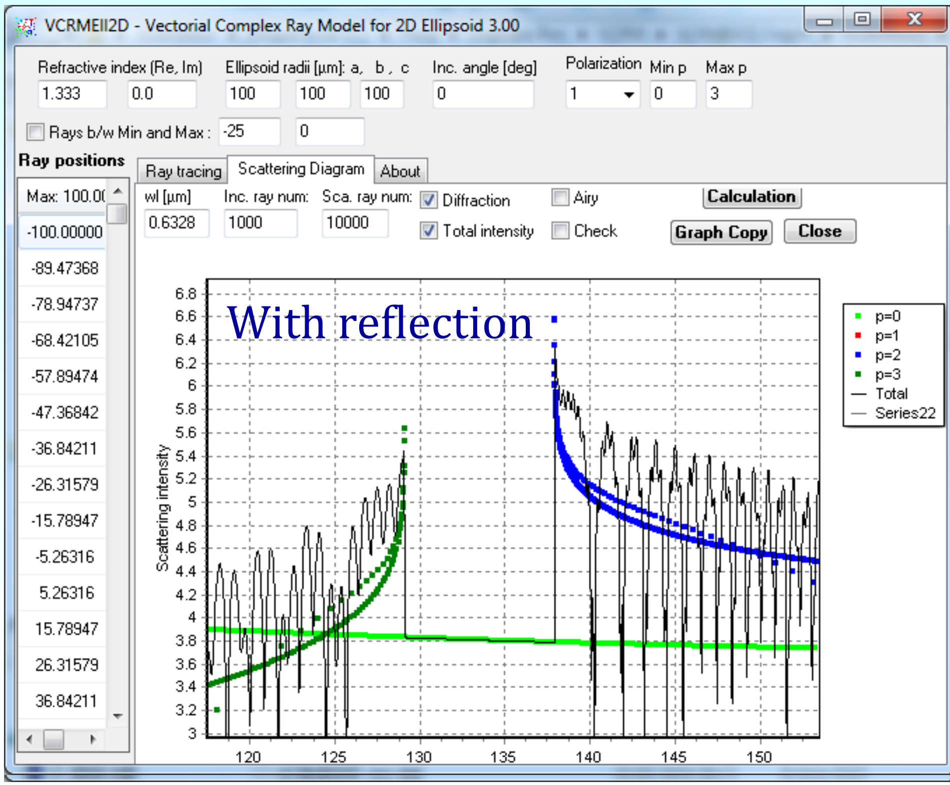
VCRMEll2D : Vectorial Complex Ray Model (VCRM) for Scattering of plane wave by an Elliptical particle in the 2D plane

2. Module for scattering diagrams



Raw data in VCRMEll2D_raw.dat

Input parameters:

a= 50.0000, b= 60.0000, c= 100.0000, theta0[deg]= 30.0000

m= 1.3330 + 0.0000i, wl= 0.63280, ki= 9.9292, kr= 13.2356

Ray n°: 0

Incident (x,z): -32.7327 75.5929, eki0: 0.5000 0.8660

Order p : 0

Ce: -1.158E-002 -1.837E-002, Ci: 1.000E-014 1.000E-014

Cl: -1.873E+005 -4.541E-009, Cr: 1.751E-002 1.215E-002

ekl: 0.50000 0.86603, ekr: 0.94773 0.31907

kint: 0.00000 9.92918, krnt: 8.75171 9.92918, rfr: -1.00000 0.0

Amplitude: 3.404E+002 Phase: 9.782E+002

Order p : 1

(x,z): 18.5546 92.8596, ekl: -0.10234 -0.99475

ekr: 0.99588 0.09070, phase:-228.7430

Ce: 2.381E-002 2.337E-002, Ci: -3.324E-001 -3.547E-002

Cl: 3.890E-001 7.477E-002, Cr: -6.744E-001 -5.729E-002

kint: 6.87711 -7.16198, krnt: 11.13046 -7.16198, rfr: 0.23620 0.0

Amplitude: 2.737E-001 Phase: 4.150E+001

Order p : 2

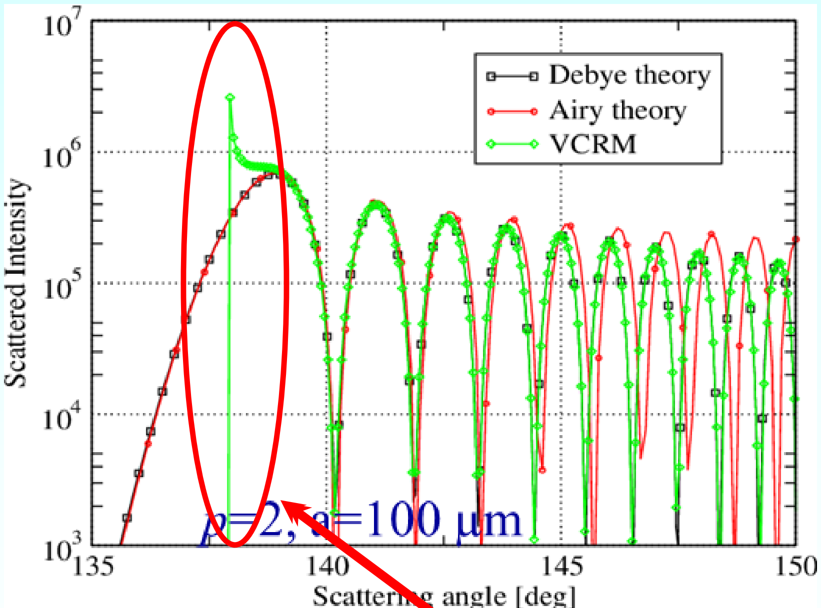
(x,z): -1.2843 -99.9670, ekl: 0.00016 1.00000

… …

Vectorial Complex Ray Model

Toward the ***Ray theory of wave***

VCRM can predict much better Airy structure than the Airy theory and can be applied directly to non-spherical particle.



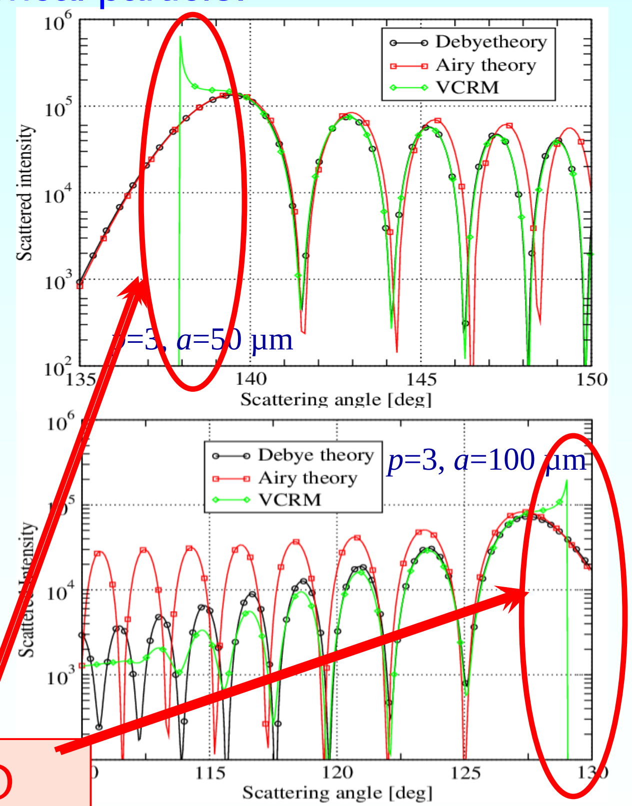


Fig. 3 Comparison of Airy structure calculated with the three methods.

Caustics of GO to be corrected !

Vectorial Complex Ray Model

Experimental set-up results

Oblate:

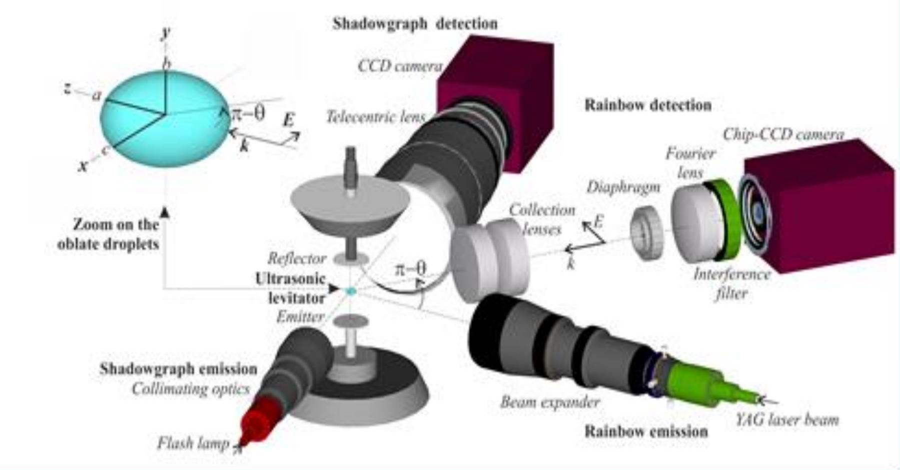
$a = c = 139.17$ µm

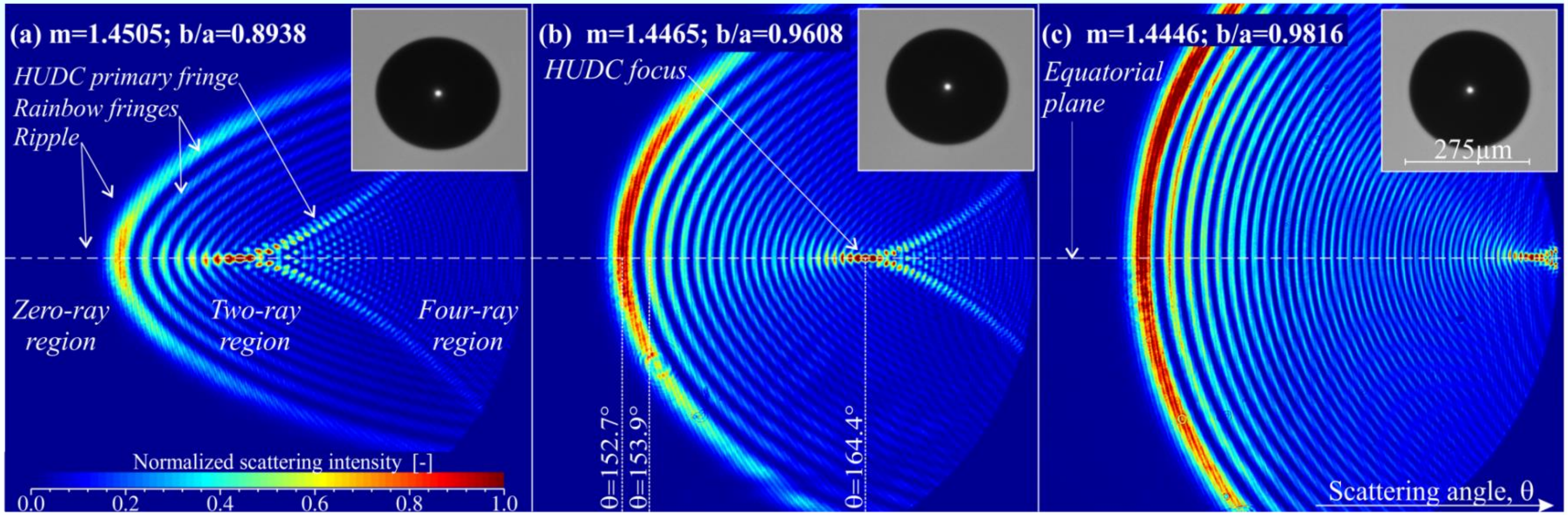
$b = 133.86$ µm,

a/b = *0.8938*, 0.9608, *0.9816 (eq. Vol)*

$m = 1.4465$

$\lambda = 0.6328$ µm





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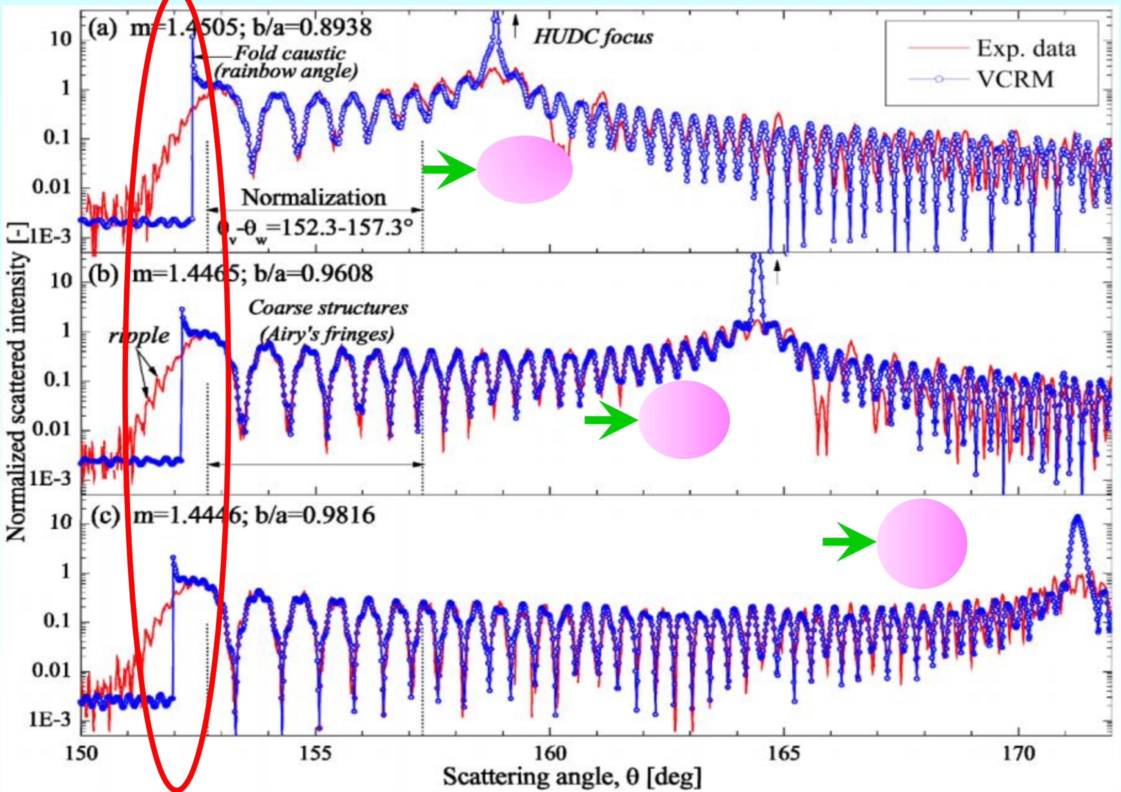
Comparison with VCRM

Fig. 3 in *Opt. Exp.* 2015
Onofri, Ren *et al*

<u>DEHS:</u>
Di-Ethyl-Hexyl-Sebacat

<u>HUDC:</u>
Hyperbolic Umbilic Diffraction Catastrophe



Comparison of VCRM and experimental normalized equatorial scattering diagrams for the droplets of 3 different aspect ratios. From (a) to (c), the droplet's aspect ratio *b/a* increases and refractive index decreases when the amplitude of the acoustic field is reduced.

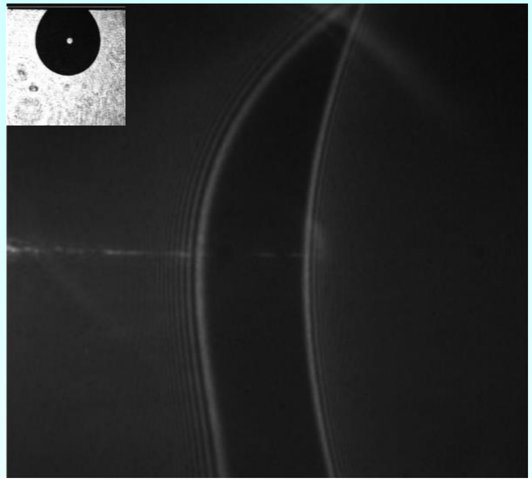
Caustics of GO to be corrected !

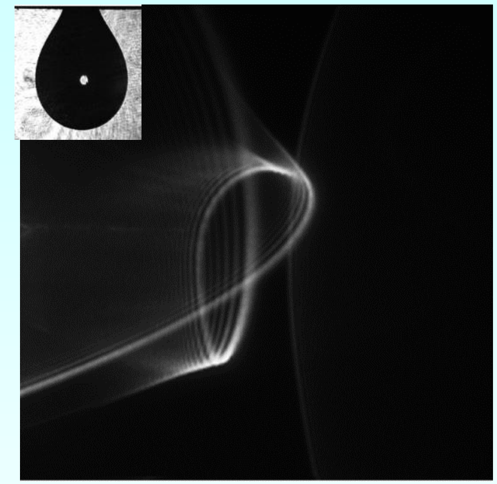
Vectorial Complex Ray Model

Scattering of a pendent droplet

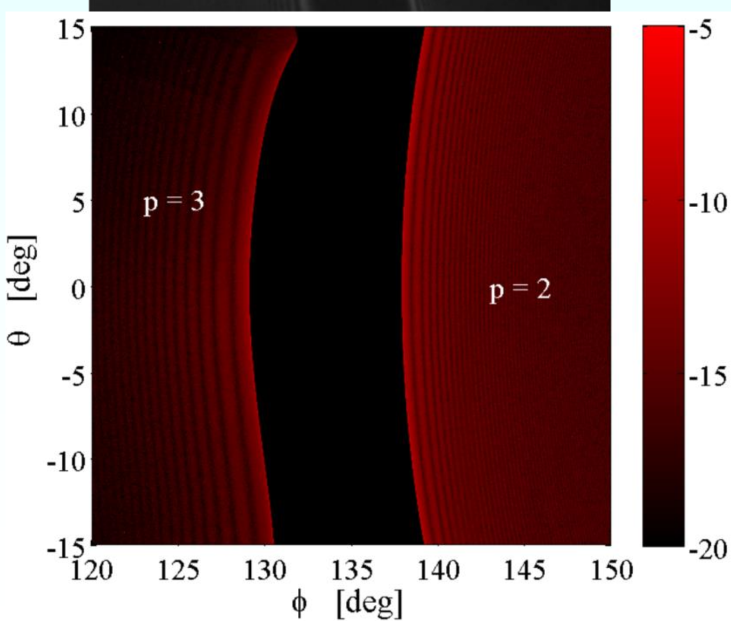
Scattering patterns around rainbow angles

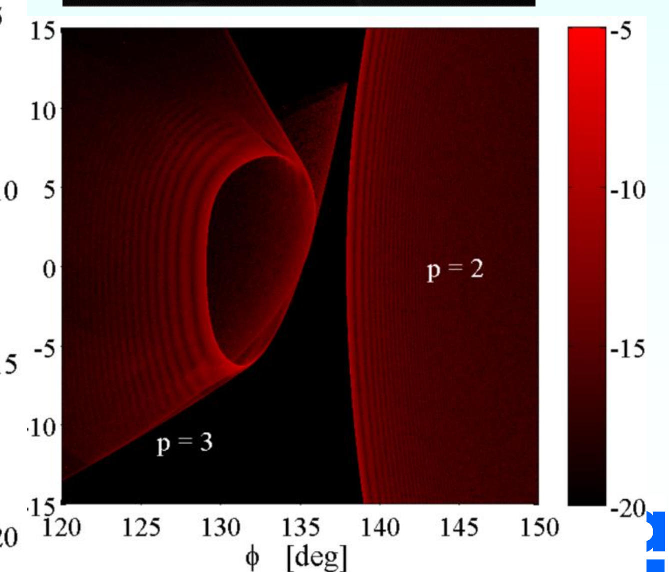
Experiment





Simulation with VCRM

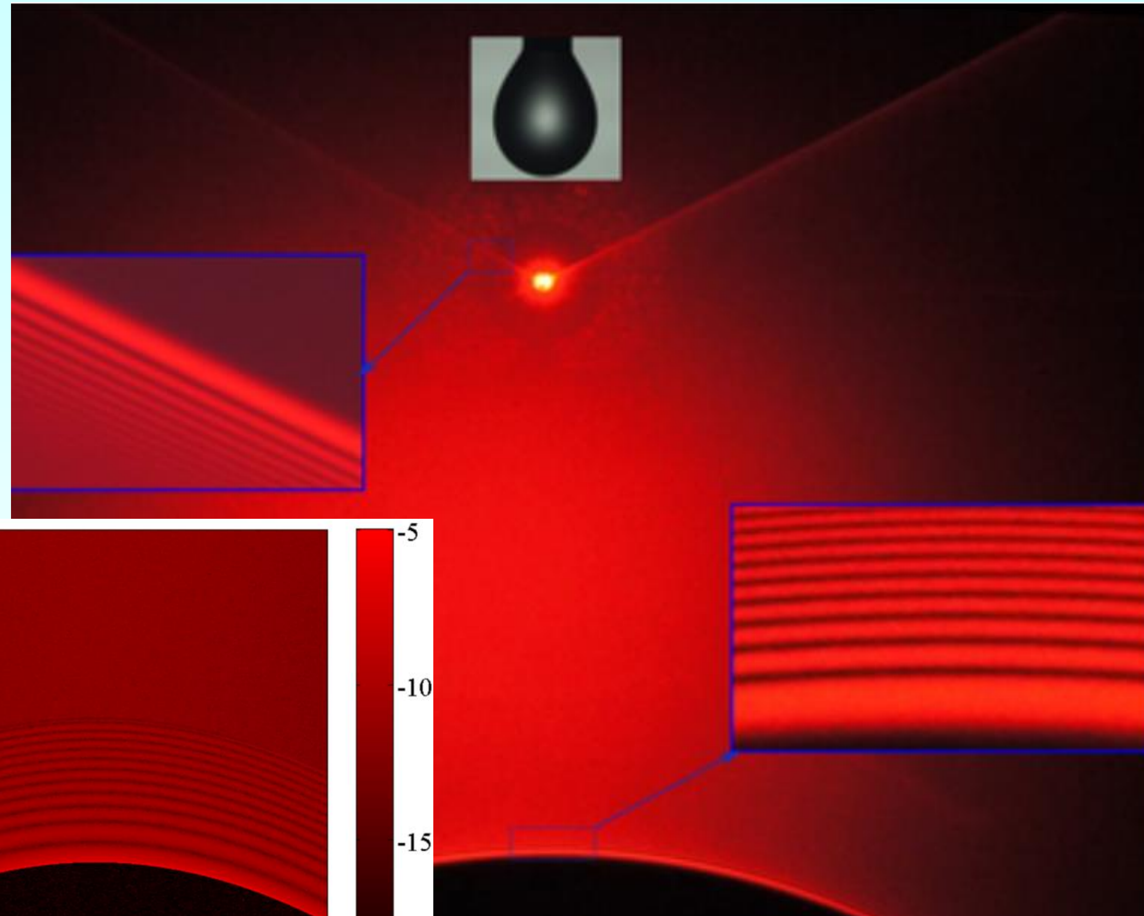




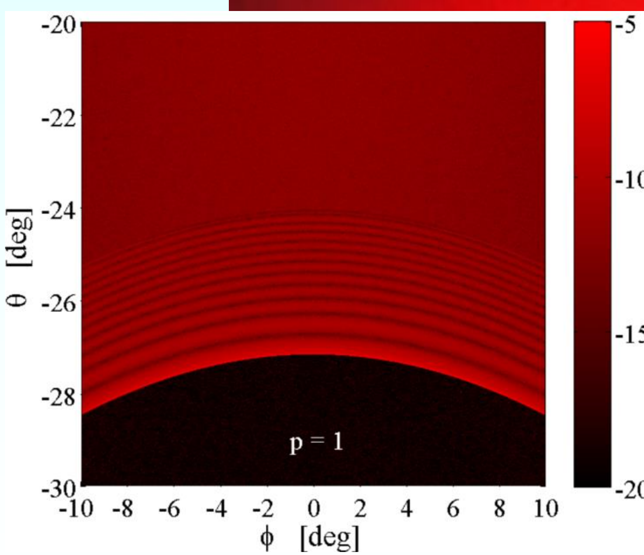
Vectorial Complex Ray Model

Scattering patterns in forward direction

Experiment ➡



Simulation with VCRM ➡



Vectorial Complex Ray Model

Application to Characterization of non-spherical droplets

- Morphology

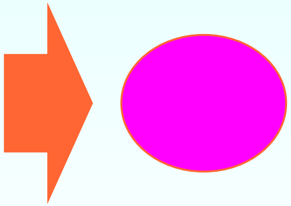
  (a, b)

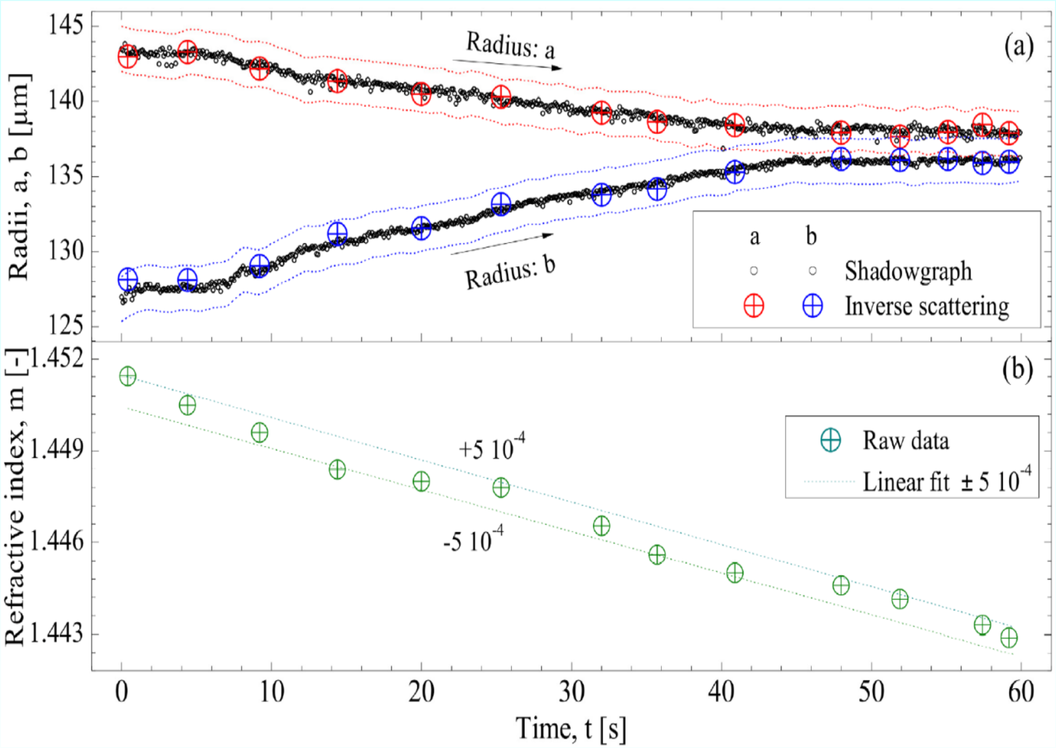
- Refractive index

  m

- Time evolution

  $a(t),\ b(t), m(t)$





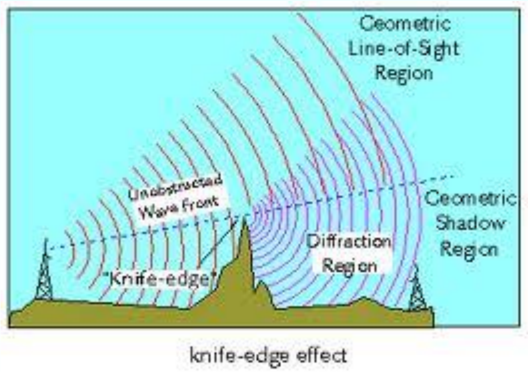
(a) Principal radii measured with the rainbow refractometer and imaging system and (b) corresponding evolution of the droplet refractive index during the course of the experiment.

Diffraction

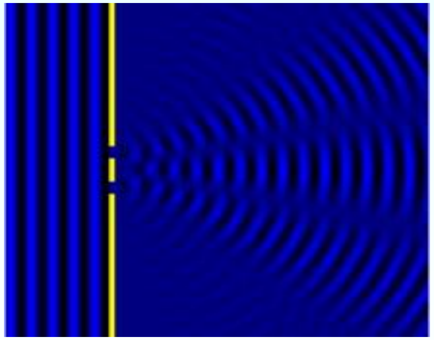
Phenomena of diffraction

The diffraction is the behavior of waves when they encounter an obstacle.

The phenomenon of diffraction is manifested when
- the dimension of the object is of the order of the wavelength,
- there is a brutal variation of the density / amplitude of the wave.



A wave diffracts around an obstacle

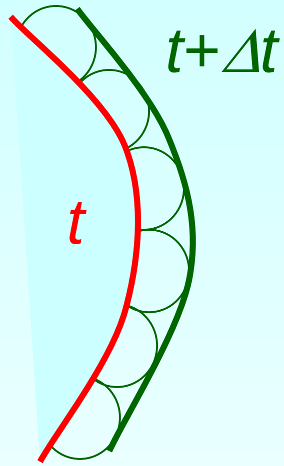


Diffraction by a double slit.

Diffraction effect in a rainbow.

Diffraction

Theory of diffraction



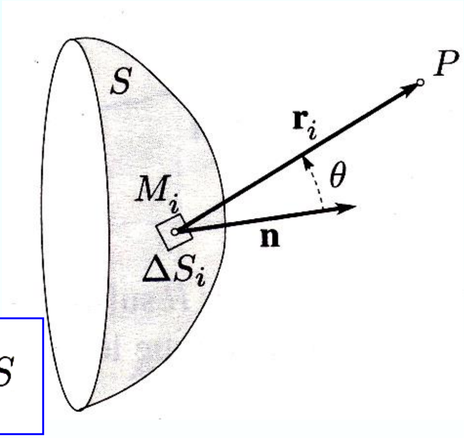
Principle of Huygens–Fresnel

(1) **The contribution of Huygens** (1678)

The wave of light propagates step. Each surface element reached by it behaves as a secondary source that emits spherical wavelets. The new ***wavefront is the envelop*** of the wavelets and the amplitude of each wavelet is proportional to the size of the element.

(2) **The contribution of Fresnel** (1818)

The ***complex amplitude*** of the wave at one point is the ***sum of the complex amplitudes*** of all the secondary sources at that point. All these waves interfere to form the wave at the considered point.



$$\underline{\psi}(P) = \sum_i \underline{\psi}_i(M_i)\, Q_i \frac{\exp(ikr_i)}{r_i} \Delta S_i$$

$$\underline{\psi}(P) = \int_S \underline{\psi}_0(M)\, Q \frac{\exp(ikr)}{r} \mathrm{d}S$$

Diffraction

Theory of diffraction

Let $\vec{R} = \overrightarrow{OM}, \vec{\rho} = \overrightarrow{OP}$, $\vec{r} = \overrightarrow{PM}$ $\quad r^2 = R^2 + \rho^2 - 2\ \vec{R}.\vec{\rho}$

$$r \cong R + \frac{x^2 + y^2}{2R} - \frac{\xi x + \eta y}{R}$$

$$\psi(M) = Ke^{ikR} \iint_S \psi(P)\, e^{-ik(\xi x + \eta y)/R}\, \boxed{e^{ik(x^2 + y^2)/2R}}\, dS$$

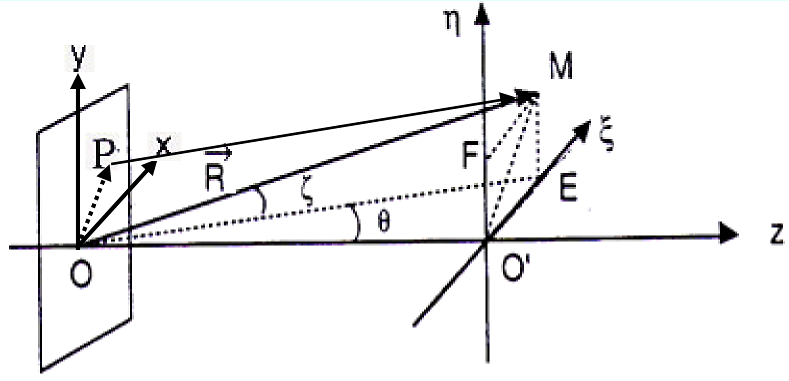
Fresnel diffraction (R is small):

$$\psi(M) = Ke^{ikR} \iint_S \psi(P) e^{ik(x^2 + y^2)/2R}\, dS$$

Fraunhofer diffraction ($R \to$ infinity):

$$\psi(M) = Ke^{ikR} \iint_S \psi(P)\, e^{-ik(\alpha x + \beta y)} dS$$

$$\alpha = \frac{\xi}{R} = \cos\zeta\sin\theta\ ;\ \beta = \frac{\eta}{R} = \sin\zeta\ ;\ \gamma = \cos\zeta\cos\theta$$



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Diffraction

Applications

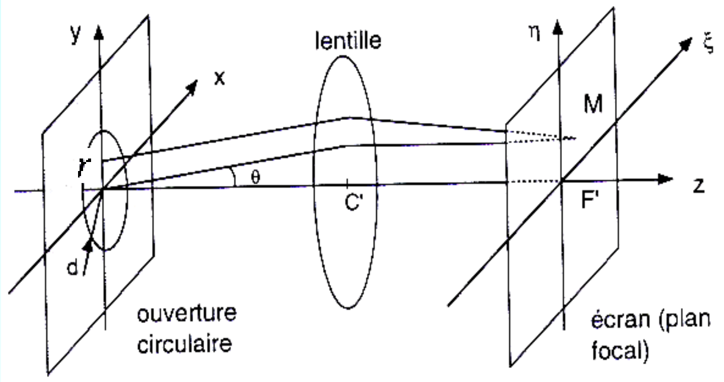
Diffraction by circular disk (sphere):

A ***circular hole*** of radius r :

$$x = \rho\cos\phi,\ y = \rho\sin\phi,\ dS = dxdy = \rho d\rho d\phi$$

Be cause of the revolution symmetry, we can place in the plane of the disk with the origin at the center:

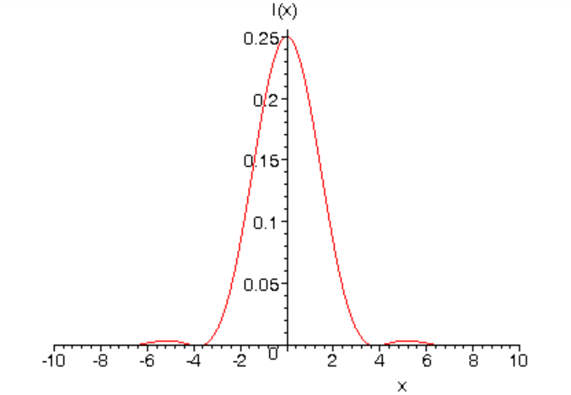
$$a = \sin\theta,\ \beta = 0$$



$$\Psi(\alpha,\beta) = C\iint e^{-ik(\alpha x+\beta y)}dxdy = C\int_0^{2\pi}\int_0^{d} e^{-ik\rho\sin\theta\cos\varphi}\rho d\rho d\varphi = C\frac{J_1(\kappa\sin\theta)}{\kappa\sin\theta}$$

$$I = I_0\left[\frac{J_1(\kappa\sin\theta)}{\kappa\sin\theta}\right]^2$$

$$\kappa = 2\pi r/\lambda$$



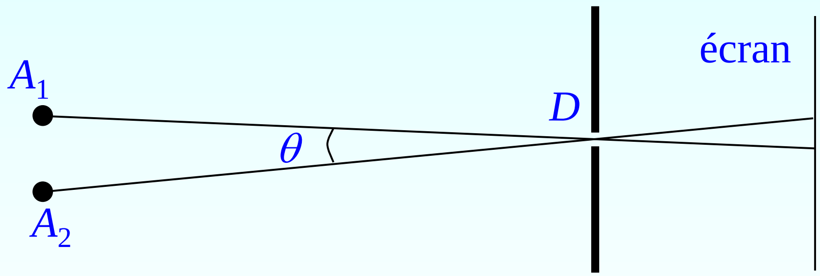
Diffraction

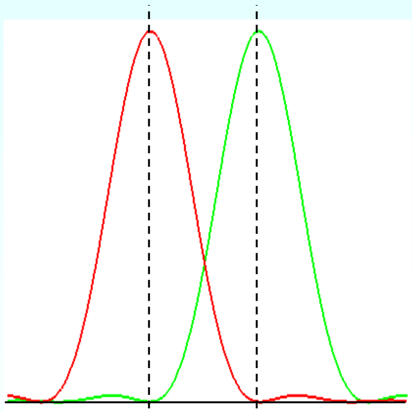
Applications

Rayleigh criterion:

The first minimum of $J_1(x)$ is located at x=3.832, i.e.:

$$\boxed{\theta = \frac{1.22\lambda}{D}}$$





So it is interesting to have an large opening for a better resolution (telescope, camera, ...)

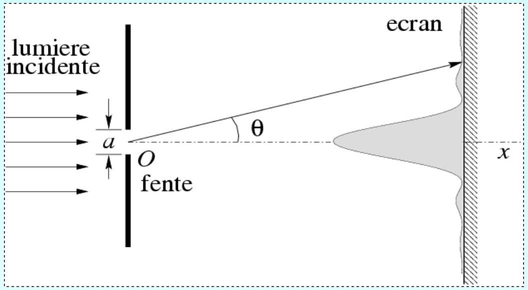
Diffraction and Interference

Applications

Diffraction by a single slit:

$$I = I_0 \frac{\sin^2 u}{u^2}$$

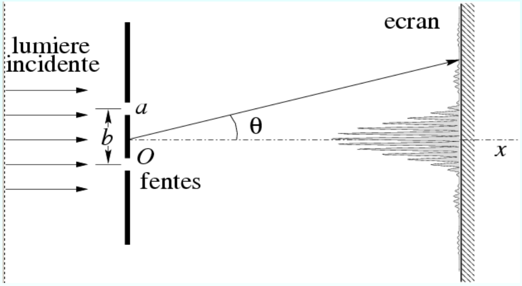
Width of the central spot: $$L_0 = \frac{2D\lambda}{a}$$



Yong's slits: N=2

$$I = I_0 \frac{\sin^2 u}{u^2} \cos^2\left(\frac{b}{a} u\right)$$

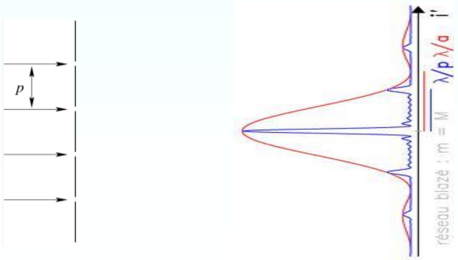
fringe : $$i = \frac{D\lambda}{b}$$



N slits:

$$I = I_0 \frac{\sin^2 u}{u^2} \frac{\sin^2 Nv}{\sin^2 v}$$

$$u = \frac{\pi a \sin\theta}{\lambda}, \quad v = \frac{\pi b \sin\theta}{\lambda}$$



VCRM + Phys. Opt. ➔ Reay theory of waves

VCRM + Phys. Opt. ➔ Reay theory of waves

➢ Physical optics

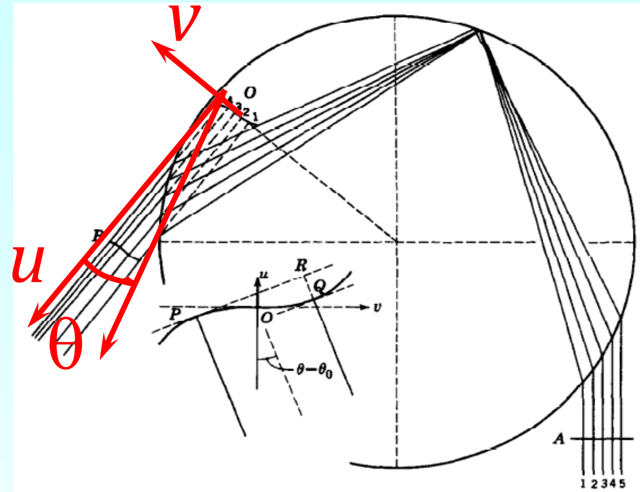
- Recall of Airy theory (1838)

Sir George Airy (1801-1892)

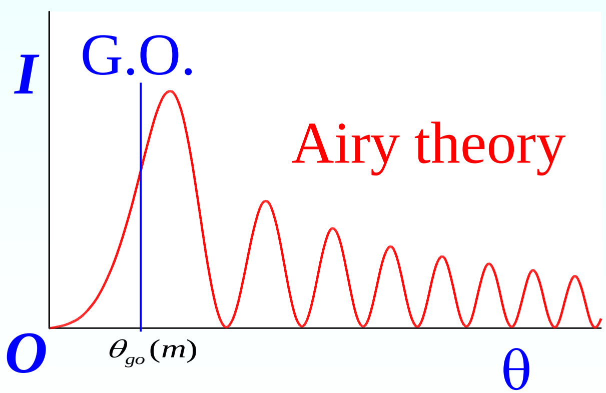
» with approximation **the vicinity of θ_R** the phase is a cubic function:

$$u = \frac{hv^3}{3a^2} \quad \text{with} \quad h = \frac{(p^2-1)^2}{p^2}\frac{(p^2-m^2)^{1/2}}{(m^2-1)^{3/2}}$$

» And assuming a constant amplitude for all emergent rays, the amplitude of the scattered wave is give by (extending v to infinity):



$$\int_{-\infty}^{\infty} e^{-ikv(\theta-\theta_0)+ikhv^3/3a^2}\,dv$$



- The version of van de Hulst (1957) permits to predict the profile,
- But the absolute or relative intensity?

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Ray theory of waves

➢ Physical optics

- Reexamination of Airy theory
  - » Some typical results in the literature:
  - » Tricker: in the book "*Introduction to meteorological optics*" 1970

$$A \sim \sqrt{a/\lambda}$$

  - » Nussenzveig: JOSA 1979 :

$$A \sim (a/\lambda)^{7/6}$$

  - » Hovenac & Lock: JOSA A 1992
  - » R. Lee: Appl. Opt. 1998

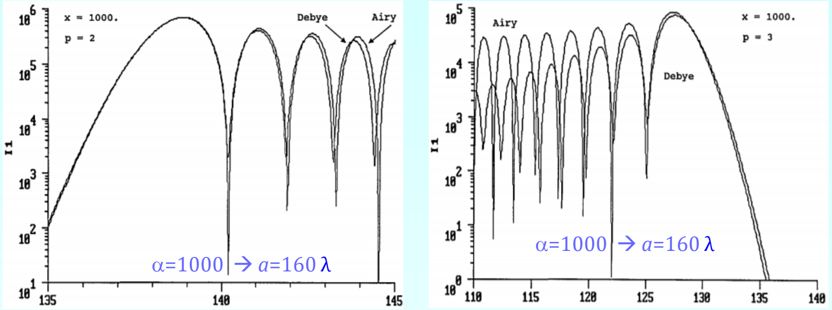
20 March 1998 / Vol. 37, No. 9 / APPLIED OPTICS 1507

Airy & Mie For a given λ, *r*, and range of θ, the two theories can yield quite different intensity maxima, so I must normalize one theory's results in order to compare them with the other's.

Perpendicular polarization

Hovenac & Lock 1992

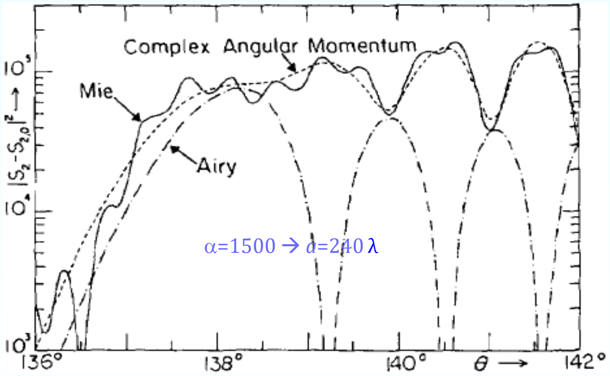
Airy differs from Debye and difference increases with p



Perpendicular polarization

Nussenzveig PRL (1974), JOSA A (1979)

Airy fails for pparallel polarization!!



Ray theory of waves

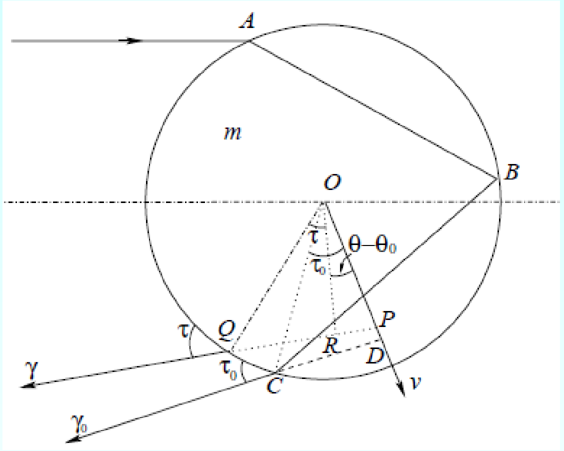
➢ Physical optics

- Airy theory in RTW to remove the two approximations

» Phase: $\Delta\Phi = \Phi(Q) - \Phi(C) - k\overline{PR}$

with

$$\begin{aligned}
\overline{OD} &= \mathbf{r}_0 \cdot \hat{\mathbf{k}}_{r\perp} \\
\overline{OR} &= \mathbf{r} \cdot \hat{\mathbf{k}}_{\perp} \\
\overline{OP} &= \frac{\overline{OR}}{\hat{\mathbf{k}}_r \cdot \hat{\mathbf{k}}} = \frac{\overline{OR}}{\hat{\mathbf{k}}_{r\perp} \cdot \hat{\mathbf{k}}_{\perp}} \\
\overline{PR} &= \overline{OP}(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}_{r\perp}) \\
v &= \overline{OP} - \overline{OD}
\end{aligned}$$



» Amplitude: $\boldsymbol{A_P} = \boldsymbol{A}_Q \frac{\kappa'_{GQ}}{\kappa_{GP}}$

κ_{GX} and κ'_{GX} are the Gaussian curvatures at point X before/after interaction.

Conclusions:

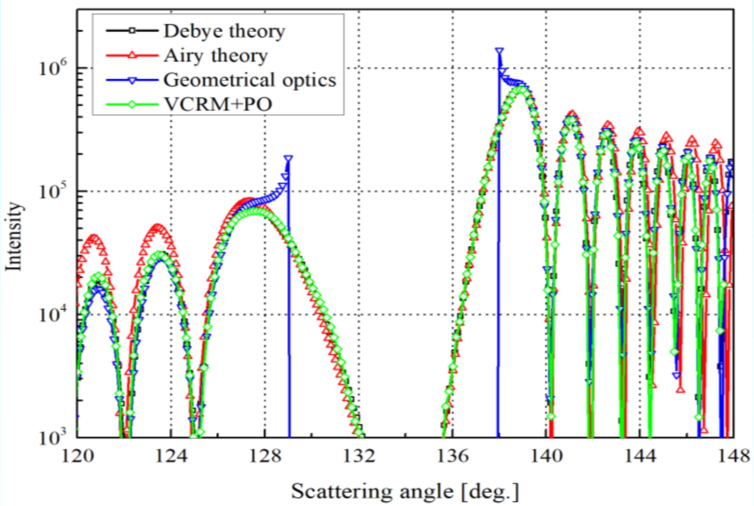
- Both phase and amplitude are calculated **numerically, so *rigorously***,
- **No any hypothesis,**
- Same method for **non-spherical particle.**

Ray theory of waves

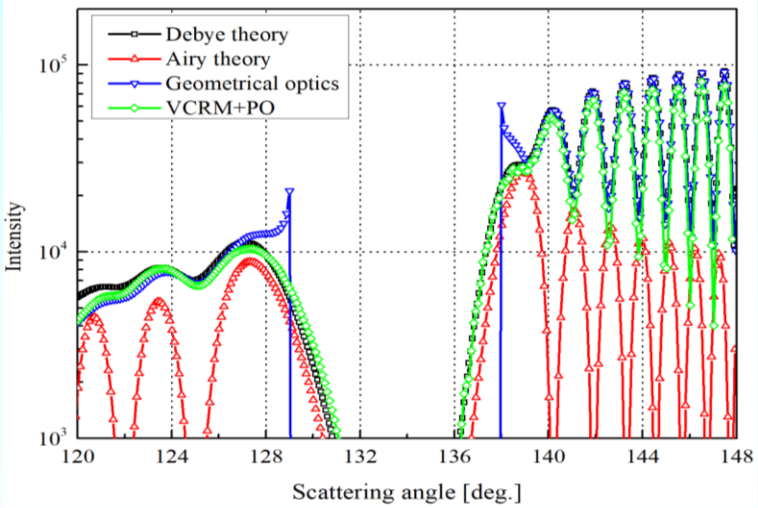
➢ Physical optics

- Airy theory in RTW – Validation of the results:

Comparison of the total intensities calculated by four methods



perpendicular polarization



parallel polarization

The results of Ray Theory of Waves (green curves) are in very good agreement with Debye (black curves).

Application of RTW

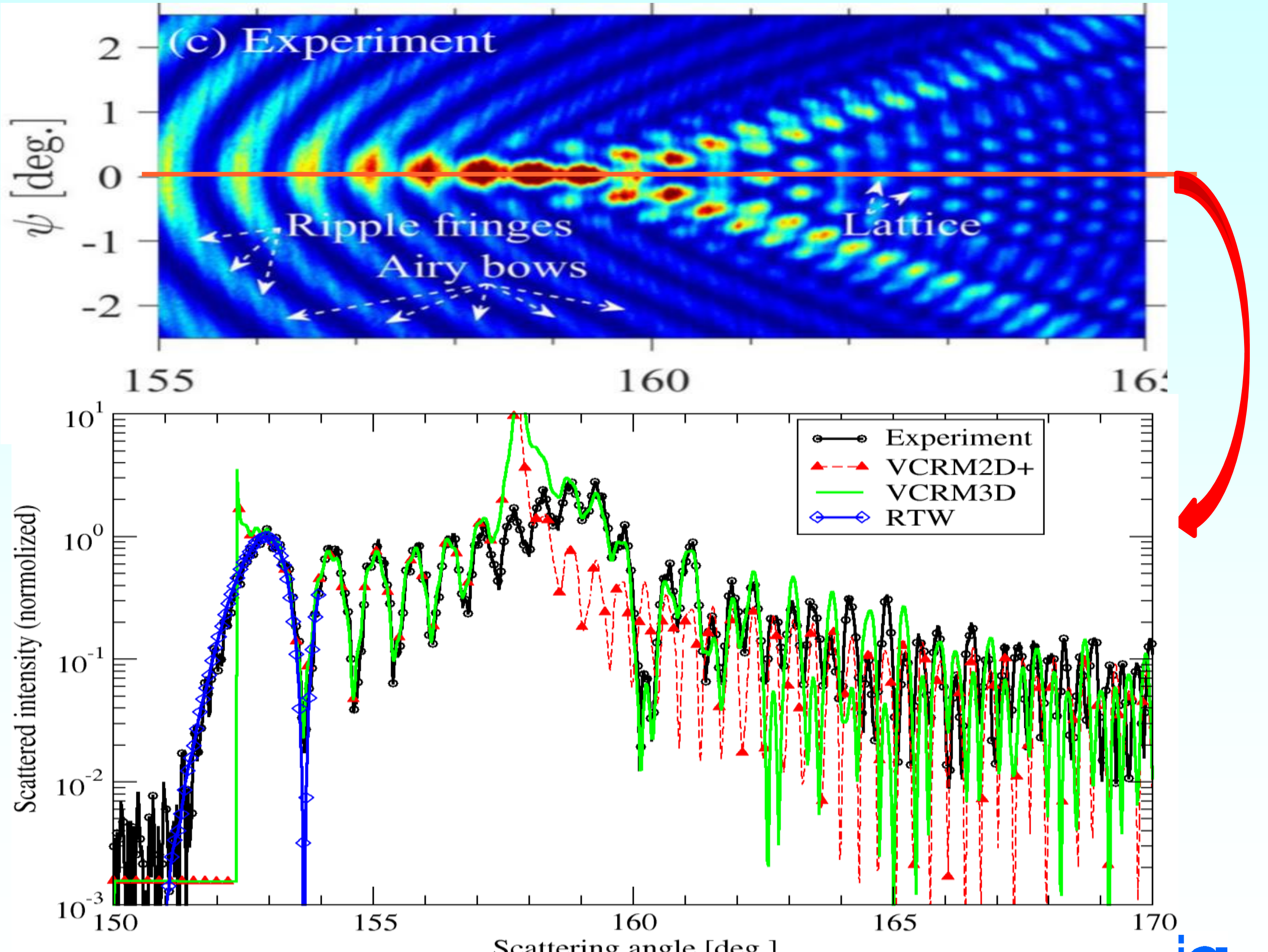
➤ **Generalized rainbow patterns**

Duan *et al* OL 2021

Quantitative comparison of **VCRM** with experiment.

Excellent agreement **in all range**.

Zhang, Rozé & Ren, ELS 2021



Rayleigh theory

Static field of a dipole

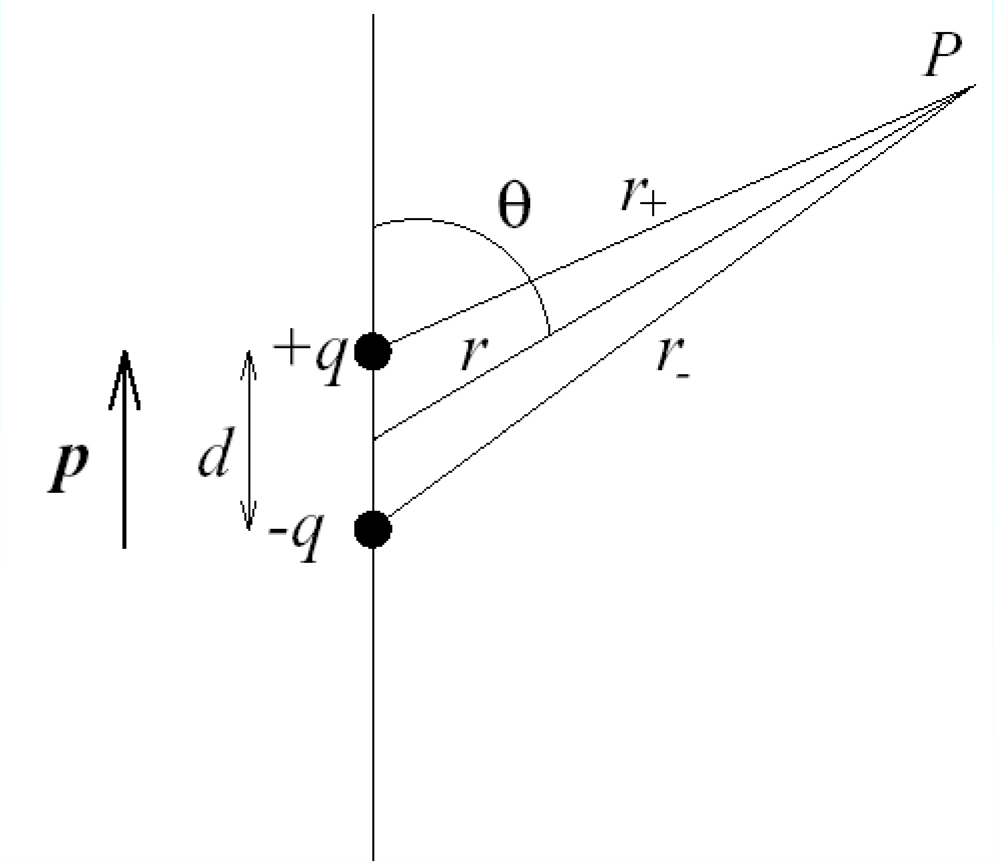
Electric fields

- Two charges +q et – q
- Distance b/t them: *d*
- Dipole moment : $p=qd$

$$\Phi_{dipôle} = \frac{q}{4\pi\varepsilon_{ext}}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

$$\approx \frac{p}{4\pi\varepsilon_{ext} r^2}\cos\theta$$

$$\frac{1}{r_\pm} \approx \frac{1}{r \mp \frac{d}{2}\cos\theta} \approx \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right)$$



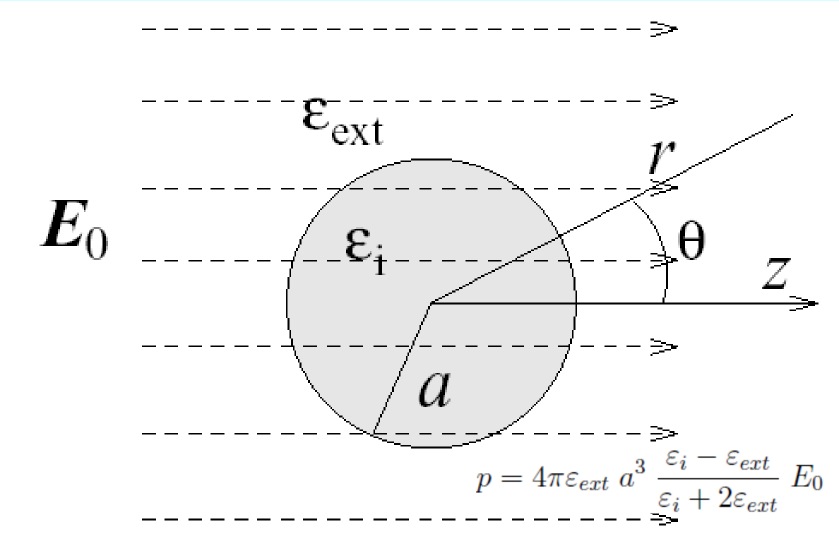
Rayleigh theory

Field scattered by an induced dipole

Electrostatic approximation

Conditions:

1. Electric field
   - static
   - uniform
2. Size of the particle: $l << \lambda$



Potential of the induced field:

$$\varphi_{ext} = \frac{a^3}{r^2} \frac{\varepsilon_i - \varepsilon_{ext}}{\varepsilon_i + 2\varepsilon_{ext}} E_0 \cos\theta$$

"Scattered" field:

$$\vec{E} = \frac{4\pi^2}{r\lambda^2} a^3 \left(\frac{\varepsilon_i - \varepsilon_{ext}}{\varepsilon_i + 2\varepsilon_{ext}} \right) E_0 \sin\theta\, \vec{e}_\theta$$

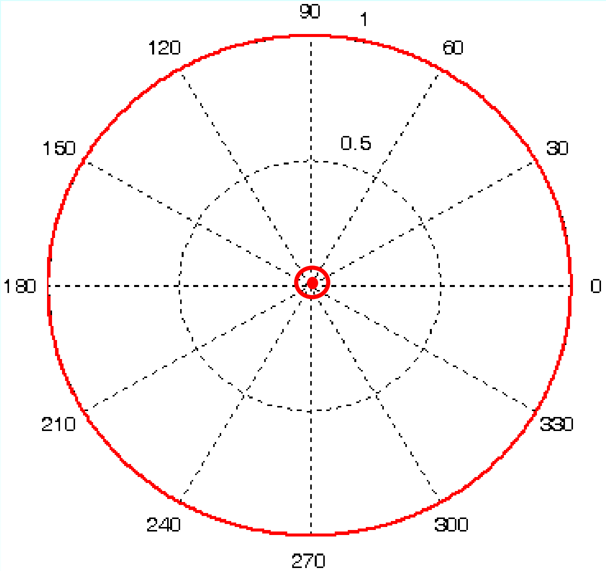
$d << \lambda$,

$$Q_{ext} = \frac{8}{3} \left(\frac{\pi d}{\lambda} \right)^4 \mathrm{Re} \left(\frac{m^2 - 1}{m^2 + 2} \right)$$

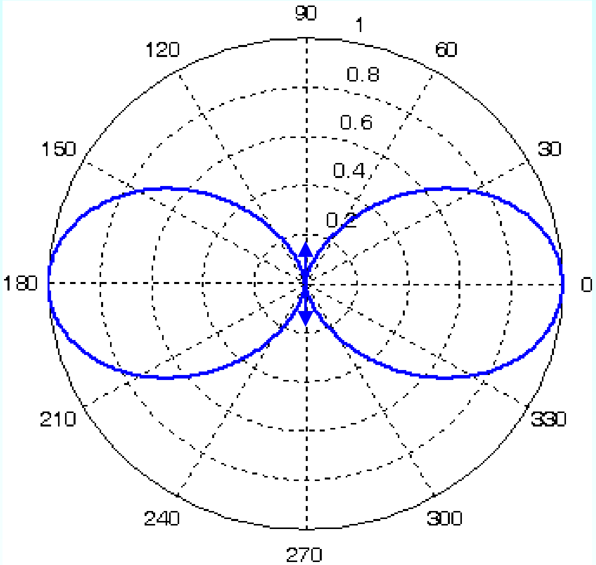
Rayleigh theory

Field scattered by an induced dipole

Scattering diagram



Perpendicular polarization



Parallel polarization

Elements of scattering matrix:

$$S_1 = -\frac{ik^3\alpha}{4\pi} \qquad S_2 = -\frac{ik^3\alpha}{4\pi}\cos\theta$$

Lorenz-Mie Theory - LMT

Conditions and principle

Conditions:

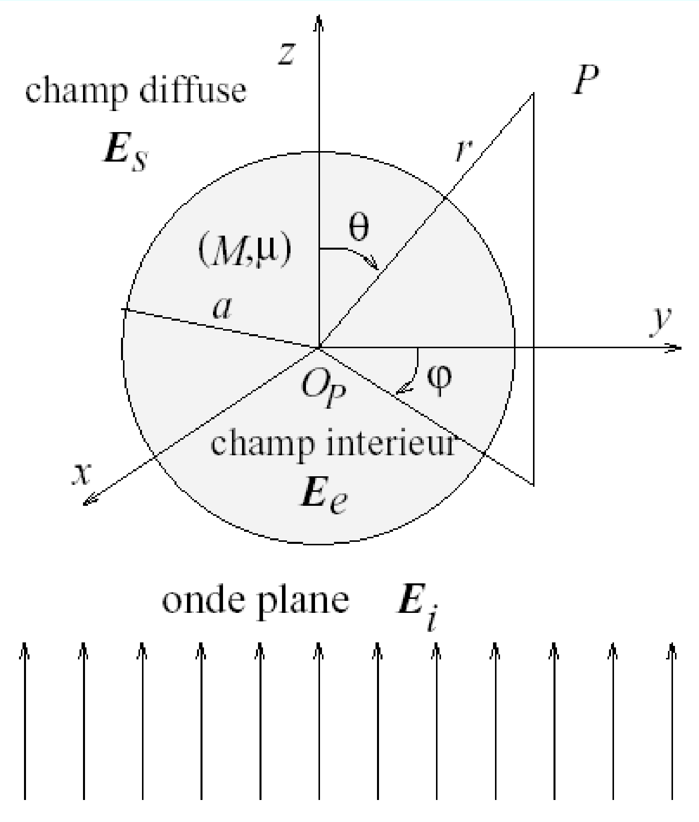
1. **Incident plane wave**
2. Particle :
   - **Spherical**
   - Homogeneous or central stratified
   - Isotropic

Electromagnetic field:

$$E_n = A_n R_n(r)\ \Theta(\theta)\ \Phi(\phi)$$
$$H_n = B_n R_n(r)\ \Theta(\theta)\ \Phi(\phi)$$

Boundary conditions :

$$E_{i,\theta} + E_{s,\theta} = E_{e,\theta}$$
$$H_{i,\theta} + H_{s,\theta} = H_{e,\theta}$$



Lorenz-Mie Theory - LMT

Conditions and principle

Incident wave:

$$\begin{pmatrix} U_{TM}^{i} \\ U_{TE}^{i} \end{pmatrix} = \frac{1}{k^2} \sum_{n=1}^{\infty} \frac{1}{i^{n+1}} \frac{2n+1}{n(n+1)} \psi_n(kr) P_n^1(\cos\theta) \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}$$

Scattered field (far region):

S_1, S_2 elements of the scattering matrix

$$E_r = H_r = 0$$

$$E_\theta = \frac{iE_0}{kr} \exp(-ikr) \cos\varphi \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[a_n \tau_n(\cos\theta) + i b_n \pi_n(\cos\theta) \right] = \frac{iE_0}{kr} \exp(-ikr) \cos\varphi \, S_2$$

$$E_\varphi = \frac{-E_0}{kr} \exp(-ikr) \sin\varphi \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[a_n \pi_n(\cos\theta) + i b_n \tau_n(\cos\theta) \right] = \frac{-E_0}{kr} \exp(-ikr) \sin\varphi \, S_1$$

$$H_\varphi = \frac{H_0}{E_0} E_\theta$$

$$H_\theta = -\frac{H_0}{E_0} E_\varphi$$

a_n, b_n **scattering** Mie coefficients dépending on the proprieties of the particle

τ_n, π_n angular functions of Legendre

Lorenz-Mie Theory - LMT

Physical quantities

Scattered intensities:

$I_{\perp}(\theta)=|S_1|^2$

$I_{||}(\theta)=|S_2|^2$

$$S_1 = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[a_n \pi_n(\cos\theta) + i b_n \tau_n(\cos\theta)\right]$$

$$S_2 = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[a_n \tau_n(\cos\theta) + i b_n \pi_n(\cos\theta)\right]$$

Efficiency sections :

$C_{ext}=C_{sca}+C_{abs}$

$$C_{sca} = \frac{\lambda^2}{2\pi} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2)$$

$$C_{ext} = \frac{\lambda^2}{2\pi} \sum_{n=1}^{\infty} (2n+1)\mathrm{Re}(a_n + b_n)$$

Radiation pressure :

$C_x=C_y=0$

$$C_{pr,z} = \frac{\lambda^2}{2\pi} \mathrm{Re}\left[\sum_{n=1}^{\infty} (2n+1)\frac{(a_n+b_n)}{2} - \frac{2n+1}{n(n+1)} a_n b_n^* - \frac{n(n+2)}{n+1}(a_n a_{n+1}^* + b_n b_{n+1}^*)\right]$$

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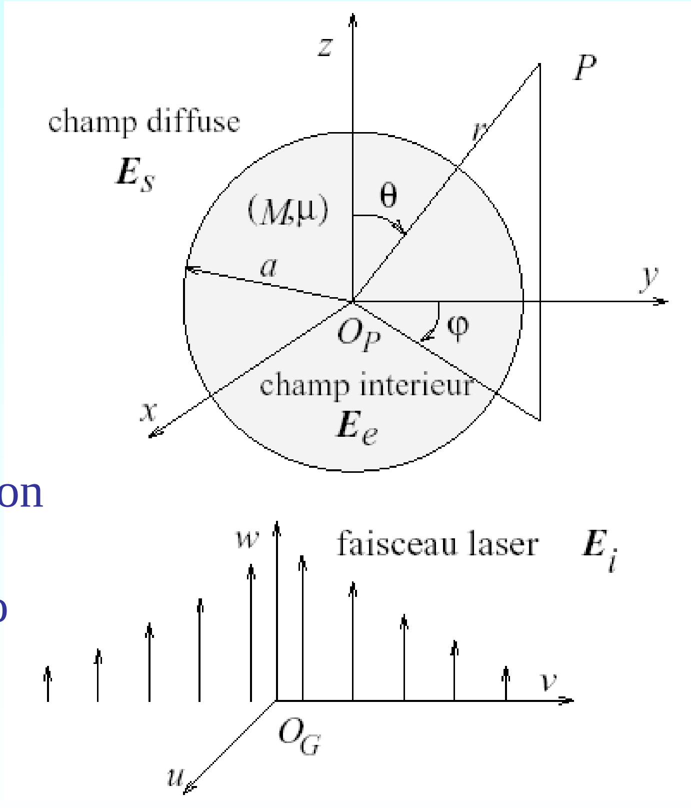
Generalized Lorenz-Mie Theory - GLMT

Conditions and principle

Conditions:

1. Incident wave: ***any shape***
2. Particle :
   - Spherical
   - Homogeneous or centrally stratified
   - Isotropic



Particularities:

1. When the object is big, the illumination is not uniform,
2. The incident wave is described by two series of coefficients :

$$g_{n,TM}^{m} \qquad g_{n,TE}^{m}$$

Generalized Lorenz-Mie Theory - GLMT

Expressions of scattered fields

Scattered field

$$
\begin{aligned}
E_\theta &= \frac{iE_0}{kr}\exp(-ikr)S_2\exp(im\varphi)\\
E_\varphi &= \frac{-E_0}{kr}\exp(-ikr)S_1\exp(im\varphi)\\
H_\varphi &= \frac{H_0}{E_0}E_\theta\\
H_\theta &= -\frac{H_0}{E_0}E_\varphi
\end{aligned}
$$

$$\sum_{n=1}^{\infty} \rightarrow \sum_{n=1}^{\infty}\sum_{m=-n}^{m=+n}$$

$$a_n \rightarrow a_n g_{n,TM}^m$$

$$b_n \rightarrow b_n g_{n,TE}^m$$

$$\pi_n(\cos\theta) \rightarrow \pi_n^m(\cos\theta)$$

$$\tau_n(\cos\theta) \rightarrow \tau_n^m(\cos\theta)$$

Elements of scattering matrix:

$$S_1 = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} C_n\left[m a_n g_{n,TM}^m \pi_n^{|m|}(\cos\theta) + i b_n g_{n,TE}^m \tau_n^{|m|}(\cos\theta)\right]$$

$$S_2 = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} C_n\left[a_n g_{n,TM}^m \tau_n^{|m|}(\cos\theta) + i m b_n g_{n,TE}^m \pi_n^{|m|}(\cos\theta)\right]$$

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Generalized Lorenz-Mie Theory - GLMT

Scattered field – plane wave/Gaussian beam

Particle :

a = 5 μm

m = 1.33

Gaussian beam:

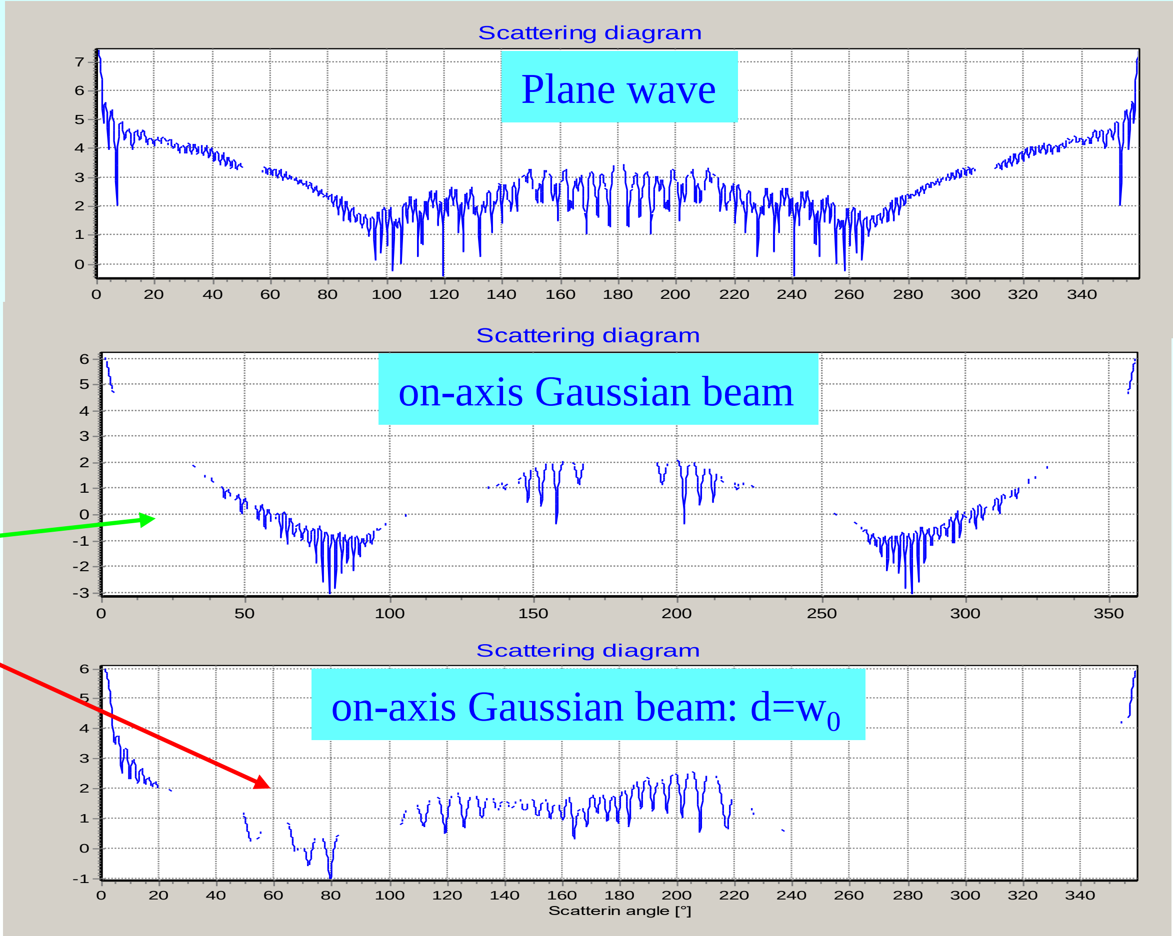
λ = 0.6328 μm

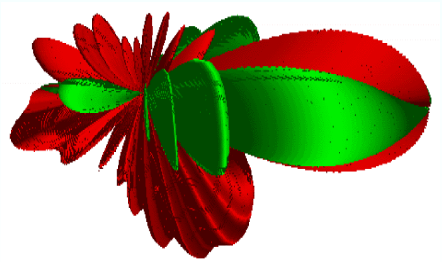
w_0 = 5 μm

Scattering diagram:

On axis - symmetric

Off axis - non-symmetric

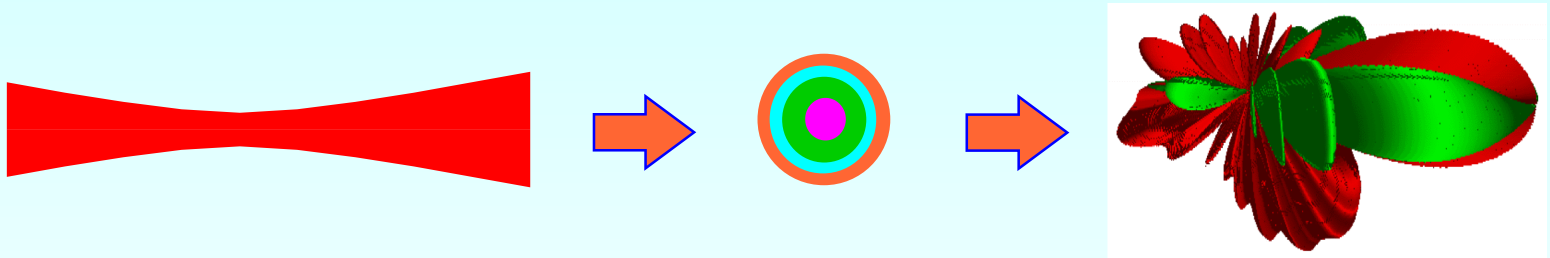




TLM and TLMG for a sphere

Structure of TLMG

Case of a sphere



E_i $\longleftrightarrow$ $g^{m}_{n,TM}$

H_i $\qquad$ $g^{m}_{n,TE}$

$+$

a_n

b_n

$\rightarrow$

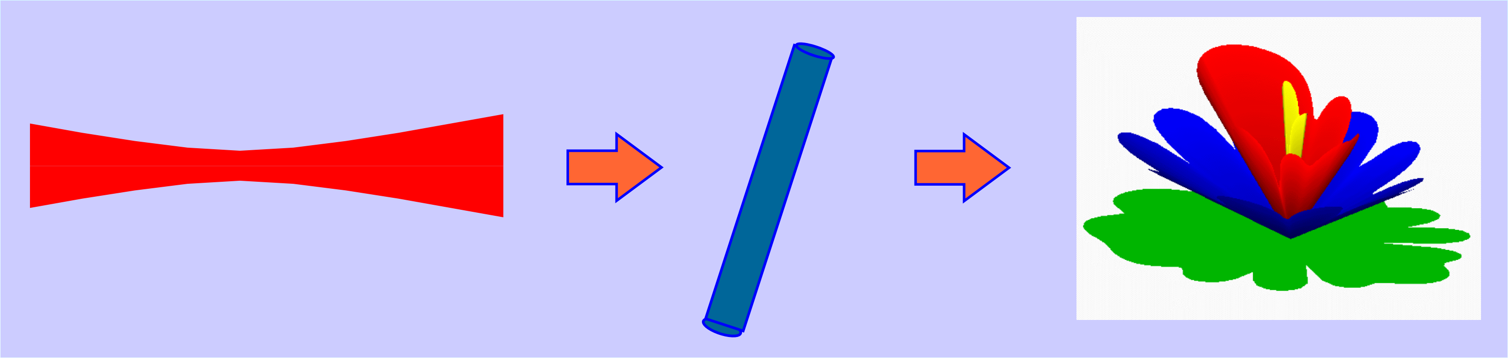
E_s $\quad$ I

H_s $\quad$ P

GLMT for a cylinder

Structure of TLMG

Case of an infinite cylinder



$\mathbf{E}_i$, $\mathbf{H}_i$ $\leftrightarrow$ $I_{n,TM}(\gamma)$, $I_{n,TE}(\gamma)$ + $a_{n\mathrm{I}}, a_{n\mathrm{II}}$, $b_{n\mathrm{I}}, b_{n\mathrm{II}}$ $\rightarrow$ $\mathbf{E}_S$, $\mathbf{H}_S$, I

Introduction of the method plane wave expansion

Debye theory

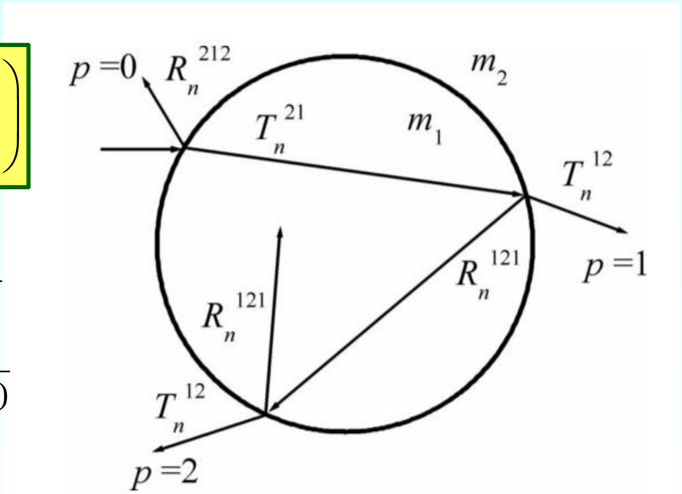
Structure of Debye theory

The Mie coefficients are developed in series, so it is

a) **rigorous**,

b) applied to any wave.

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \frac{1}{2}\left[1 - R_n^{22} - T_n^{12}T_n^{21}\sum_{p=1}^{\infty}\left(R_n^{11}\right)^{p-1}\right] = \frac{1}{2}\left(1 - R_n^{22} - \frac{T_n^{12}T_n^{21}}{1-R_n^{11}}\right)$$



$$R_n^{11} = -\frac{\alpha\xi_n'(x)\xi_n(y) - \beta\xi_n(x)\xi_n'(y)}{\alpha\xi_n'(x)\zeta_n(y) - \beta\xi_n(x)\zeta_n'(y)} \qquad R_n^{22} = -\frac{\alpha\zeta_n'(x)\zeta_n(y) - \beta\zeta_n(x)\zeta_n'(y)}{\alpha\xi_n'(x)\zeta_n(y) - \beta\xi_n(x)\zeta_n'(y)}$$

$$T_n^{12} = \frac{2i}{\alpha\xi_n'(x)\zeta_n(y) - \beta\xi_n(x)\zeta_n'(y)} \qquad T_n^{21} = \frac{n_1}{n_2}\frac{2i}{\alpha\xi_n'(x)\zeta_n(y) - \beta\xi_n(x)\zeta_n'(y)}$$

$$T_n^{21}T_n^{12} = \frac{n_1}{n_2}\frac{-4}{\left[\alpha\xi_n'(x)\zeta_n(y) - \beta\xi_n(x)\zeta_n'(y)\right]^2}$$

$$\alpha = \begin{cases} n_1/n_2 & \text{pour } a_n \\ 1 & \text{pour } b_n \end{cases} \qquad \beta = \begin{cases} 1 & \text{pour } a_n \\ n_1/n_2 & \text{pour } b_n \end{cases}$$

An **rigorous theory** interprets the scattering in the language of GO.

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ABSphere – software for all physical quantities

To be download from:
www.amocops.eu

- ✓ Scattering diagrams
- ✓ Radiation pressure
- ✓ Efficiency factors

- Internal field
- External field
- Far field

ABSphere

- **Plane wave**
- **Gaussian beam**
- Bessel beam
- Dognut beam

- Total field
- Field of each mode
- Effect of shaped beam

- Rainbow refractometry
- Extinction spectroscopy
- Holography
- PDA, PIV,

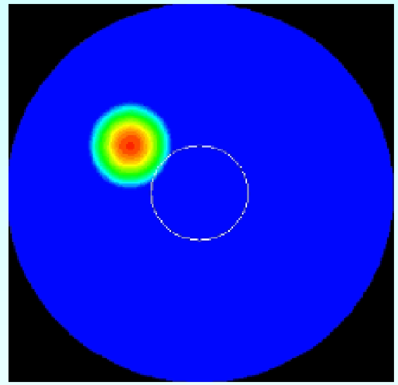
…

- Fluid mechanics
- Combustions
- Environment science
- Life science
- … …

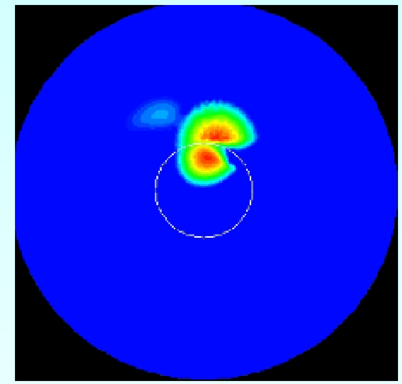
Scattering of a pulse by a sphere

Scattering of a Gaussian beam by a sphere

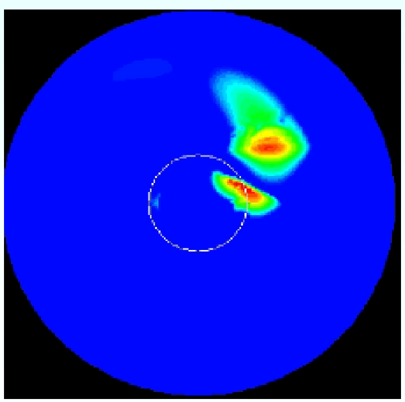
Internal fields
Homogeneous sphere
d=40 µm, τ=50 fs
Gaussian beam



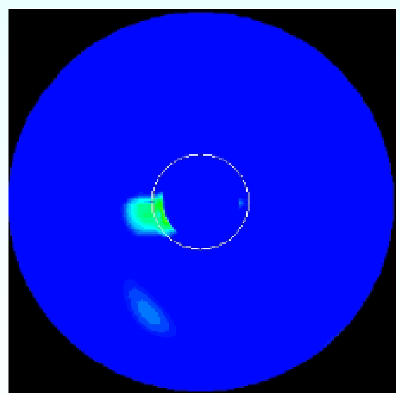
t = – 120



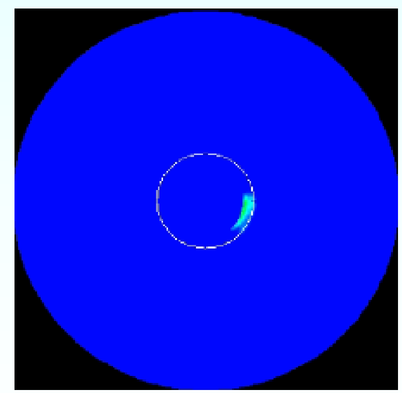
t = 20



t = 120



t = 1080



t = 3400

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Scattering of a pulse by a sphere

Scattering of a plane wave by a coated sphere

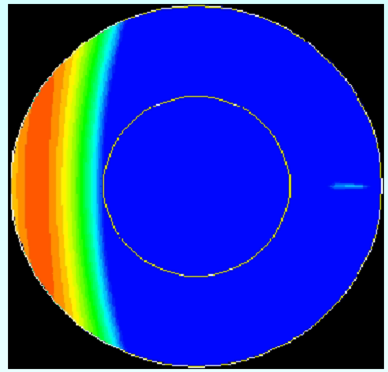
Internal fields

coated sphere

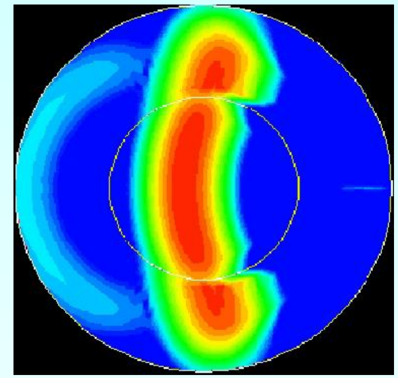
d_e=40 µm, d_i=20 µm

τ=50 fs

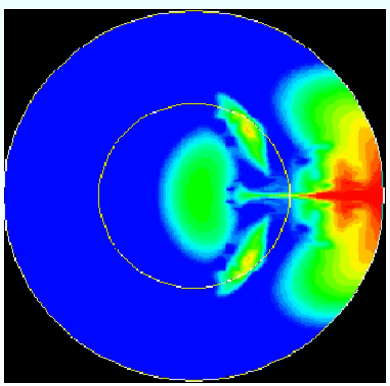
Plane wave



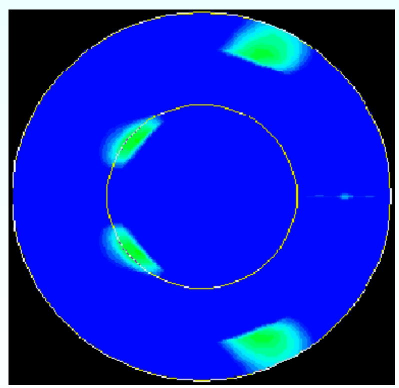
t = – 120



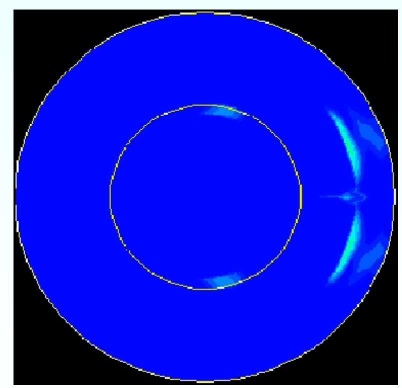
t = 20



t = 120



t = 1060



t = 3400

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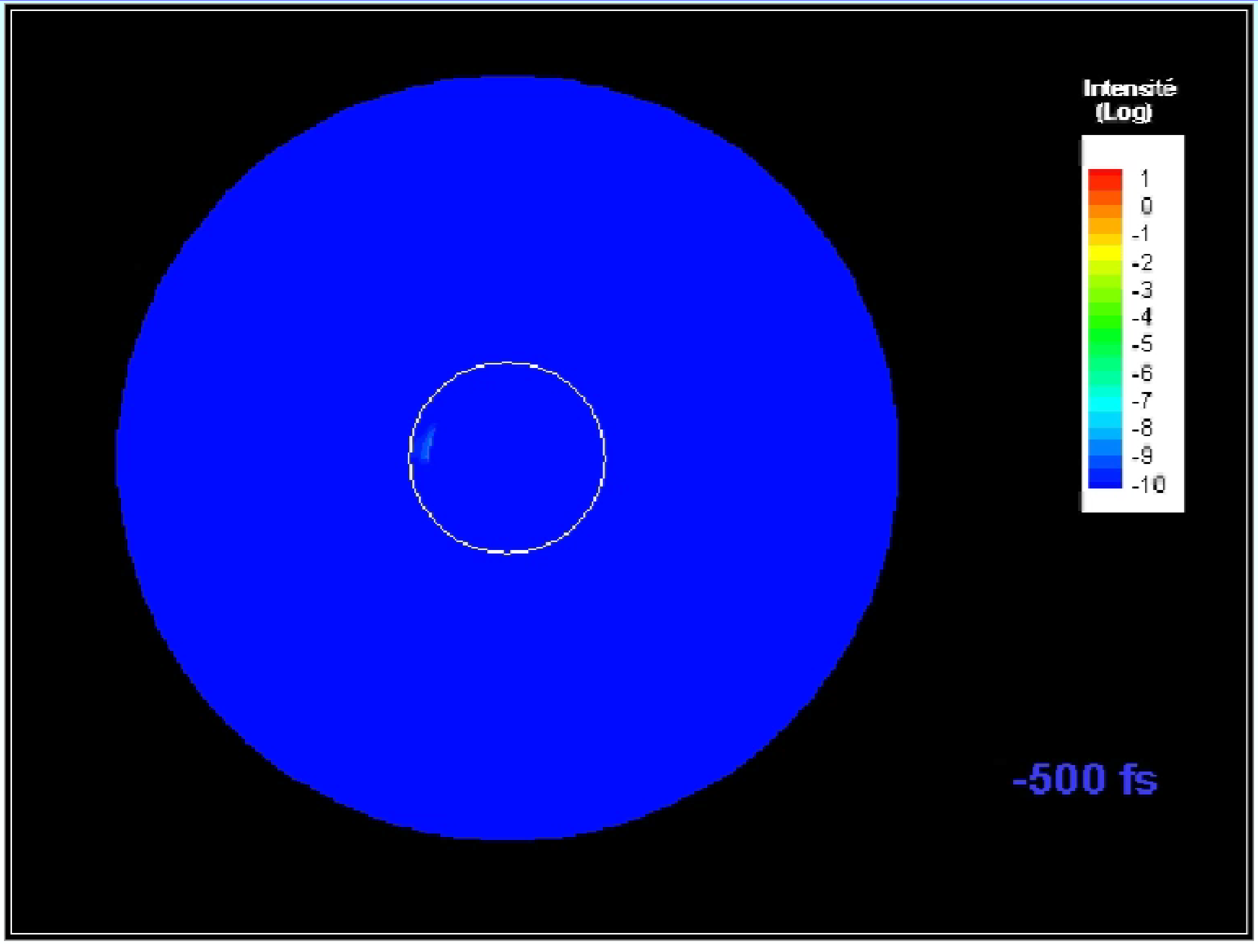
Scattering of a pulse by a sphere

Scattering of a **Gaussian** pulse by a sphere



Exercise on geometrical optics

1. Under what condition can the geometrical optics be used?
2. A light ray arrives on a surface separating two media of different refractive indices. The two figures below represent the reflection coefficients as a function of the incident angle *i* in the two cases: from a more refractive medium to a less refractive and from a less refractive medium to a more refractive.
   - To which case does each of these two figures correspond?
   - What does each of the two curves represent in the figures?
   - Why is there a plateau in the right figure from 41.8°? Deduce the ratio of the refractive indices of the two media.
   - What is the angle of 56.3° in the left figure and 33.7° in the right figure? Explain this phenomenon.
3. A light ray of wavelength λ= 0.488 µm penetrates into a medium of index $m = 1.35–0.001i$. What is the speed of light in this medium? What is the characteristic depth of penetration defined by $I(\delta) = I(0)/e^{2}$.