Lecture at Xidain University
On frontiers of modern optics

# Scattering of shaped beam by particles and its applications

## IV. Ray theory of wave and its applications

Kuan Fang REN

CORIA/UMR 6614 CNRS - Université et INSA de Rouen
School of physics and optoelectronic Eng., Xidian University

西安电子科技大学
现代光学前沿专题

# 波束散射理论和应用

## 第四讲：波的射线理论及其应用

任宽芳

法国鲁昂大学 — CORIA研究所
西安电子科技大学物理与光电学院

个人网页：http://ren.perso.neuf.fr
电话：中国 8820 2673， 法国 02 32 95 37 43

# Plan of lecture

I. Fundamentals of geometrical optics

II. Geometrical optics for light scattering

III. VCRM - Vectorial complex ray Model

UNIVERSITÉ DE ROUEN

coRia UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

# Fundamentals of geometrical optics

## What is it used for ?

- **Imaging in daily life:**
  - Image by reflection: mirror, water surface, summer route …
  - Image by refraction: fish, stick in water, mirage in the nature …
- **Optical instruments:**
  - Camera, telescope, microscope, ..
  - Industrial measurement systems,
  - Optical fiber for telecommunication
- **Scientific research:**
  - Fluid mechanics: PIV, holography, LDV, PDA,
  - Measurement of temperature, size distribution, …
  - Biological imaging, …

**Advantages:**

- Simple
- Object of arbitrary shape

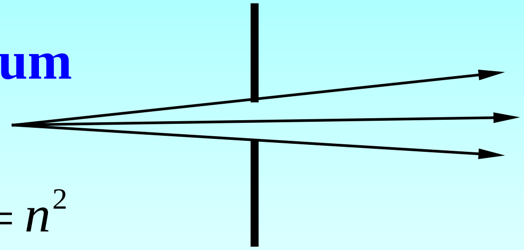
# Fundamentals of geometrical optics

## Condition of application

$$\lambda \ll l$$

***Wavelength λ much smaller the dimension of the object l.***

1. **Straight line in homogeneous medium**



2. **Eikonal eq. (程函方程):** $(\nabla S)^2 = n^2$

3. **Eq. of ray:** $$\nabla n = \frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right)$$

The rays are perpendicular to the wave front.

# Fundamentals of geometrical optics

## Reflection and refraction laws

**Reflection and refraction on a surface between two media**

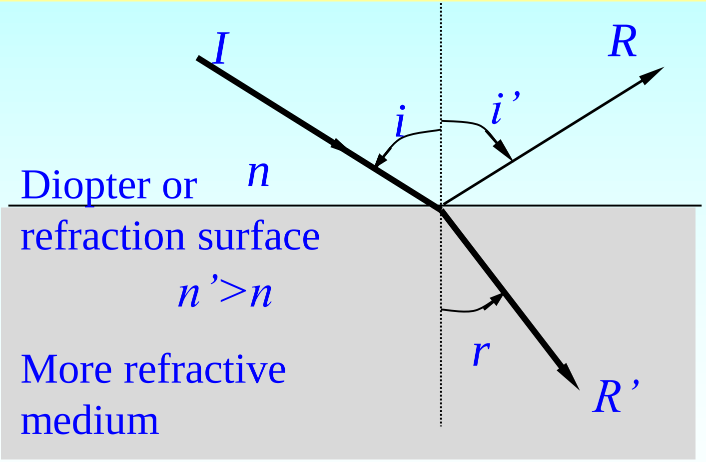
**Law of Snell-Descartes**

**Reflection:** $i = i'$

**Refraction:** $n \sin i = n' \sin r$

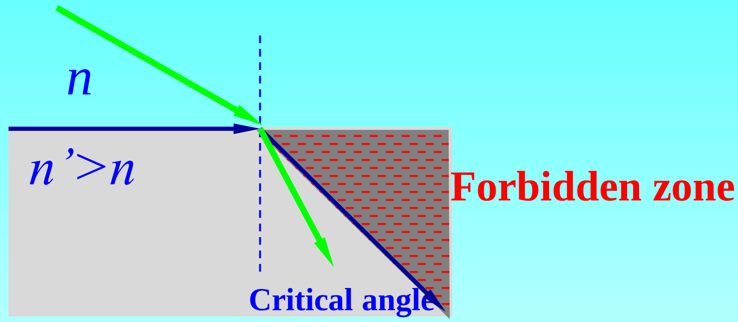
The velocity of light $v$ depends on the refractive index $n$ of the medium:

$$n = \frac{c}{v}$$



# Fundamentals of geometrical optics

## Total reflection

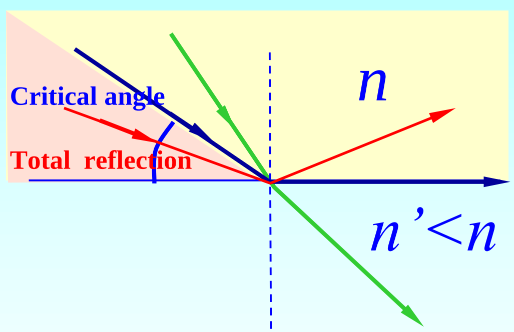



**Critical angle:**

$$i_l = \arcsin\left(\frac{n'}{n}\right)$$

**Total reflection :**

$$i \geq i_l$$




*Examples* :

$n_2 / n_1 = 1/1.333 \quad i_l = 48.6°$

$n_2 / n_1 = 1/1.500 \quad i_l = 41.8°$

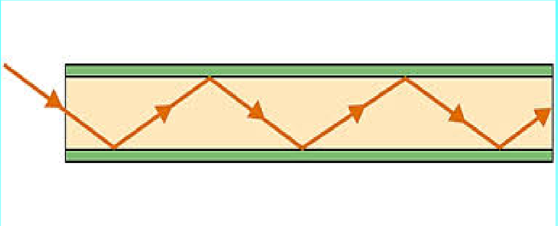
**Applications**

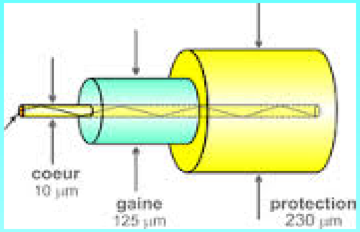
- Optical fiber
  - Telecommunication
  - Transport de laser beam
- Critical angle for measuring bubbles
- Optical gauge
- Natural mirage

# Fundamentals of geometrical optics

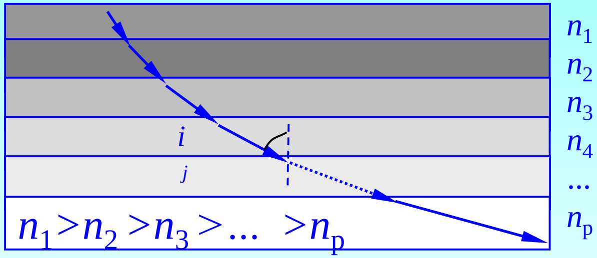
## Total reflection - applications

Application 1:
*Optical fiber*






Application 2:
*Effect of mirage*

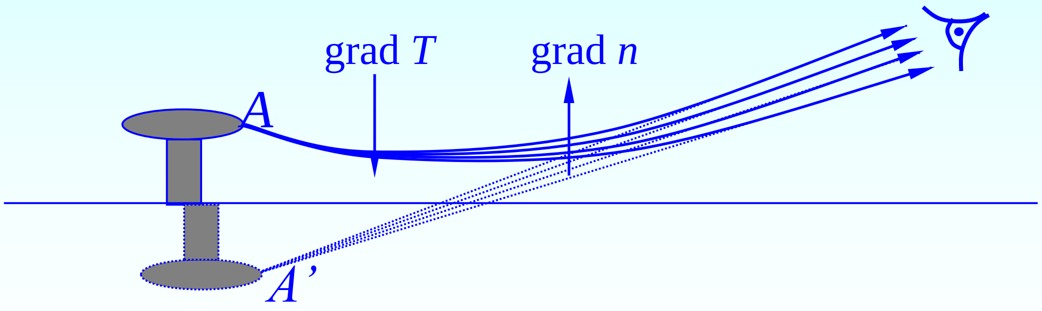




Propagation of a light beam in a parallel stratified environment:

$$n_j \sin i_j = C$$

$n_i \downarrow$ until $i_j$=90°,
i.e. total reflection.



# Fundamentals of geometrical optics

## Relation of amplitudes and intensities

**Fresnel formulas –** relation between the amplitudes of the reflected/refracted and incident waves

$$r_X = \frac{E_X^r}{E_X^i}, \qquad t_X = \frac{E_X^t}{E_X^i}$$

$X = \perp$ ou $\parallel$ polarisation state

$E_X^i$: amplitude of incidente wave

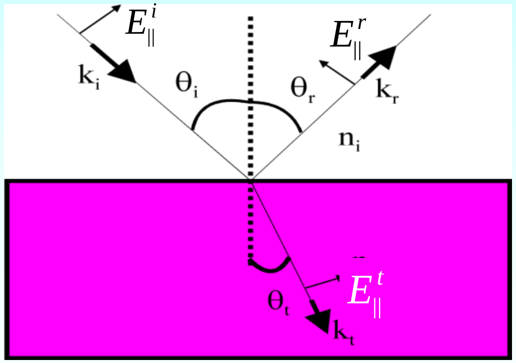
$E_X^r, E_X^r$ amplitudes of reflected/refracted waves

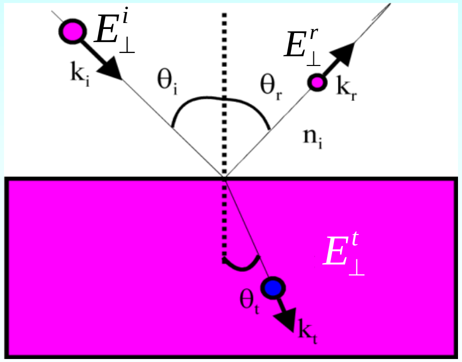
$$r_{//} \equiv \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$r_\perp \equiv \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$t_{//} \equiv \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

$$t_\perp \equiv \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$



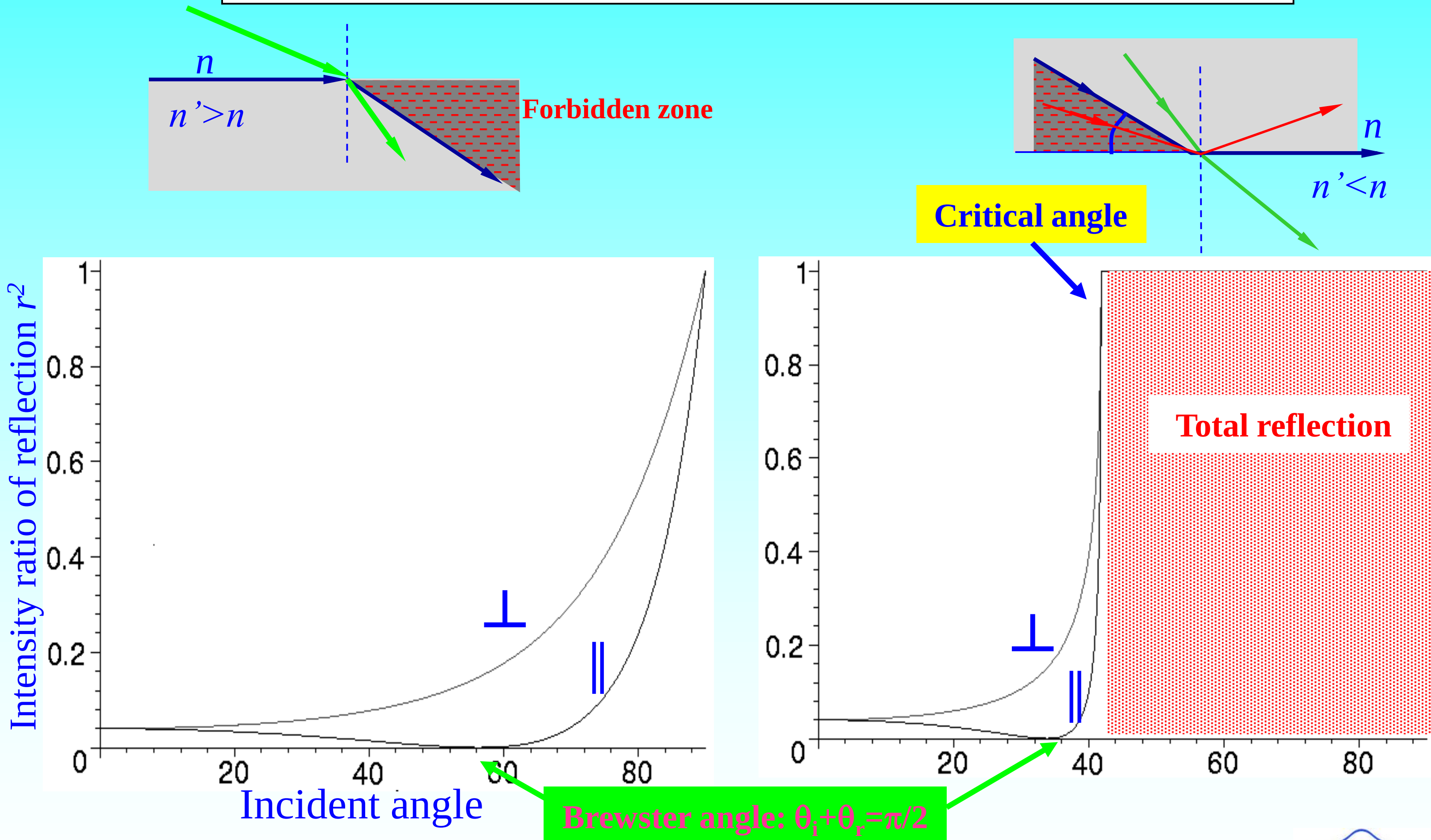




UNIVERSITÉ DE ROUEN

CORIA UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

# Fundamentals of geometrical optics

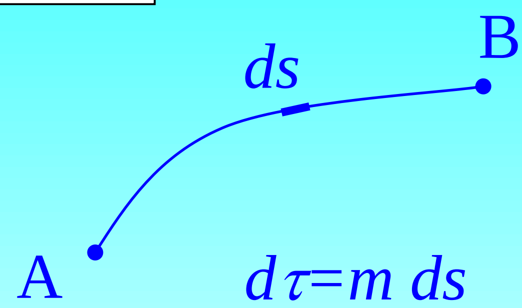
## Relation of amplitudes and intensities



# Fundamentals of geometrical optics

## optical path, phase and intensity

**Propagation of light in a medium:**




$d\tau = m\, ds$

**Optical path:**

$$\tau(AB) = \int_A^B m(s)ds$$

**Complex refractive index:**

$$\tilde{m} = m_r - i m_i$$

- **Phase difference :** $\Delta\varphi(AB) = k\tau_r = k m_r \Delta S$
- **Absorption:** $I(B) = I(A)\exp(-2k\tau_i) = I(A)\exp(-2k m_i \Delta S)$
- **If** $m$ **is constant:** $\Delta\phi = k m_r \Delta S$

$$I = I_0 \exp(-2k m_i \Delta S)$$

# Geometrical optics for light scattering

## Can GO be applied to light scattering ?

**YES to the simple shaped objects/particles:**

- Homogeneous sphere,
- Homogeneous circular cylinder

**Possible for objects of complex shape:**

- Pure ray model, precision is very limited
- Ray model + electromagnetic integration, much better
- Ray model + wave properties → **Vectorial Complex Ray Model**

→ **New concept, very precise and easy to use.**

# Geometrical optics for light scattering

## Application of GO to light scattering

### Light scattering by a sphere:

- **Intensity：**

$$\epsilon_X = r_X^2 \qquad p = 0$$
$$\epsilon_X = r_X^{2(p-1)}(1 - r_X^2)^2 \qquad p \geq 1$$

where $p$ is the scattering order, $X$ represents the polarization sate ($\perp$ or $\parallel$), $r_X$ Fresnel coefficients.

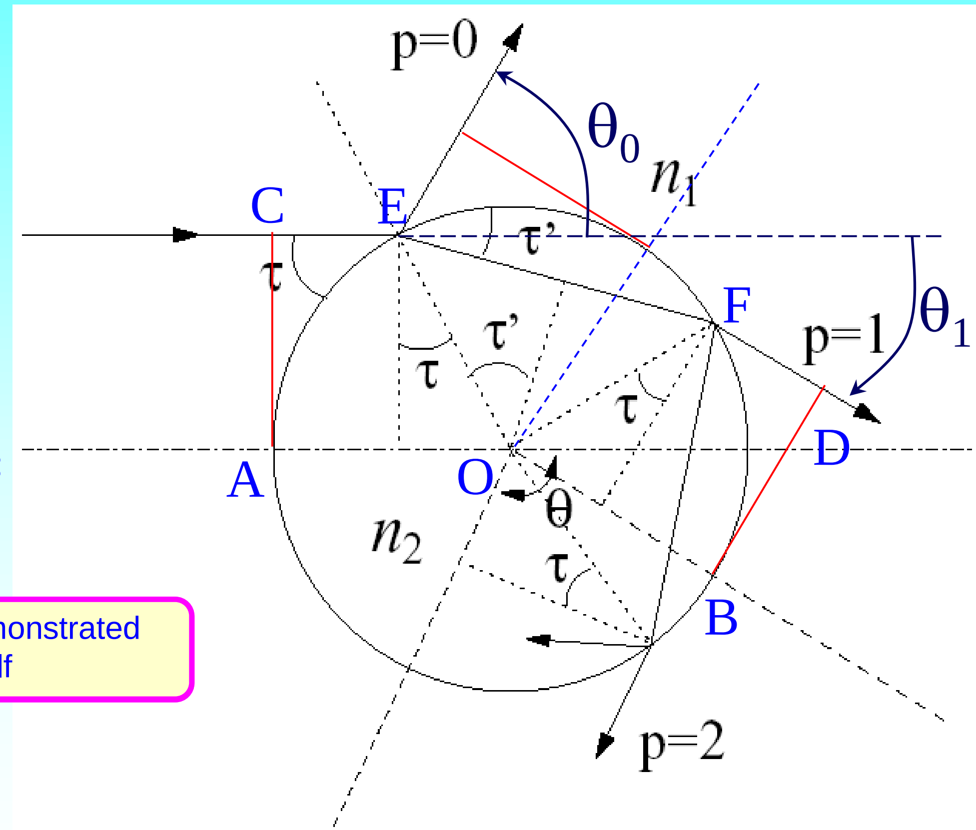
- **Length of the path in the particle:**

$$\Delta_p = 2pa\sin\tau'$$

- **Deviation angle de :**

$$\theta_p = 2\tau - 2p\tau'$$

To be demonstrated by yourself



# Geometrical optics for light scattering

## Application of GO to light scattering

### ➢Phases:

- Phase difference due to the difference of the optical paths :

$$\Delta\phi = \frac{2\pi d}{\lambda}|(\sin\tau - pm\sin\tau')$$

- Jump of phase due to reflection on the surface and the focal lines.

1) Reflection: phase of the complex number: $r_X$,
2) Focal lines: phase jump π/2 at each focal line.

### ➢ Divergence factor :

$$D_p = \frac{\cos\theta_i \sin\tau}{\sin\theta_p \left|\frac{d\theta'_p}{d\tau}\right|}$$

**Summary**

These equations are enough to calculate the intensity of each order and the total field.

$$I_{s,p} = \frac{I_0 \varepsilon_X dS_i}{dS_s} = \frac{I_0 \varepsilon_X a^2 \cos\tau \sin\tau d\tau d\varphi}{r^2 \sin\theta_p d\theta_p d\varphi} = \frac{a^2}{r^2} I_0 \varepsilon_X D_p$$

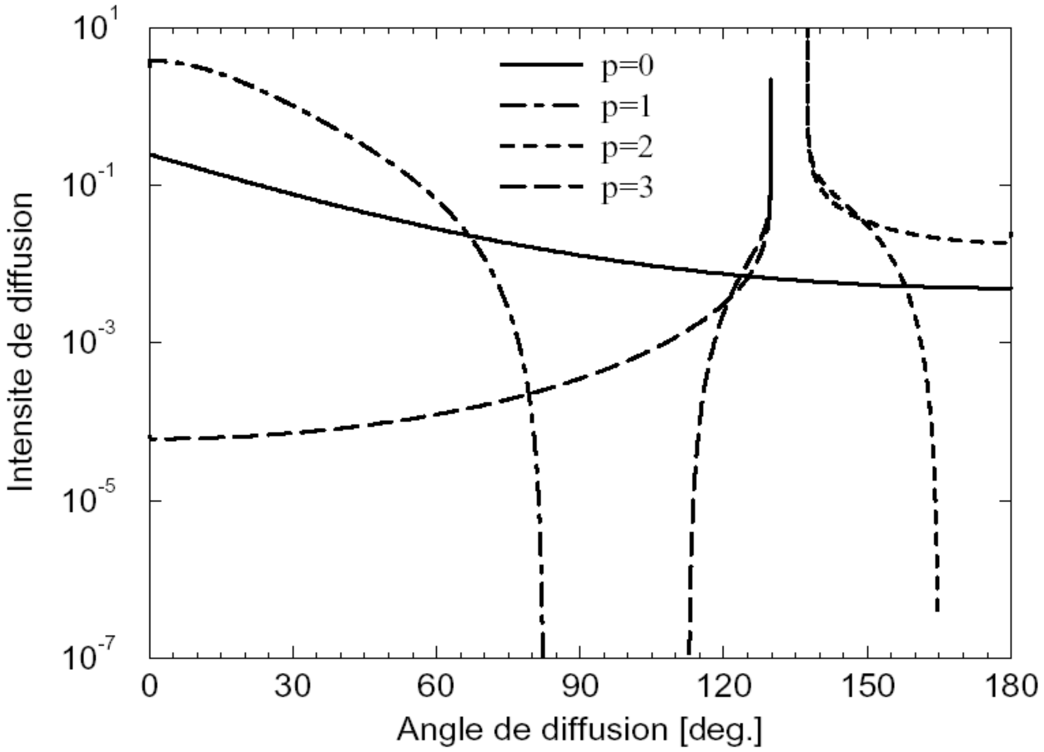
$$I_{s,t} = \sum_{p=0}^{N} \sqrt{I_{s,p}} e^{-ik\Delta\phi_P}$$

# Geometrical optics for light scattering

## Application of GO to light scattering

➢ **Scattering diagram:** **without** interference between different modes

- **Sphere of water**
- **Polarization ⊥**
- **Intensity → ∝ at rainbow angle**



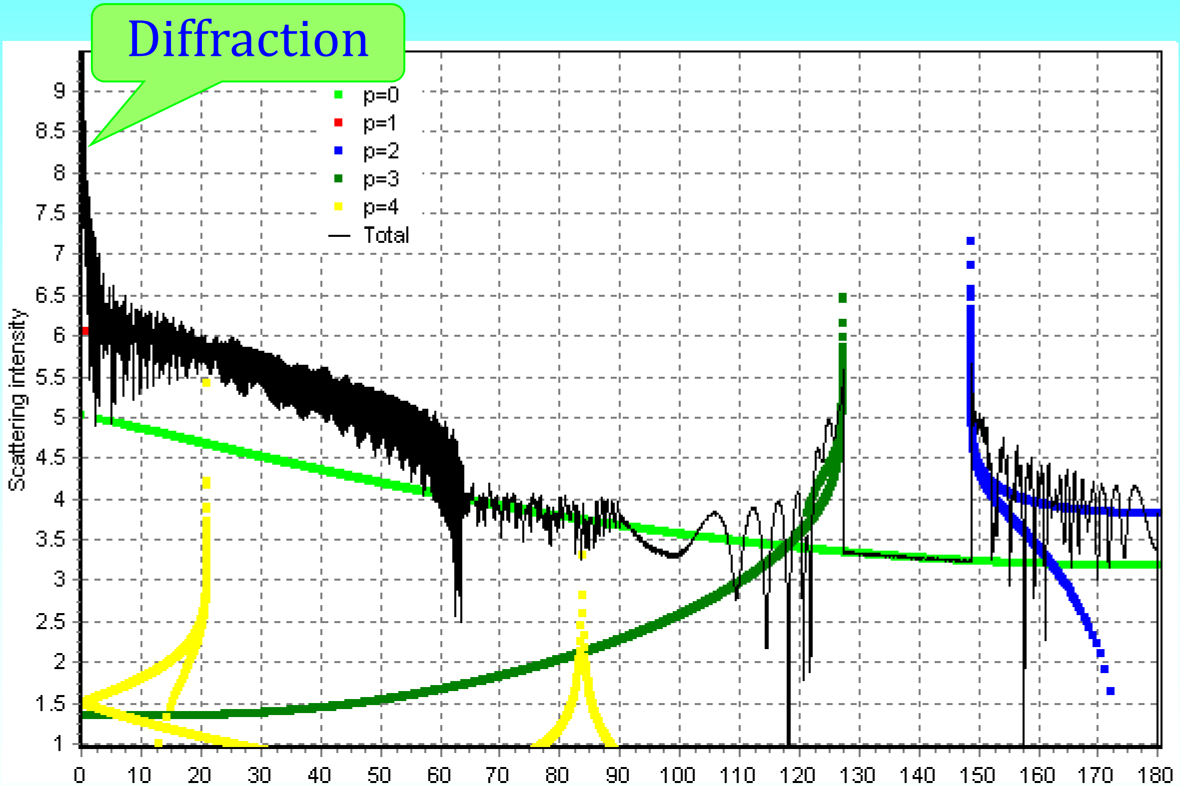

UNIVERSITÉ DE ROUEN

coRia UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

# Geometrical optics for light scattering

## Application of GO to light scattering

➢ **Scattering diagram:** **With** interference between different modes



# Geometrical optics for light scattering

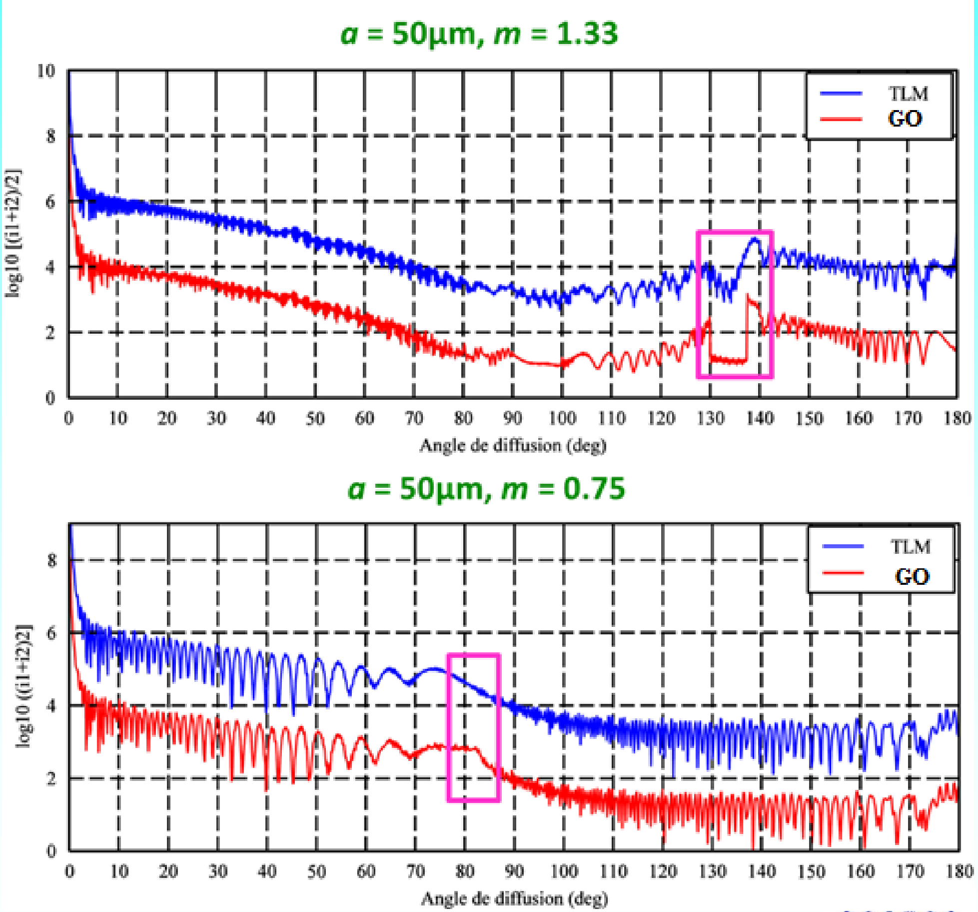
## Application of GO to light scattering

➢ **Scattering diagram:**

**Comparison with rigorous theory** for a homogeneous sphere

**Total intensity with interference:**

A sphere of water or a air bubble in the water illuminated by a plane wave of wavelength of 0.6328 µm.



# Geometrical optics for light scattering

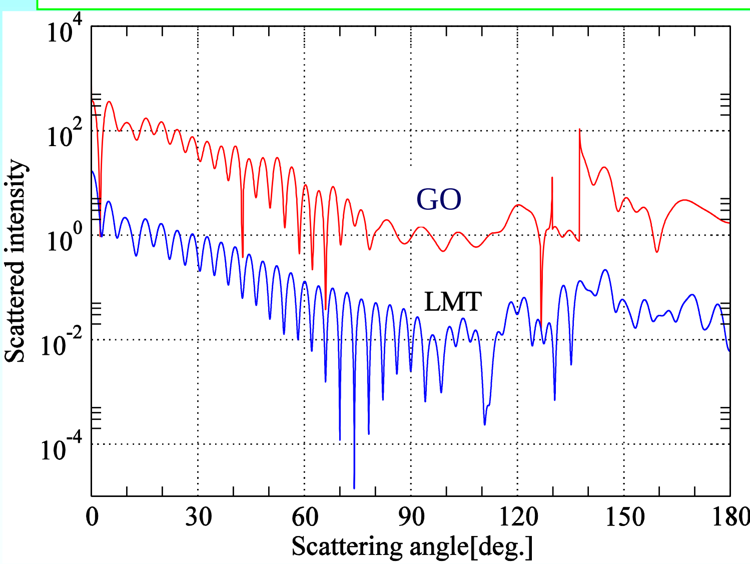
## Application of GO to light scattering

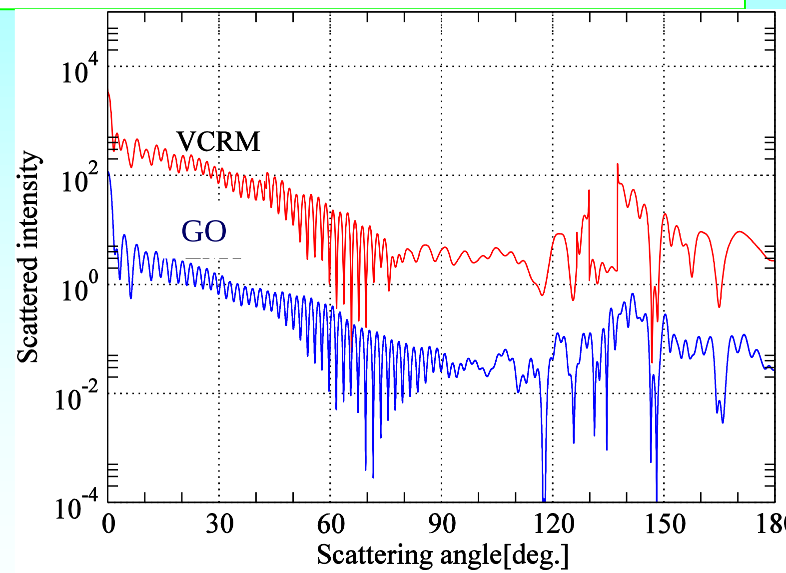
➢ **Scattering diagram:** Comparison with rigorous theory for a cylinder

### Total intensity with interference

A circular cylinder of water with a radius of 5 μm (left) or 10 μm (right) illuminated by a plane wave with a wavelength of 0.6328 μm.

**Geometric optics still work very well for *a* ~ 10λ.**

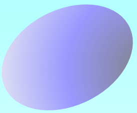


# Geometrical optics for light scattering

## Preliminary conclusions

- By taking into account correctly the interferences, the Ray model can predict the scattering diagram in **ALL directions**.
- It can be applied to the scattering of **any shaped beam**.
- It works also for a circular infinite **cylinder**.



## Limitations

NOT appropriate for a spheroid or an ellipsoid

NO for any irregular shaped particles.

The key problem is **the divergence factor.**

**How to improve the model ?**

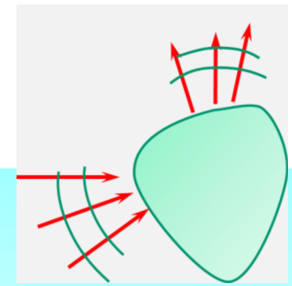
UNIVERSITÉ DE ROUEN

coria UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

# Vectorial Complex Ray Model

## Possible candidate?

- ✗ ~~Rigorous theories:~~ the particle shape must correspond to a coordinates system
- ✗ ~~Numerical methods:~~ size limited, very time communing
- ✓ **Ray models: precision to be improved**

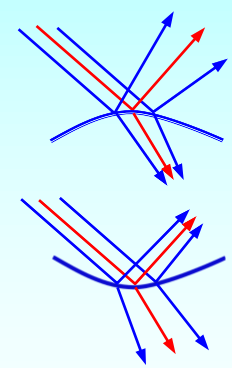


**Key problem: lack of wave properties**

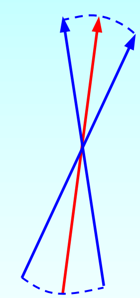
**Our strategy: Extension of ray model**

- **Inclusion of wave front curvature**
- Interference between all the rays.
- Diffraction.

**Vectorial Complex Ray Model**



Divergence/ Convergence



Phase in focal lines

# Vectorial Complex Ray Model

**Geometrical optics + wave form**

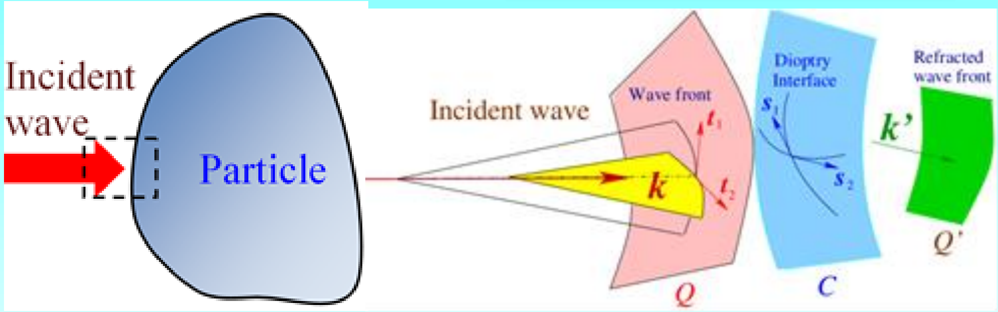
important

- **Vectorial Complex Ray Model – new**

For details:
- Ren et al, *Opt. Lett.* 36(3), 2011
- http://www.amocops.eu

✓ **5 properties of a ray:**

Classical optics
1. direction,
2. amplitude,
3. phase,
4. polarization




**New 5. Wave front curvature**

✓ **Advantages:**
- Objects of any shape with smooth surface,
- Incident wave of any form,
- Sufficiently precise – scattering in all directions,
- All scattering properties of the objet.

UNIVERSITÉ DE ROUEN · CORIA UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN · XIDIAN UNIVERSITY

# Vectorial Complex Ray Model

## Basic laws of geometrical optics

### 1. Snell-Descartes law:

Reflection: $i = i'$

Refraction: $n \sin i = n' \sin r$

### 2. Fresnel's Equations :

$$r_{//} \equiv \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$t_{//} \equiv \frac{2 n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{2 \sin\theta_i \cos\theta_t}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

$$r_{\perp} \equiv \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$t_{\perp} \equiv \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{2 \sin\theta_i \cos\theta_t}{\sin(\theta_i + \theta_t)}$$

**VCRM**

**The tangent component of wave vector is continuous**

$$k_\tau = k'_\tau$$

$$r_\perp = \frac{k_{in} - k_{rn}}{k_{in} + k_{rn}}$$

$$t_\perp = \frac{2k_{in}}{k_{in} + k_{rn}}$$

$$r_\parallel = \frac{m^2 k_{in} - k_{rn}}{m^2 k_{in} + k_{rn}}$$

$$t_\parallel = \frac{2m k_{in}}{m^2 k_{in} + k_{rn}}$$

UNIVERSITÉ DE ROUEN

CORIA UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

# Vectorial Complex Ray Model

## Summary of VCRM

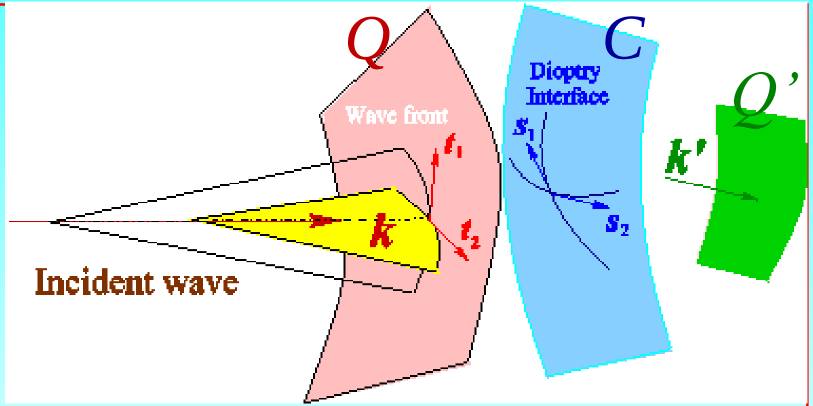
- **Fundamental laws**

1. Wave front equation:

$$\left(k_n^i - k_n^t\right) C = k' \Theta'^T Q' \Theta' - k \Theta^T Q \Theta$$

2. Law of Snell-Descartes **in vectors**:

$$k_\tau^i = k_\tau^t$$




- Amplitude:

$$A = \sqrt{D}\varepsilon$$

- Phase:

$$\Phi = \Phi_{inc} + \Phi_{fl} + \Phi_{path} + \Phi_\varepsilon$$

- Total field:

$$E = S_{diff} + \sum_{i=1}^{N} S_i$$

Divergence factor:

$$D = R'_{11} R'_{21} \frac{R'_{12} R'_{22}}{R_{12} R_{22}} \cdots \frac{R'_{1q} R'_{2q}}{R_{1q} R_{2q}}$$

Fresnel coefficients: ε

$$\tilde{r}_\perp = \frac{k_n - \tilde{k}'_n}{k_n + \tilde{k}'_n} \qquad \tilde{r}_\parallel = \frac{\tilde{m}^2 k_n - \tilde{k}'_n}{\tilde{m}^2 k_n + \tilde{k}'_n}$$

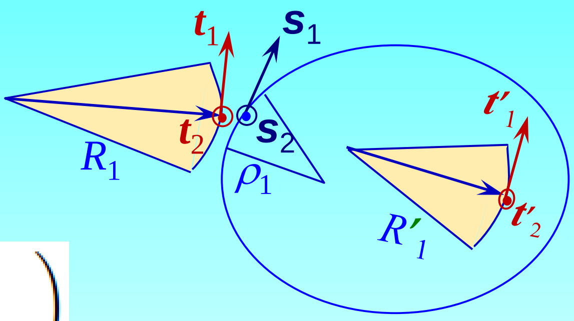
*All expressed in wave vector components.*

# Vectorial Complex Ray Model

## Special case of the wave front equation:

When the rays remain in the same plane – a main direction of the wave front and particle surface:

- Spherical particle
- Infinite cylinder at normal incidence
- Ellipsoidal particle in the symmetric plane.




➢ Curvature matrix:

$$C = \begin{pmatrix} \frac{1}{\rho_1} & 0 \\ 0 & \frac{1}{\rho_2} \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{1}{R_1} & 0 \\ 0 & \frac{1}{R_2} \end{pmatrix}$$

$$Q' = \begin{pmatrix} \frac{1}{R'_1} & 0 \\ 0 & \frac{1}{R'_2} \end{pmatrix}$$

➢ Wave front equation:

$$\frac{{k'_n}^2}{k' R'_1} = \frac{k_n^2}{k R_1} + \frac{k'_n - k_n}{\rho_1}$$

$$\frac{k'}{R'_2} = \frac{k}{R_2} + \frac{k'_n - k_n}{\rho_2}$$

**Rewrite the wave front equations in this simple form for the special case.**

UNIVERSITÉ DE ROUEN

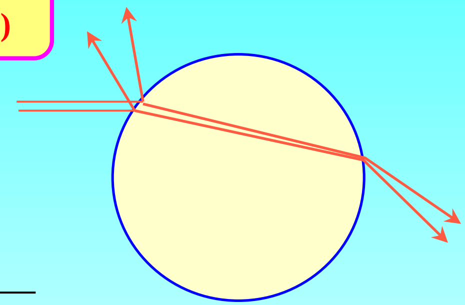
coria UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

# Vectorial Complex Ray Model

## Applications to a sphere and a cylinder

Derive these results from the wavefront equations (see below)

- **Sphere**
  - Reflection: $R_1' = -\frac{a\cos\alpha}{2} \quad R_2' = -\frac{1}{2a\cos\alpha} \rightarrow D = \frac{1}{4}$
  - Refraction $p=1$:



After 1st refraction: $R'_{11} = -\frac{am\cos^2\beta}{m\cos\beta - \cos\alpha} \quad R'_{12} = -\frac{am}{m\cos\beta - \cos\alpha}$

Check these by yourself.

After 2nd refraction:

$$R'_{21} = \frac{m\cos\beta - 2\cos\alpha}{2(m\cos\beta - \cos\alpha)}\cos\alpha \quad R'_{22} = \frac{2a\cos\beta(m\cos\beta - \cos\alpha) - m}{2(m\cos\beta - \cos\alpha)(\sin\alpha\sin\beta - \sin\alpha\sin\beta)}$$

Divergence factor:

$$D = \frac{m\sin(2\alpha)\cos\beta}{4\sin[2(\beta-\alpha)](\cos\alpha - m\cos\beta)}$$

Identical to the classical one.

- **Cylinder: $R_2 = \infty$**
  - Reflection : $D = \frac{a\cos\alpha}{2}$
  - Refraction p=1: $D = \frac{m\cos\alpha\cos\beta}{2(\cos\alpha - m\cos\beta)}$

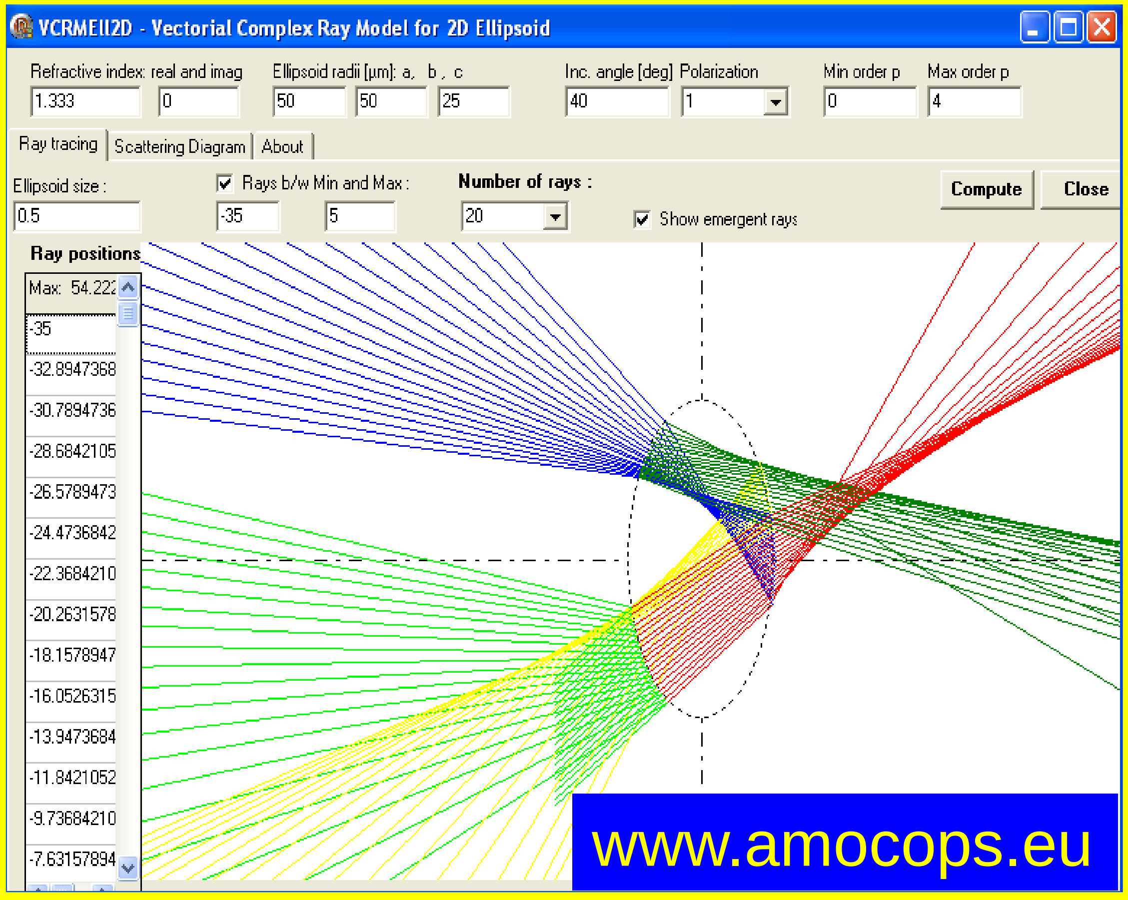
$$\frac{k_n'^2}{k' R_1'} = \frac{k_n^2}{k R_1} + \frac{k_n' - k_n}{\rho_1}$$
$$\frac{k'}{R_2'} = \frac{k}{R_2} + \frac{k_n' - k_n}{\rho_2}$$

# Vectorial Complex Ray Model

## Software for an ellipsoid

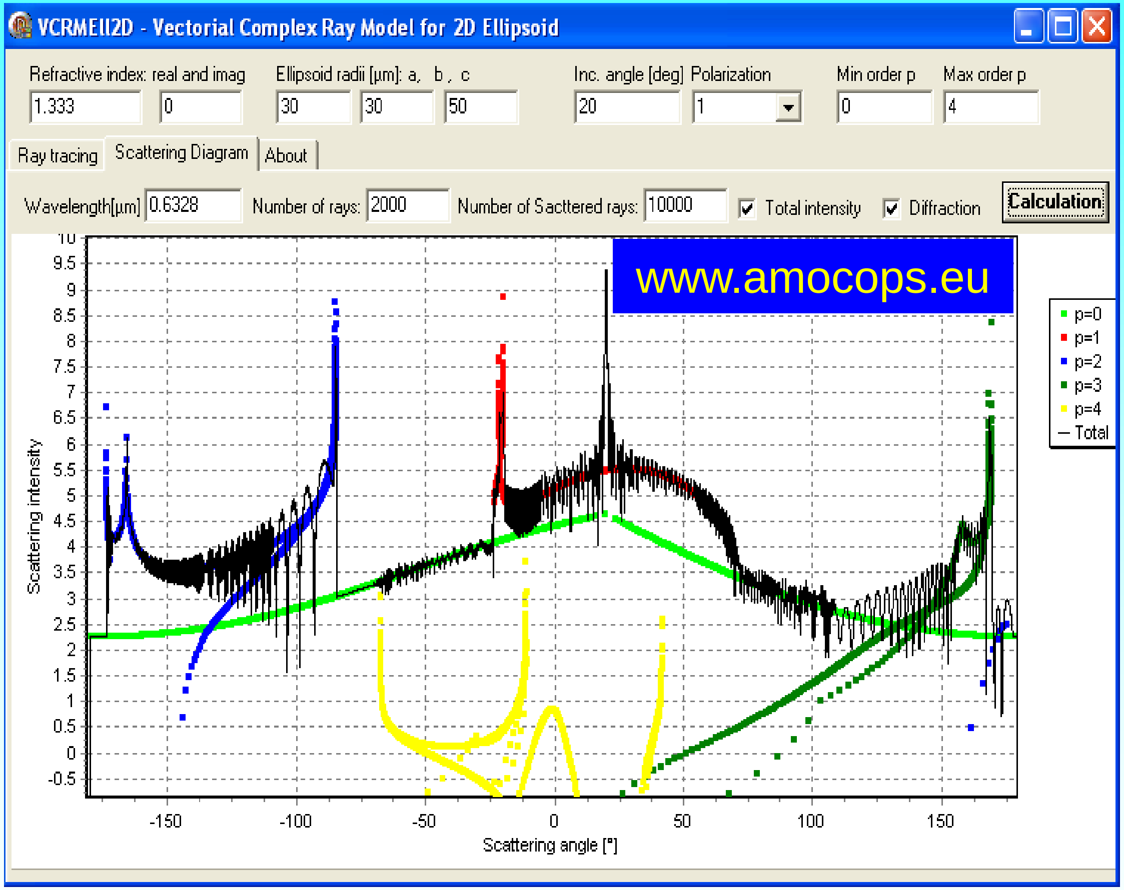
**Ray tracing Module**

- Tracing of all the rays or a part of them
- Any number of rays
- Any position of rays



# Vectorial Complex Ray Model

## Software for an ellipsoid

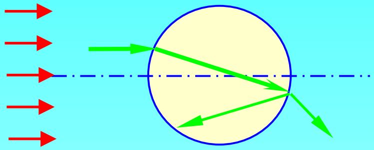



**Scattering diagram module**

- Intensity of each order
- total intensity of all the rays:
  - Interference
  - Diffraction

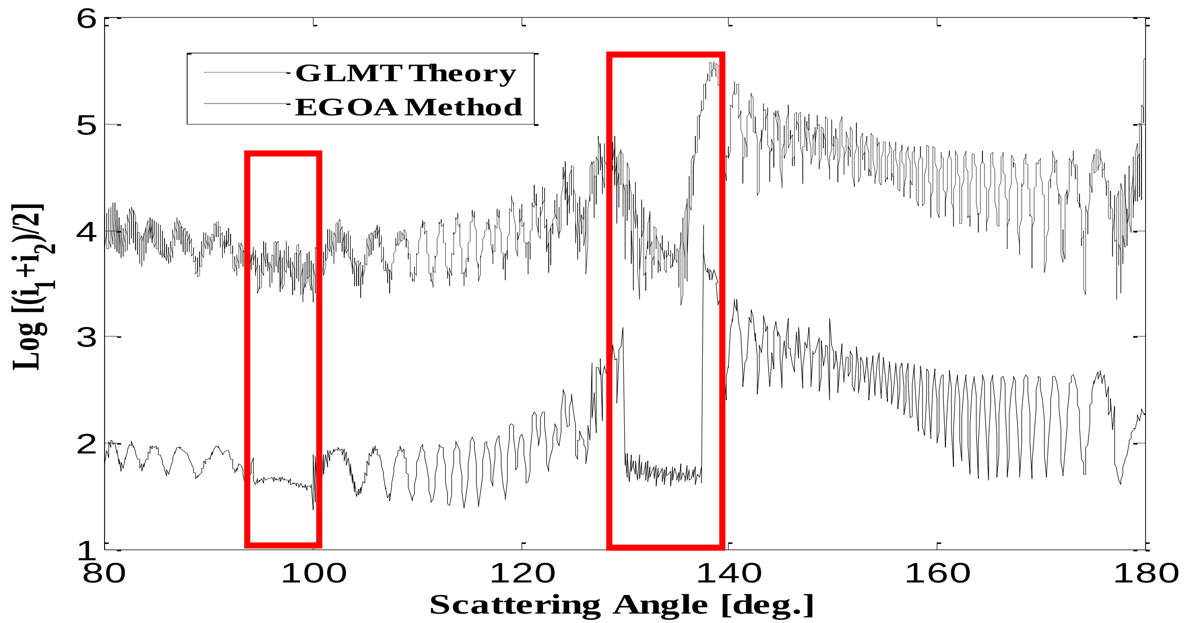
# Vectorial Complex Ray Model

## Comparison with Lorenz-Mie Theory



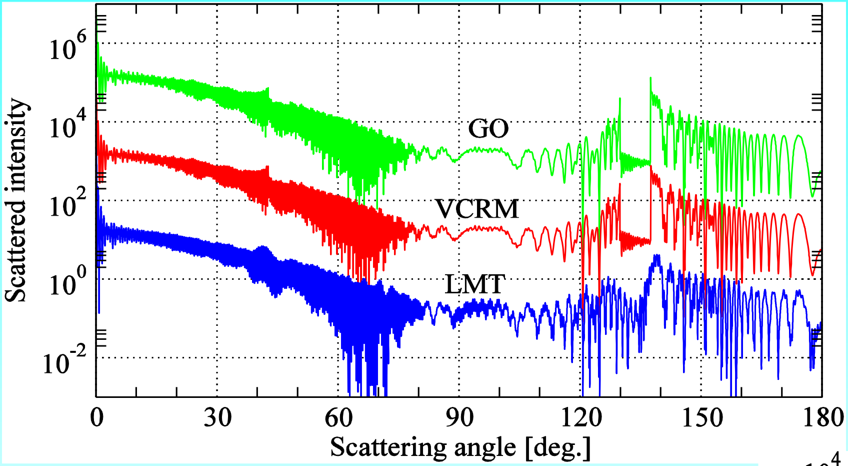
Sphere a=100 µm **plane wave**

Discrepancy found near rainbow angles



# Vectorial Complex Ray Model

## Comparison with Lorenz-Mie theory

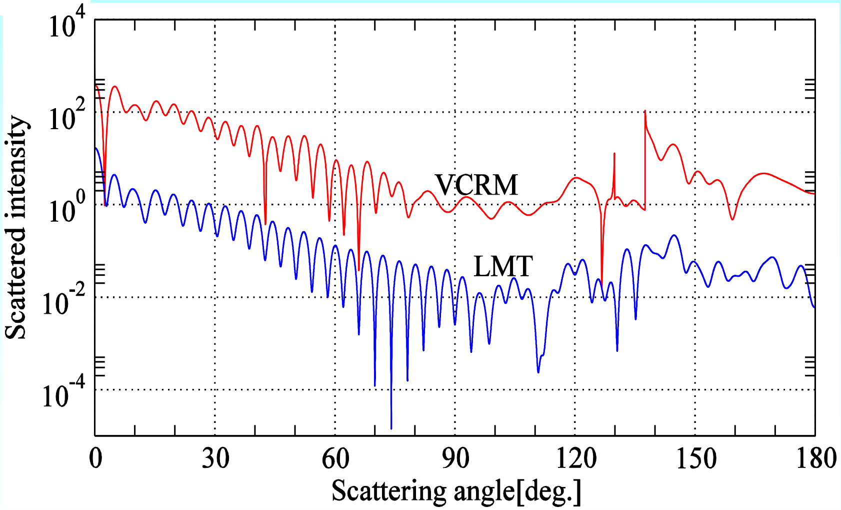



← Scattering diagrams by LMT, GO and VCRM for an infinite circular cylinder :

- refractive index: m = 1.33,
- **radius a = 50 μm**
- wavelength λ = 0.6328 μm.

→ Scattering diagrams by LMT, GO and VCRM for an infinite circular cylinder :

- refractive index: m = 1.33,
- **radius a = 5 μm**
- wavelength λ = 0.6328 μm.




UNIVERSITÉ DE ROUEN

coria UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

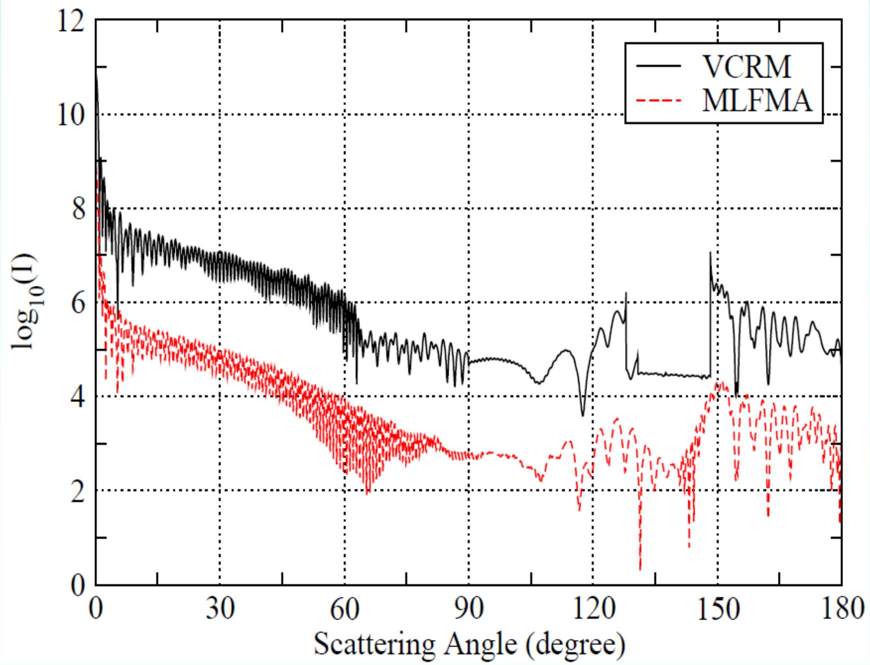
# Vectorial Complex Ray Model

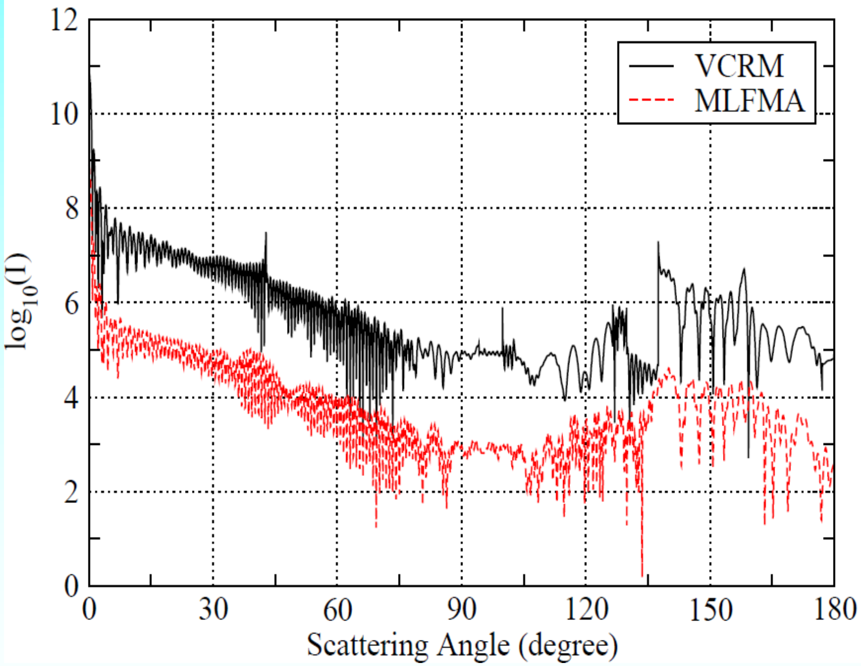
## Comparison with exact numerical method MLFMA

A spheroidal water droplet (*m*=1.33) *a* = **30 μm** and a plane wave of wavelength λ=0.785 μm.

prolate: $a = b$, $c = 1.1a$

oblate: $a = 0.9b$, $c = a$




UNIVERSITÉ DE ROUEN

coria UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

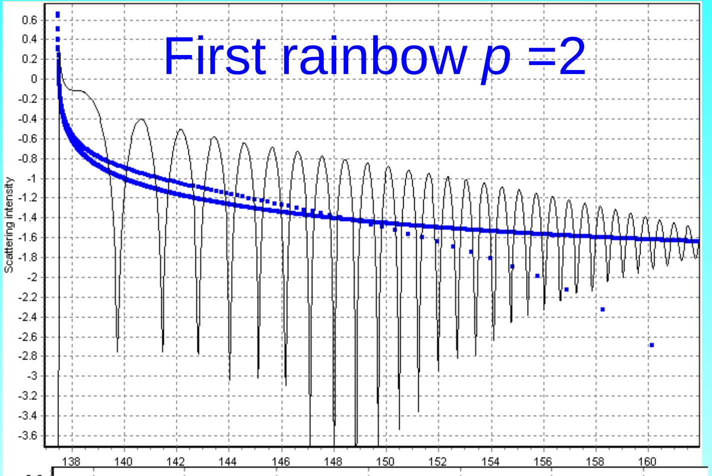
# Vectorial Complex Ray Model

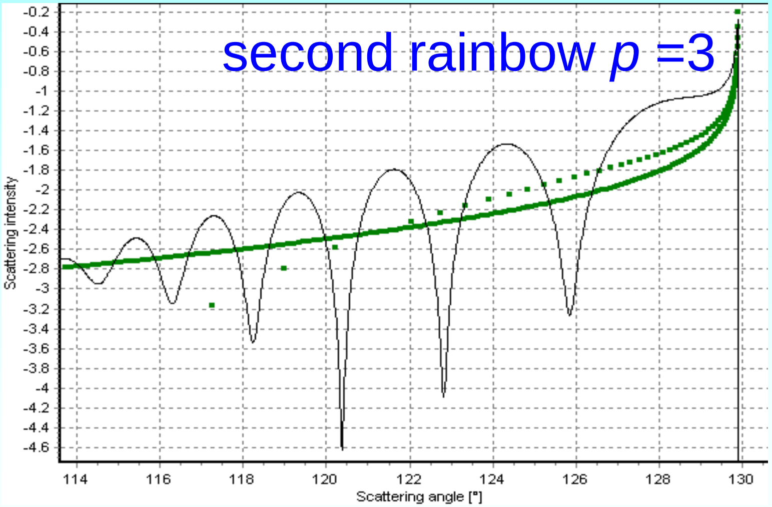
## Interference near rainbow angles

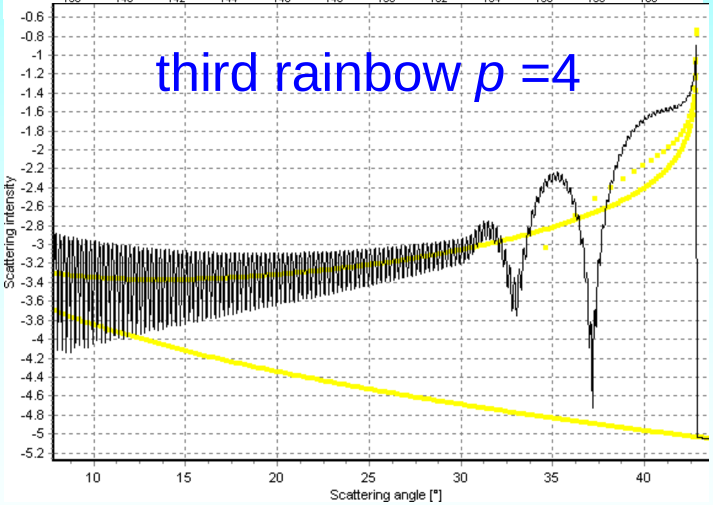
Plane wave incident on a sphere:

λ=0.6328 μm,

$a$ = 100 $\mu$m,   $m$ = 1.33



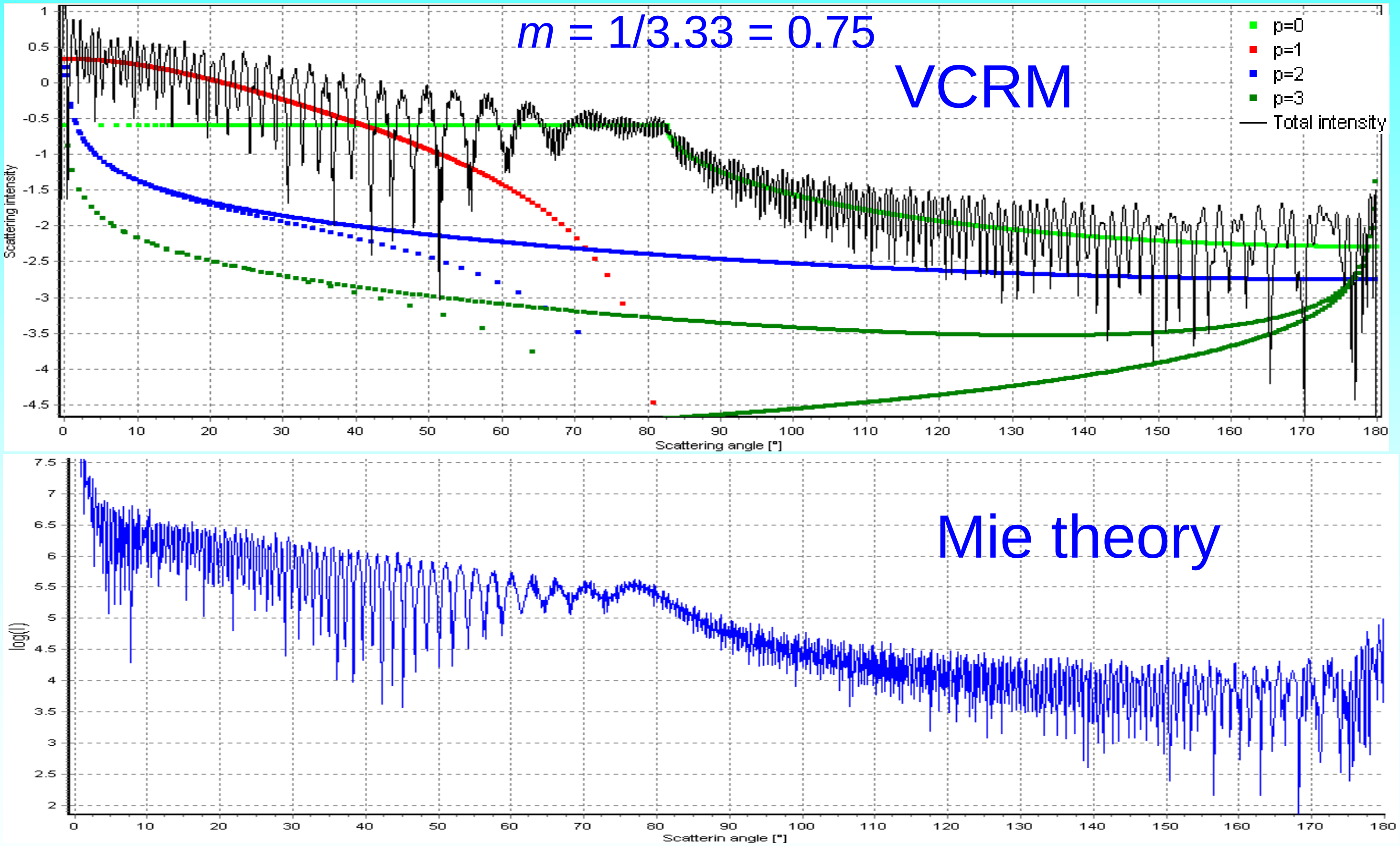




UNIVERSITÉ DE ROUEN

coria
UMR 6614
CNRS - UNIVERSITE et INSA de ROUEN

# Vectorial Complex Ray Model

## Comparison with Mie theory for a bubble




UNIVERSITÉ DE ROUEN

coria
UMR 6614
CNRS - UNIVERSITE et INSA de ROUEN

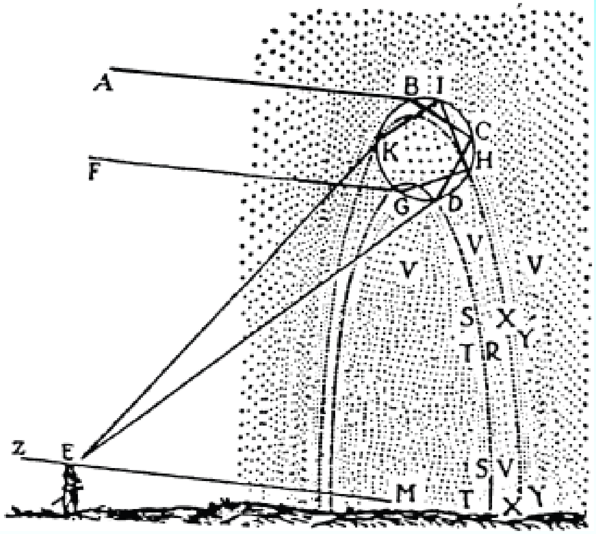
XIDIAN UNIVERSITY

# Ray Theory of Wave

***The rainbow is a marvel of nature.***

*L'arc-en-ciel est une merveille de la Nature si remarquable, et sa cause a été de tout temps si curieusement recherchée par les bons esprits, et si peu connue, que je ne saurais choisir de matière plus propre à faire voir comment, par la méthode dont je me sers, on peut venir à des connaissances que ceux dont nous avons les écrits n'ont point eues.*

彩虹乃自然之奇观
自古引圣贤索其源




**The Colors of bows are due to the refractive index.**
But the intensity tends to infinity !!

UNIVERSITÉ DE ROUEN
coria UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

# Ray Theory of Wave

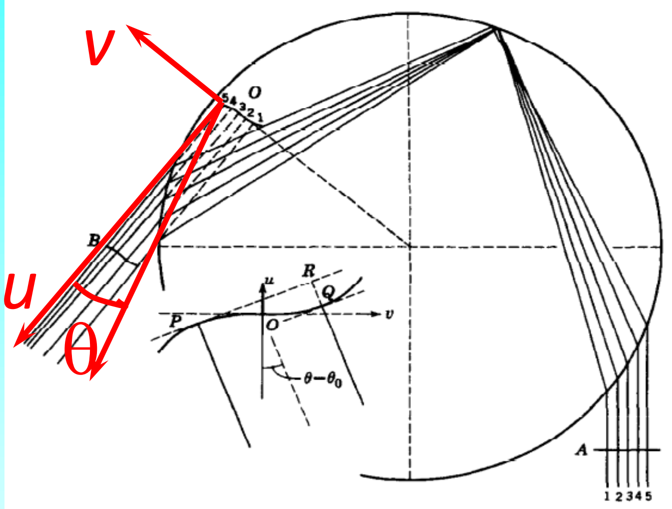
## Airy theory (1838)

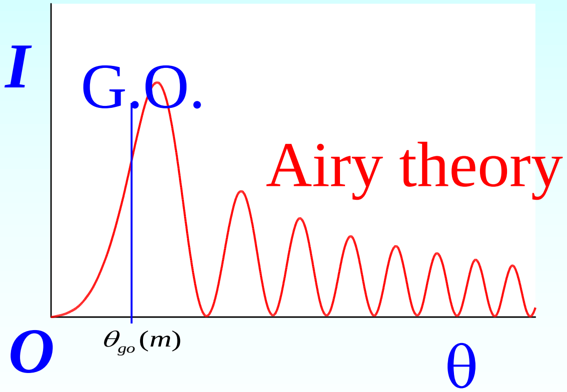

- Phase difference is:

$$\Delta\Psi = khv^3/3a^2$$

- Amplitude is constant for all emergent rays
- Amplitude of scattered field in θ direction:

$$\int_{-\infty}^{\infty} e^{-ikv(\theta-\theta_0)+ikhv^3/3a^2}\,dv$$




Airy theory predict a good profile,

Questions :

- The phase function found in $v$ -> 0 but integration to infinity ?
- The amplitude is not constant near rainbow angles!
- It is only valid for a sphere.

# Ray Theory of Wave

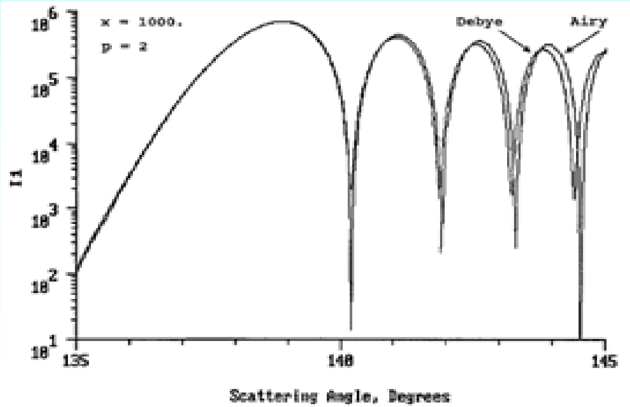
## Airy theory in 1990s'

R. Wang and van de Hulst
Appl. Opt. 30(1):106, 1991

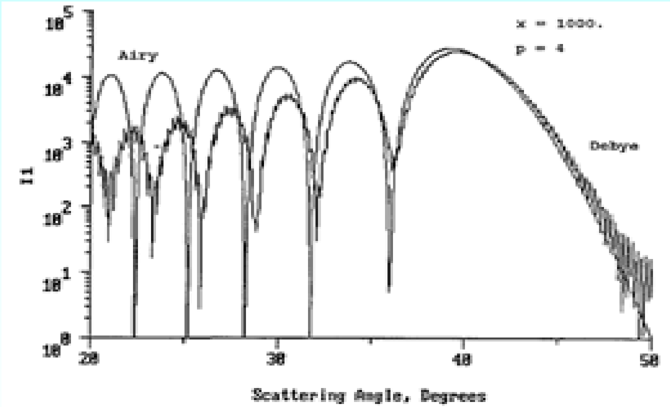
Can anything new be said about the rainbow? Yes. The insight that this phenomenon arises from the play of light in a single spherical drop is 7 centuries old, the full geometrical optics theory of Descartes 3-1/2 centuries, and its modification by Airy to take account of diffraction just 1-1/2 centuries. Exactly a century ago

- Wang, van de Hulst et Lock: same principle but with amplitude correction:

$$E^p_{Airy}(\theta) = x\left(\frac{2\pi\sin\theta_i^R}{\sin\theta^R}\right)^{1/2}\frac{x^{1/6}}{h^{1/3}}T^{21}(\theta_i^R)\times\left[R^{11}(\theta_i^R)\right]^{p-1}T^{12}(\theta_i^R)\times Ai\left(\frac{-x^{2/3}\Delta}{h^{1/3}}\right)\exp(2\pi i L^R/\lambda)\exp\left[(ix\Delta)\left(\frac{p^2-n^2}{p^2-1}\right)^{1/2}\right]$$




Airy theory compared to Debye theory (rigorous)




UNIVERSITÉ DE ROUEN

coria UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

# Ray Theory of Wave

## Airy theory in 1990s'

- **Revisit of Airy theory** to:
  1. understand the method,
  2. Extend to **non-spherical particle**.
- **Theoretical demonstration for a sphere**
  - We know for each emergent ray:

    phase: $\Phi = 2ka(\sin\tau - pm\sin\tau')$

    deviation angle: $\theta = 2\tau - 2p\tau'$
  - The derivatives: ($v \sim 0$):

$$\left.\frac{du}{dv}\right|_{\tau_0} = 0 \qquad \left.\frac{d^2u}{dv^2}\right|_{\tau_0} = 0 \qquad \frac{d^3\Delta\Phi}{d\tau^3} = -2kah\sin^3\tau_0 = -\frac{2ka(p^2-1)}{p^2}\cos\tau_0$$

so

$$h = \frac{p^2-1}{p^2\sin^2\tau_0\tan\tau_0} = \frac{(p^2-1)^2\sqrt{p^2-m^2}}{p^2(m^2-1)^{3/2}}$$

UNIVERSITÉ DE ROUEN coRia UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

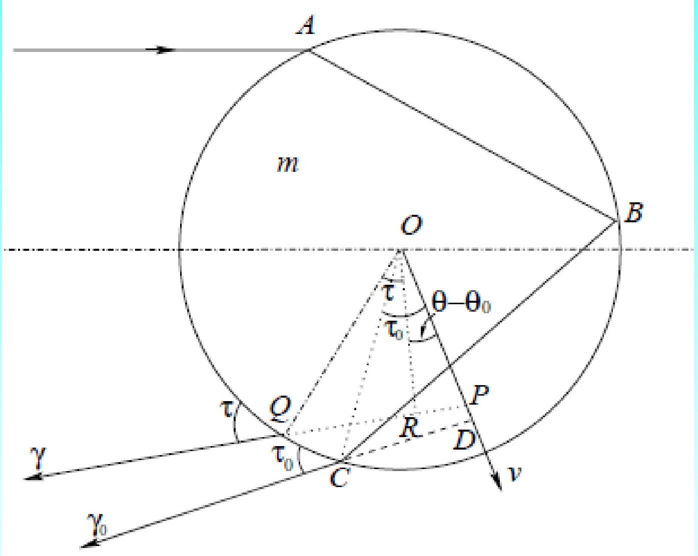
# Ray Theory of Wave

## Airy theory in 21st century

- **Phase:** $\Delta\Phi = \Phi(\boldsymbol{k}) - \Phi(\boldsymbol{k}_r) - k\overline{PR}$

with

$$\begin{aligned}
\overline{OD} &= \boldsymbol{r}_0 \cdot \hat{\boldsymbol{k}}_{r\perp} \\
\overline{OR} &= \boldsymbol{r} \cdot \hat{\boldsymbol{k}}_{\perp} \\
\overline{OP} &= \frac{\overline{OR}}{\hat{\boldsymbol{k}}_r \cdot \hat{\boldsymbol{k}}} = \frac{\overline{OR}}{\hat{\boldsymbol{k}}_{r\perp} \cdot \hat{\boldsymbol{k}}_{\perp}} \\
\overline{PR} &= \overline{OP}(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}_{r\perp}) \\
v &= \overline{OP} - \overline{OD}
\end{aligned}$$




- **Amplitude**

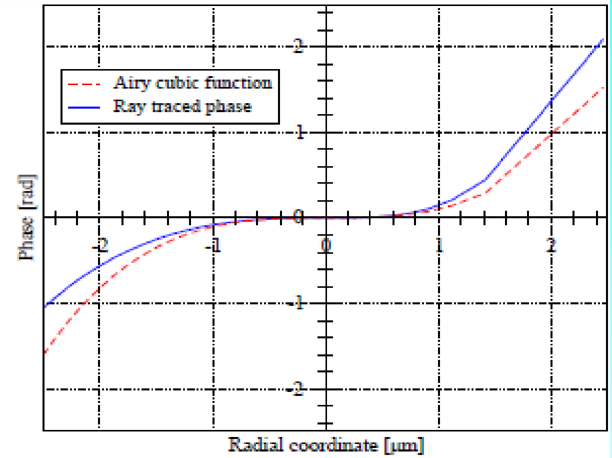
The amplitude of each ray is naturally calculated in VCRM.

- All values are calculated **numerically**,
- **Without any hypothesis/condition.**
- The model is directly applicable to **any shaped particle.**

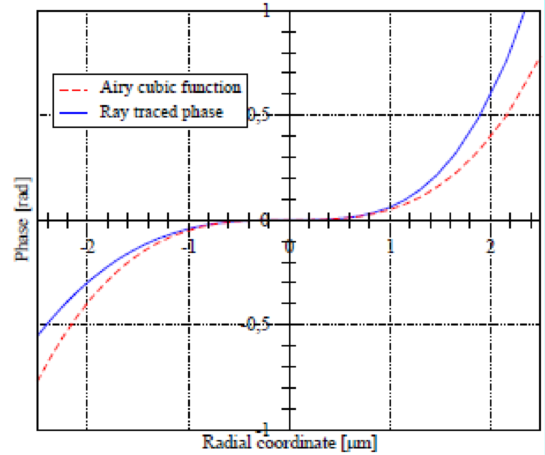
# Ray Theory of Wave

## Airy theory in 21st century

Phases calculated by Airy and VCRM

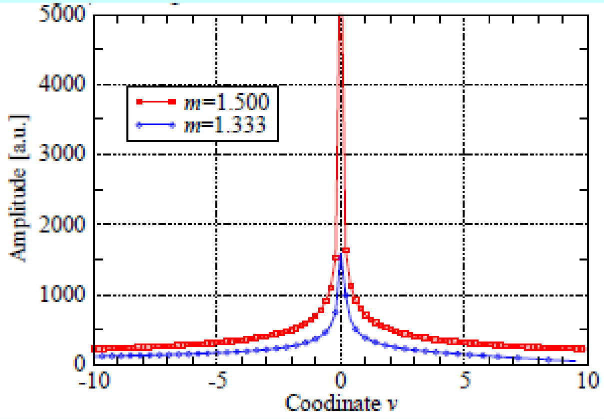



(a). $m = 1.333$




(b). $m = 1.5$

Amplitudes calculated by Airy and VCRM



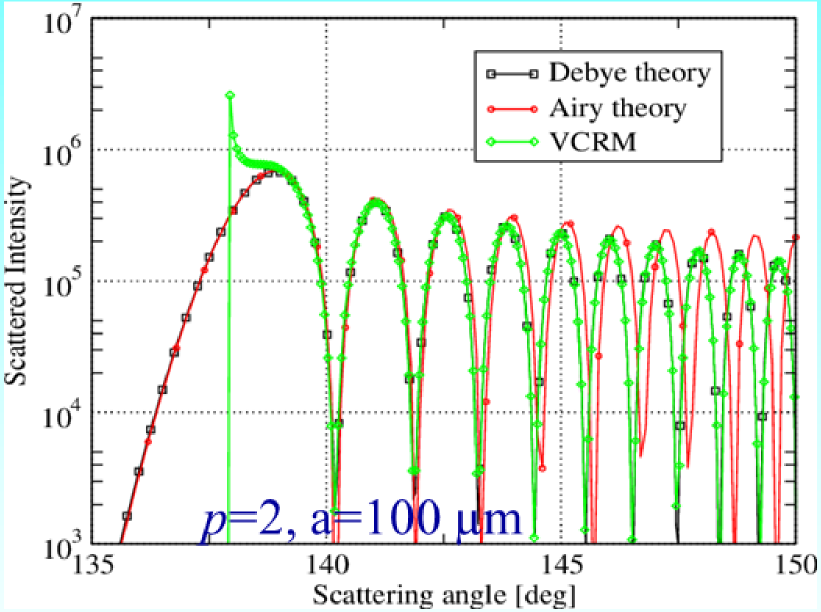

UNIVERSITÉ DE ROUEN

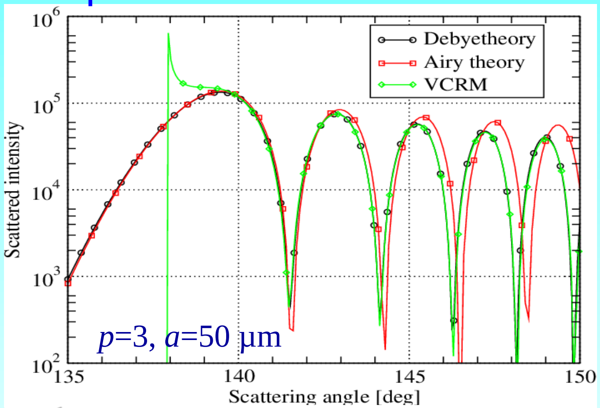
coria
UMR 6614
CNRS - UNIVERSITE et INSA de ROUEN

# Ray Theory of Wave

## Toward the ***Ray theory of wave***

VCRM can predict much better Airy structure than the Airy theory and can be applied directly to non-spherical particle.



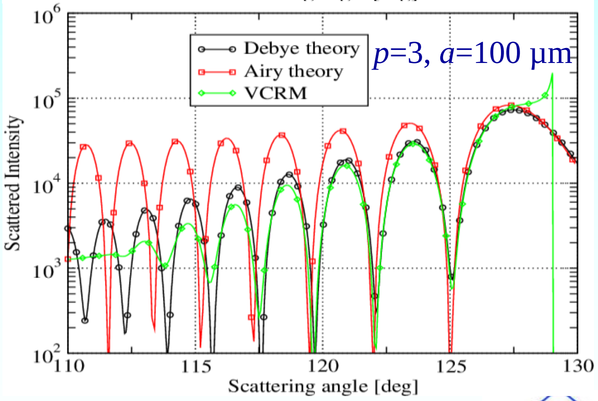




Fig. 3 Comparison of Airy structure calculated with the three methods.

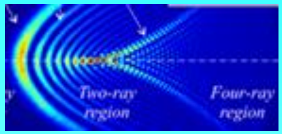
UNIVERSITÉ DE ROUEN

coria UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

# State-of-art on Ray Theory of Wave

## Motivations

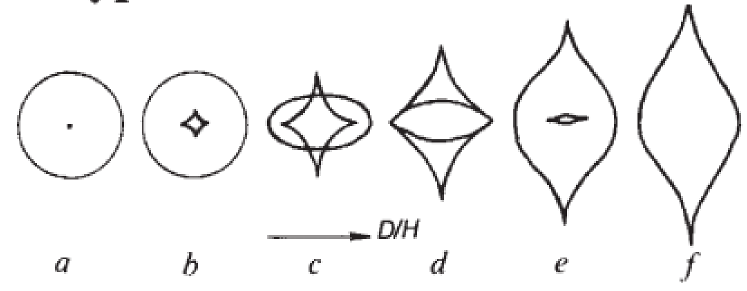
*An example*




- **Understanding of the natural phenomena:**
  - Fine structure of rainbow
  - Optical caustics, cusp and catastrophes

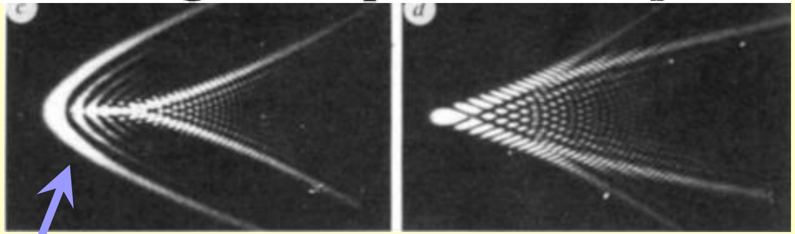
J. F. **Nye**, **Nature** (London) 312, 531 (1984)

**Rainbow scattering from spheroidal drops—an explanation of the hyperbolic umbilic foci**

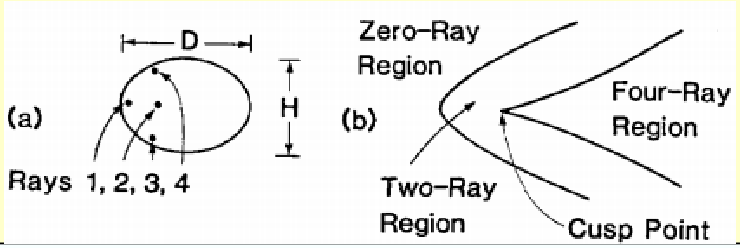



P. L. **Marston**, E. H. Trinh, **Nature** (London) 312, 529 (1984)

**Hyperbolic umbilic diffraction catastrophe and rainbow scattering from spheroidal drops**



Marston, Opt. Lett. 1985

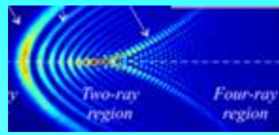



Can we **predict** the **fine** structure ?

UNIVERSITÉ DE ROUEN

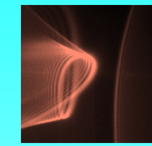
coria UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

# State-of-art on Ray Theory of Wave

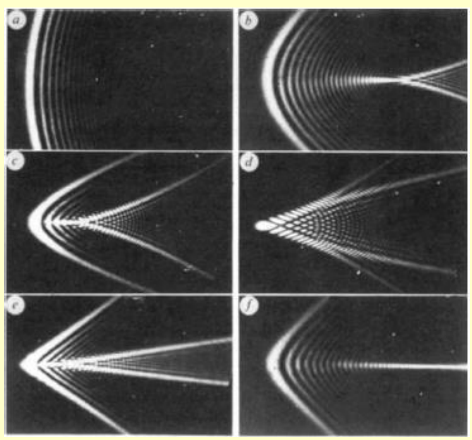



## Motivations

*An example*

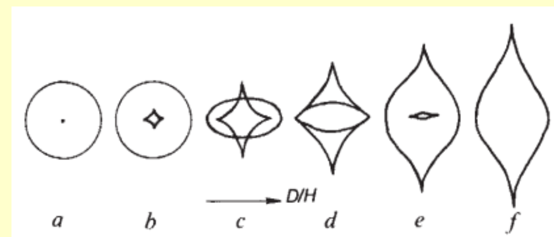


- **Application to multiphase flow:**
  - Fine structure of rainbow
  - Optical caustics, cusp and catastrophes



35 years ago

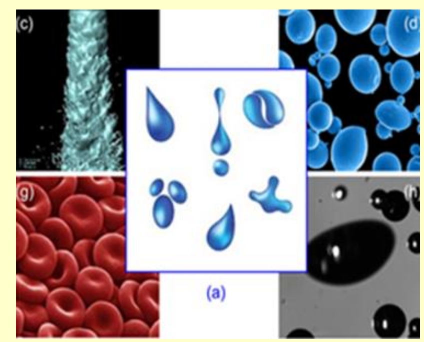
Marston → Nye




Today

1. Prediction of fine structure
2. Inversion

and

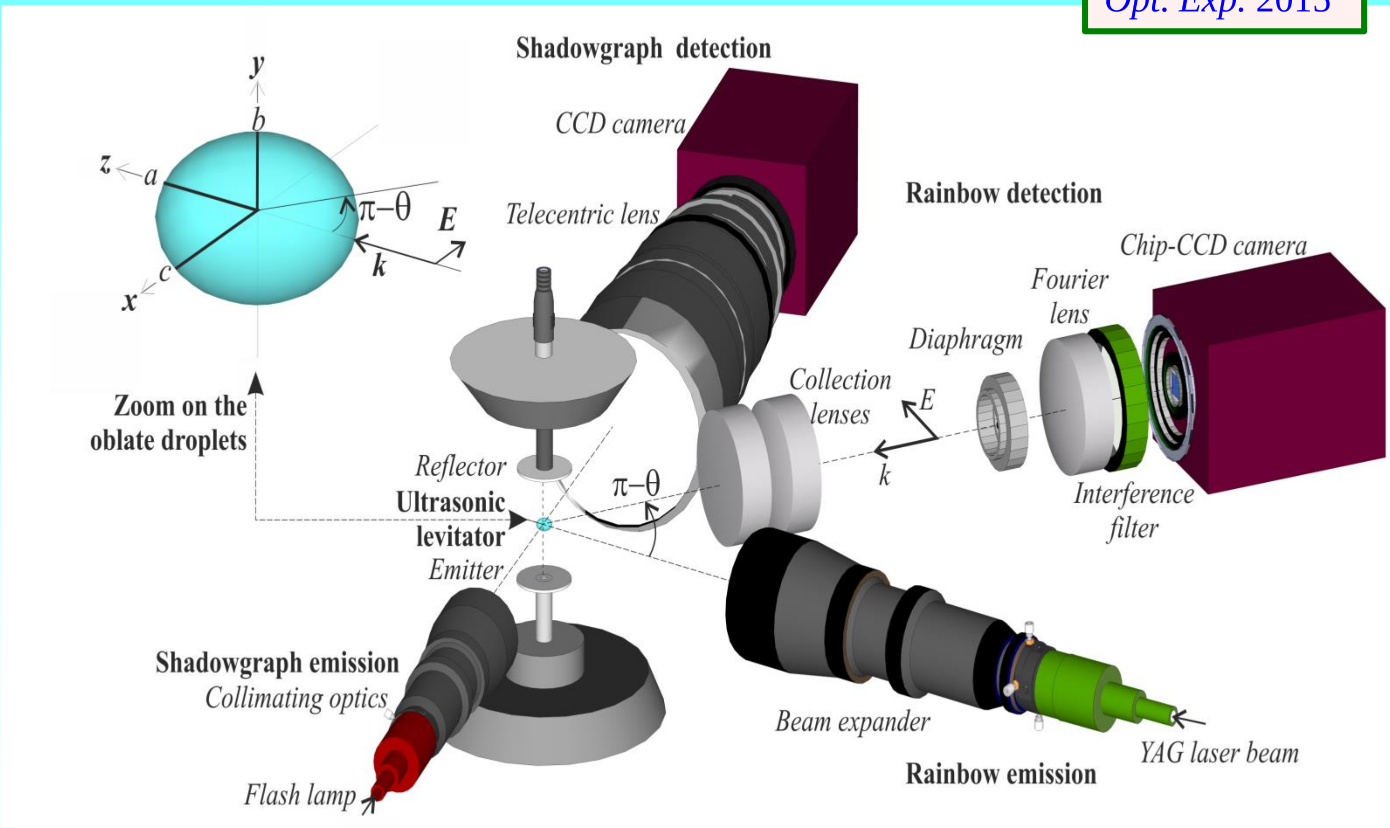


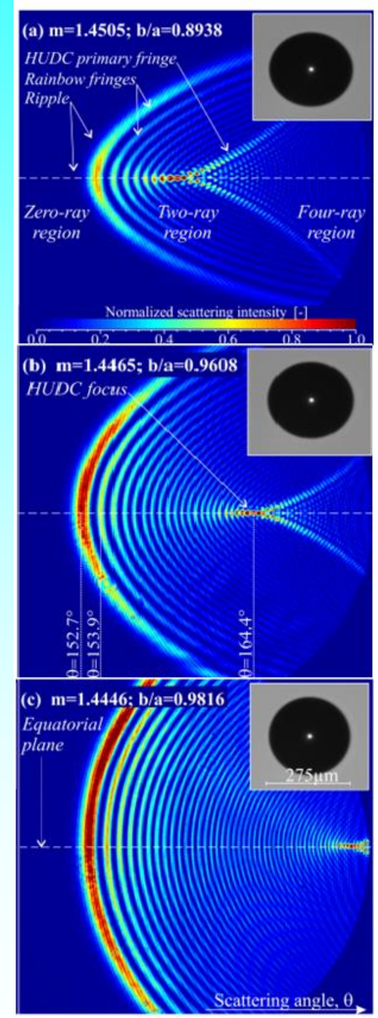
# State-of-art on Ray Theory of Wave

## Experimental validation

- **Experimental set-up**

Figure 3 in Onofri et al, *Opt. Exp.* 2015




UNIVERSITÉ DE ROUEN

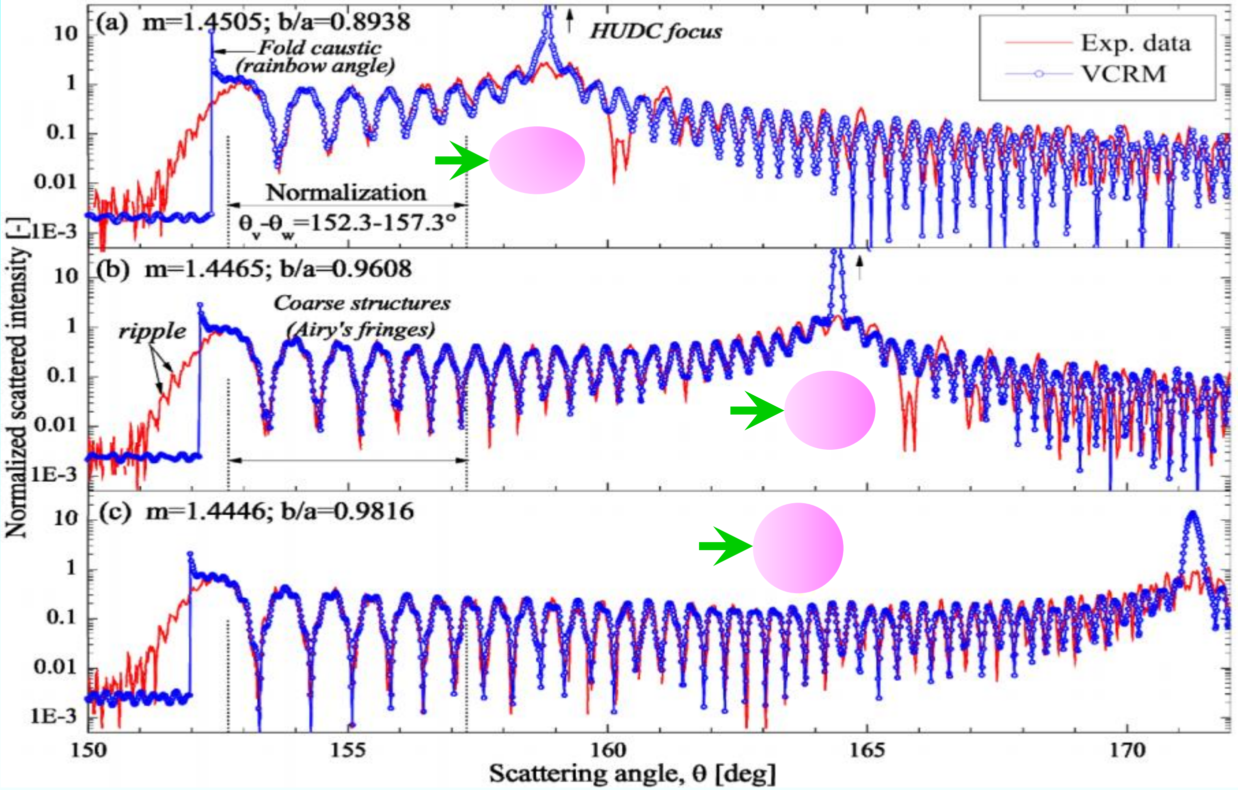
coria
UMR 6614
CNRS - UNIVERSITE et INSA de ROUEN

# State-of-art on Ray Theory of Wave

## Experimental validation

- **Comparison of the results**

Figure 3 in Onofri et al, *Opt. Exp.* 2015




DEHS: Di-Ethyl-Hexyl-Sebacat

HUDC: Hyperbolic Umbilic Diffraction Catastrophe

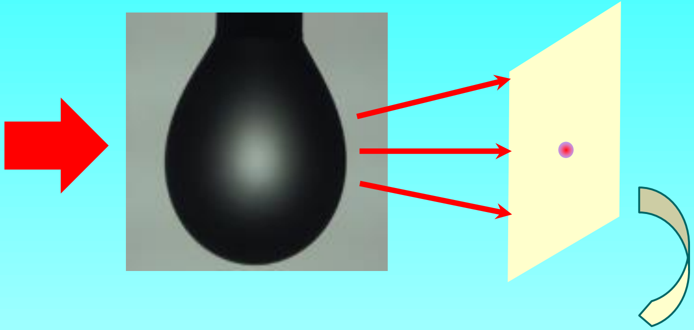
**Comparison of VCRM and experimental normalized equatorial scattering diagrams for the droplets of 3 different aspect ratios. From (a) to (c), the droplet's aspect ratio *b/a* increases and refractive index decreases when the amplitude of the acoustic field is reduced.**

UNIVERSITÉ DE ROUEN

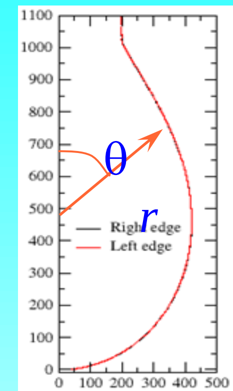
coria UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

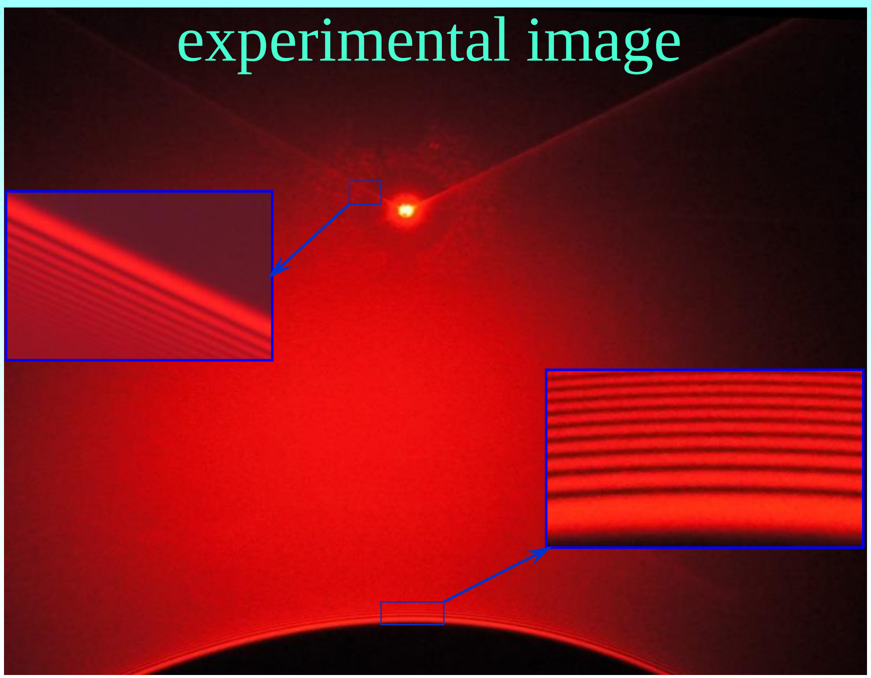
# State-of-art on Ray Theory of Wave

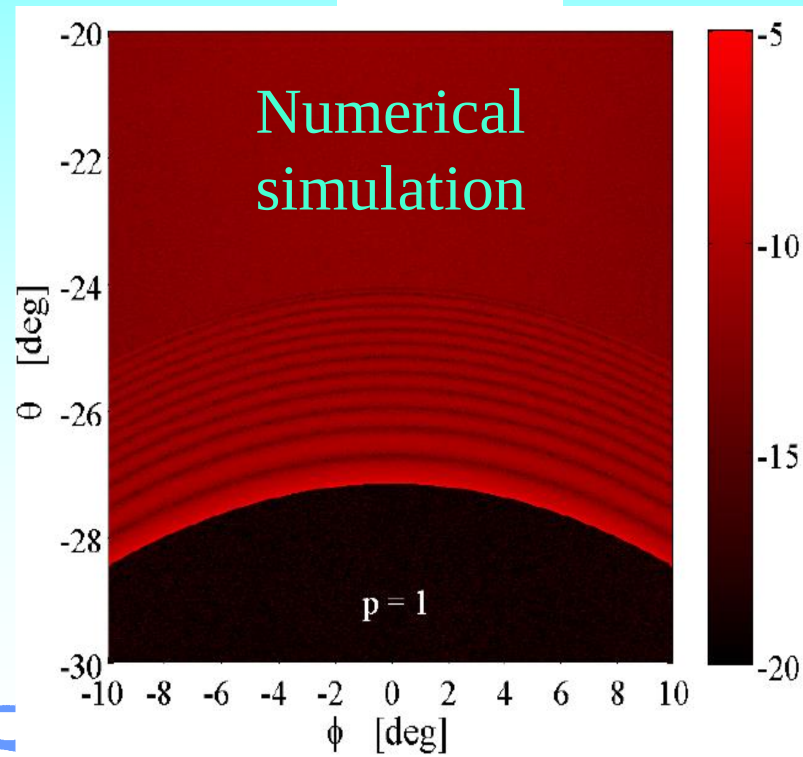
- **Light scattering by a pendant water drop**



$$r(\theta) = a_0 + \sum_{i=2}^{10} a_i \theta^i$$



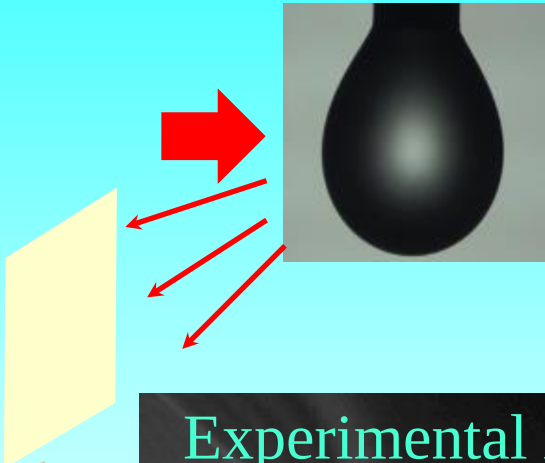




UNIVERSITÉ DE ROUEN

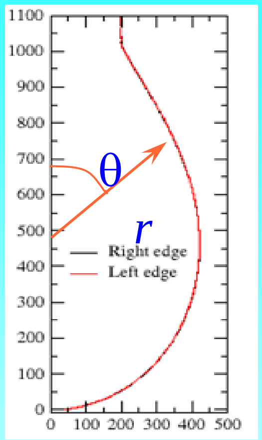
UMR 6614
CNRS - UNIVERSITE et INSA de ROUEN

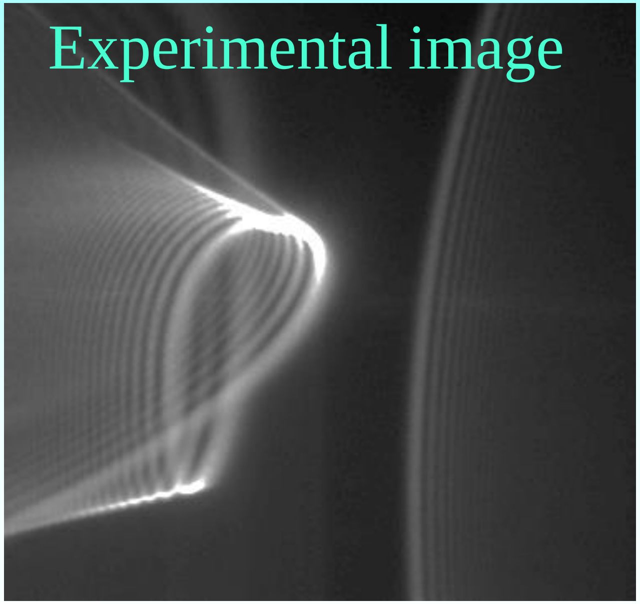
# State-of-art on Ray Theory of Wave

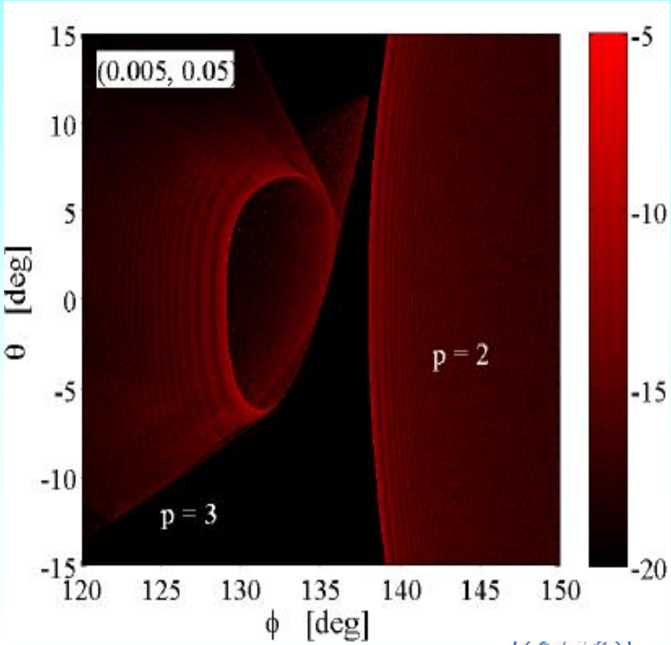
- **Light scattering by a pendant water drop**



$$r(\theta) = a_0 + \sum_{i=2}^{10} a_i \theta^i$$




UNIVERSITÉ DE ROUEN

coria
UMR 6614
CNRS - UNIVERSITE et INSA de ROUEN

# State-of-art on Ray Theory of Wave

## ➢ Shaped beams in VCRM

- Electric field of a wave:

$$\widetilde{\boldsymbol{E}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) = \boldsymbol{E}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\boldsymbol{e}^{\boldsymbol{i\varphi}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})}$$

Where $E(x,y,z)$ is the amplitude and $\varphi(x,y,z)$ the phase.

- Propagation direction

$$\hat{\boldsymbol{k}} = \frac{\nabla\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})}{\|\nabla\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\|}$$

- Curvature matrix of the wave front surface:

The wave front surface being $\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) =$Const. the principal directions and the principal curvatures are determined by the ***generalized eigenvalues problem***:

$$\begin{vmatrix} L-\kappa E & M-\kappa F \\ M-\kappa F & N-\kappa G \end{vmatrix} = 0$$

*E*,*F*,*G* and *L*,*M*,*N* are respectively parameters of the first and the second fundamental forms of $\boldsymbol{\varphi}$ =C.

# State-of-art on Ray Theory of Wave

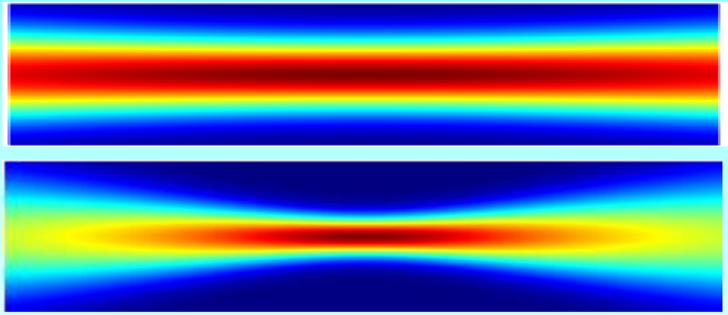
## ➢ Shaped beams in VCRM

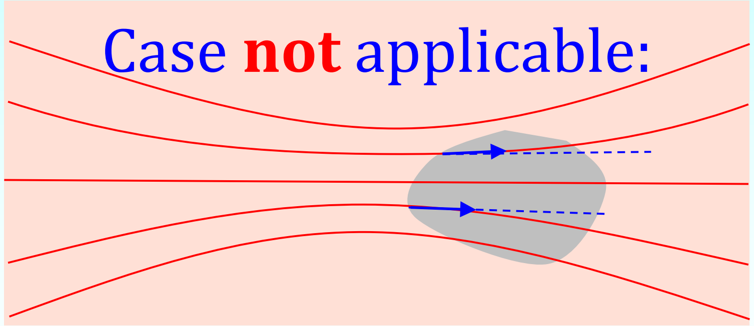
***Light beam in VCRM***

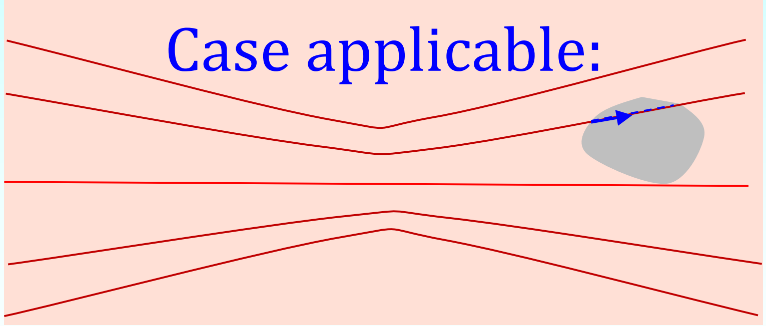
*1. The object is large compared to the wavelength.*

2. The rays propagate ***rectilinearly / straightly*** in a homogeneous medium, at least approximately in the scale of the object.

- No problem for collimated beams
- For focused beam,
  The object should be small and far from the beam waist.



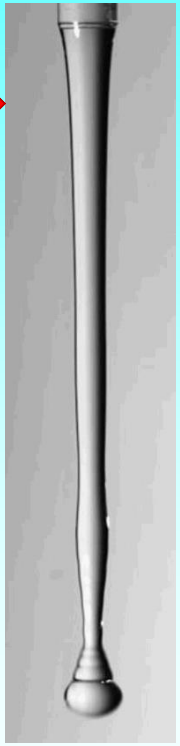


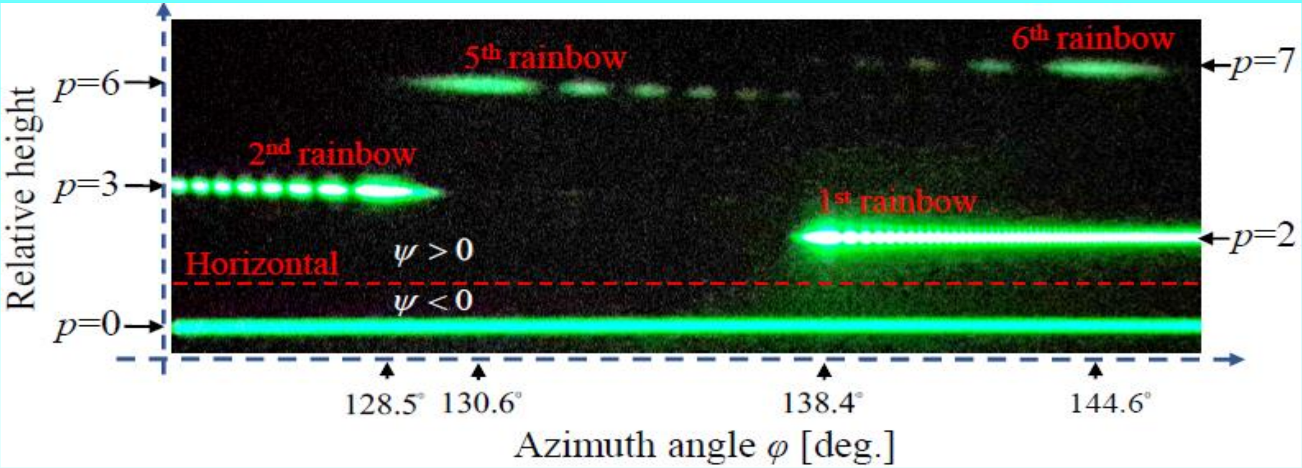



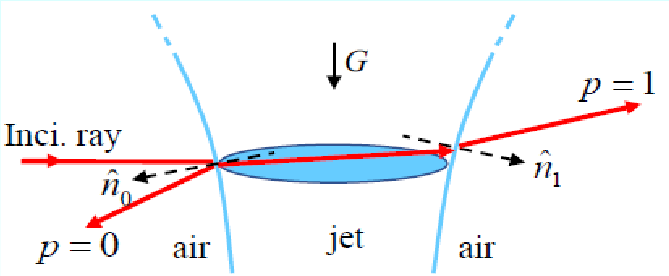
# State-of-art on Ray Theory of Wave

- **Scattering of an elliptical Gaussian beam by a real liquid jet**

Experimental scattering pattern





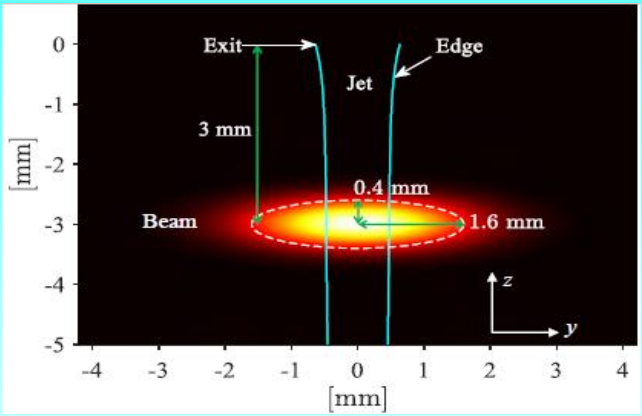

UNIVERSITÉ DE ROUEN

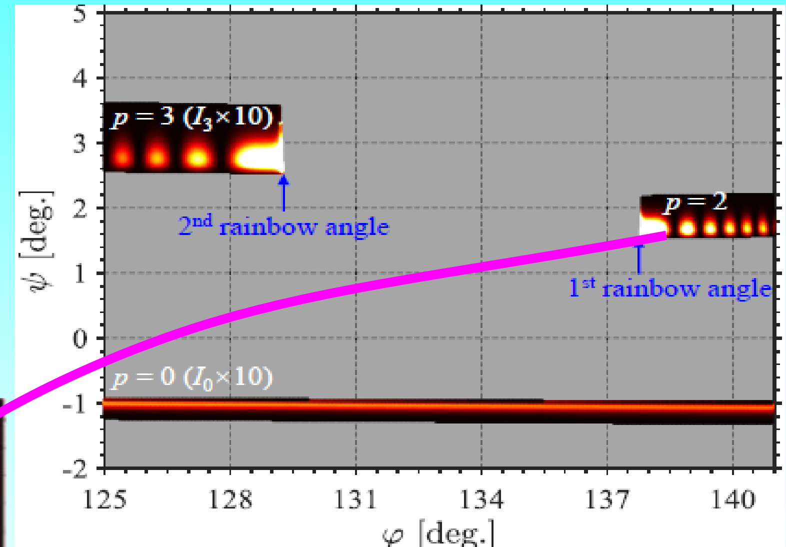
coria UMR 6614 CNRS - UNIVERSITE et INSA de ROUEN

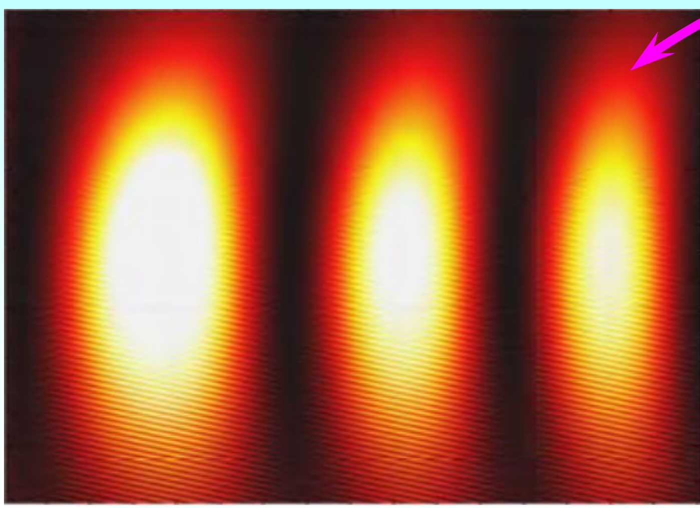
# State-of-art on Ray Theory of Wave

- **Scattering of an elliptical Gaussian beam by a real liquid jet**

Simulation with VCRM



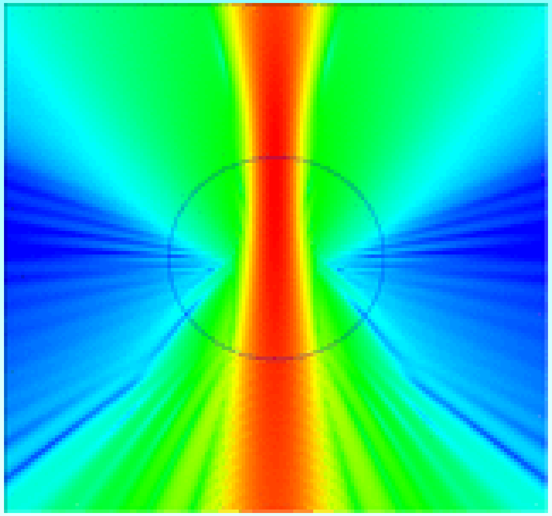



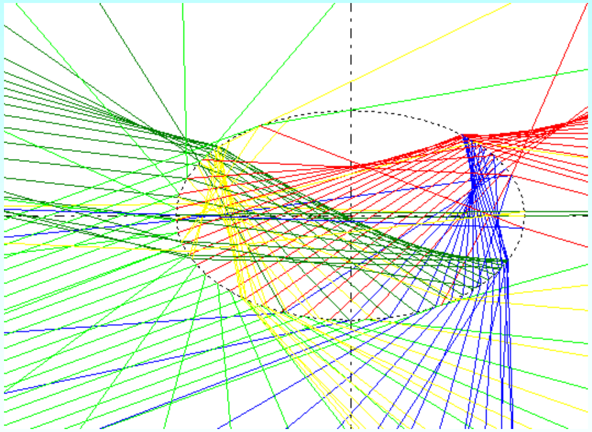
# Conclusions

- **Conclusions**
  - The Vectorial Complex Ray model (VCRM) is established.
  - VCRM has been validated :
    - theoretically for the spherical and cylindrical particle.
    - numerically for the scattering of a ellipsoid.
    - experimentally for the scattering of an oblate droplet, a real liquid jet and a pendant drop.
  - A software for 2D  ellipsoid is realized.
  - VCRM can be applied to any shaped beams,
  - First step to the Ray Theory of Wave achieved.

- **Application fields:**
  - Metrology of non-spherical particles in multiphase flow,
  - Prediction of radiation force, torque, and stress,
  - Application to freeform optical systems,
  - Diagnostics for micro-fluidics.

# Thank you for your attention





UNIVERSITÉ DE ROUEN

coRia
UMR 6614
CNRS - UNIVERSITE et INSA de ROUEN