Lecture at Xidian University
On frontiers of modern optics

Scattering of shaped beam by particles and its applications

III. Description and scattering of shaped beam

Kuan Fang REN

CORIA/UMR 6614 CNRS - Université et INSA de Rouen
School of physics and optoelectronic Eng., Xidian University

西安电子科技大学
现代光学前沿专题

波束散射理论和应用

第三讲：波束描述和散射

任宽芳

法国鲁昂大学 — CORIA研究所
西安电子科技大学物理与光电学院

个人网页：http://ren.perso.neuf.fr
电话：中国 8820 2673， 法国 02 32 95 37 43

Plan of lecture

- EM field expression of shaped beam
- Expansion of shaped beam
- Formulae of physical quantities
- Examples of calculation and Conclusions

EM expression of a shaped beam

Plane wave – the simplest wave

Propagation along z direction: $\vec{k} = k\hat{z}$

– polaraized in *x* direction:

$$\boxed{\vec{E} = \hat{x}E_0 e^{-i(\omega t - \vec{k}\cdot\vec{r})}}$$

– polarized in *y* direction:

$$\boxed{\vec{E} = \hat{y}E_0 e^{-i(\omega t - \vec{k}\cdot\vec{r})}}$$

Plane wave: Constant amplitude : $A=E_0$.

Shaped beam: $A = E(x,y,z)$

How to describe a shaped beam?

1. The fields expressions must satisfy the Maxwell equations.
2. The theoretical fields describe as precisely as possible the real fields.

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EM expression of a shaped beam

Davis' model:

Cf: K. F. Ren, Thesis

- EM field expressed in vector potential:

$$\boldsymbol{H} = \frac{1}{\mu}\nabla \times \boldsymbol{A} \qquad \boldsymbol{E} = -i\omega\left[\boldsymbol{A} + \frac{1}{k^2}\nabla(\nabla \cdot \boldsymbol{A})\right]$$

- Equation of vector potential:

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = 0$$

- We suppose for a beam propagating in z direction and polarized in x direction:

$$A_x = \frac{iE_0}{\omega}\psi(x, y, z)\exp(-ikz)$$

$$\boxed{\psi = \psi_0 + s^2\psi_2 + s^4\psi_4 + \cdots}$$

$$\boxed{\nabla^2\psi - 2ik\frac{\partial \psi}{\partial z} = 0}$$

To be demonstrated by yourself

EM expression of a shaped beam

Circular Gaussian beam:

Solution of fundamental mode

$$\psi_0 = iQ \exp\left(-iQ\frac{x^2+y^2}{w_0^2}\right)$$

$$Q = \frac{1}{i + \frac{2z}{l}} \qquad s = \frac{w_0}{l} = \frac{1}{kw_0}$$

-Local diameter of the beam:

$$w = w_0\left(1+\frac{4z^2}{l^2}\right)^{1/2}$$

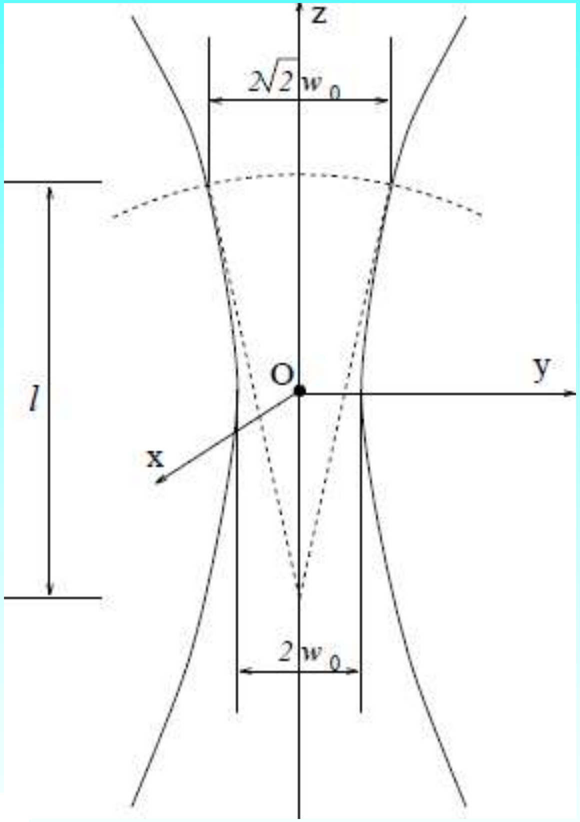
-Curvature radius of the beam at *z* on the axis:

$$R = z\left(1+\frac{l^2}{4z^2}\right)$$

- Higher orders :

$$\psi_2 = \left(2iQ + i\rho^4 Q^3\right)\psi_0$$

$$\psi_4 = \left(-6Q^2 - 3\rho^4 Q^4 - 2i\rho^6 Q^5 - 0.5\rho^8 Q^6\right)\psi_0$$



EM expression of a shaped beam

Symmetric EM field of Gaussian beam at 5th order:

$$
\begin{aligned}
E_x &= E_0\psi_0 \exp(-ikz)\{1 + s^2(-\rho^2Q^2 + i\rho^4Q^3 - 2Q^2\xi^2) \\
&\qquad + s^4[+2\rho^4Q^4 - 3i\rho^6Q^5 - 0.5\rho^8Q^6 + (8\rho^2Q^4 - 2i\rho^4Q^5)\xi^2]\} \\
E_y &= E_0\psi_0 \exp(-ikz)\{s^2(-2Q^2\xi\eta) + s^4[(8\rho^2Q^4 - 2i\rho^4Q^5)\xi\eta]\} \\
E_z &= E_0\psi_0 \exp(-ikz)\{s(-2Q\xi) + s^3[(+6\rho^2Q^3 - 2i\rho^4Q^4)\xi] \\
&\qquad + s^5[(-20\rho^4Q^5 + 10i\rho^6Q^6 + \rho^8Q^7)\xi]\} \\
H_x &= H_0\psi_0 \exp(-ikz)\{s^2(-2Q^2\xi\eta) + s^4[(8\rho^2Q^4 - 2i\rho^4Q^5)\xi\eta]\} \\
H_y &= H_0\psi_0 \exp(-ikz)\{1 + s^2(-\rho^2Q^2 + i\rho^4Q^3 - 2Q^2\eta^2) \\
&\qquad + s^4[+2\rho^4Q^4 - 3i\rho^6Q^5 - 0.5\rho^8Q^6 + (8\rho^2Q^4 - 2i\rho^4Q^5)\eta^2]\} \\
H_z &= H_0\psi_0 \exp(-ikz)\{s(-2Q\eta) + s^3[(+6\rho^2Q^3 - 2i\rho^4Q^4)\eta] \\
&\qquad + s^5[(-20\rho^4Q^5 + 10i\rho^6Q^6 + \rho^8Q^7)\eta]\}
\end{aligned}
$$

$$
\rho^2 = \xi^2 + \eta^2 \qquad \xi = \frac{x}{w_0} \qquad \eta = \frac{y}{w_0}
$$

Same comment as for Gaussian beam at 2nd order but here $O(s^5)$

EM expression of a shaped beam

Elliptical Gaussian beam:

$$\psi_0^{sh} = i\sqrt{Q_x Q_y}\exp\left(-iQ_x\frac{x^2}{w_{0x}^2} - iQ_y\frac{y^2}{w_{0y}^2}\right)$$

$$Q_x = \frac{1}{i + \frac{2z}{l_x}} \qquad Q_y = \frac{1}{i + \frac{2z}{l_y}}$$

$$l_x = kw_{0x}^2 \qquad l_y = kw_{0y}^2$$

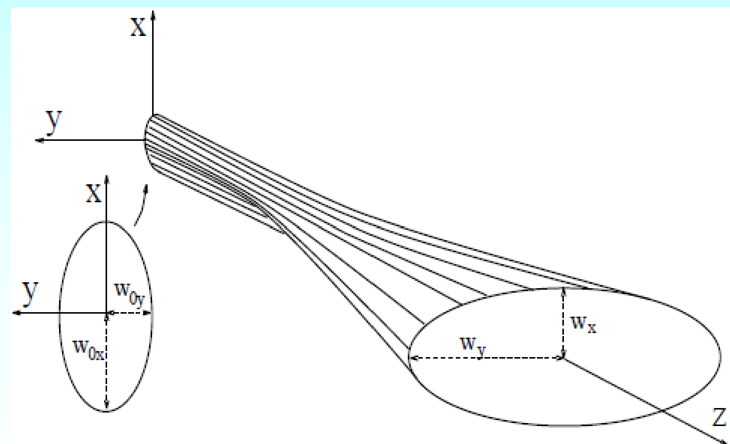
Other solution of the differential equation:

$$\nabla^2\psi - 2ik\frac{\partial\psi}{\partial z} = 0$$

- Local radii and curvature radii

$$w_x = w_{0x}\left(1 + \frac{4z^2}{l_x^2}\right)^{1/2} \qquad R_x = z\left(1 + \frac{l_x^2}{4z^2}\right)$$

$$w_y = w_{0y}\left(1 + \frac{4z^2}{l_y^2}\right)^{1/2} \qquad R_y = z\left(1 + \frac{l_y^2}{4z^2}\right)$$



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EM expression of a shaped beam

EM fields of a Gaussian beam:

$$A_x = \frac{iE_0}{\omega}\psi(x,y,z)\exp(-ikz)$$

$$\boldsymbol{E} = -i\omega\left[\boldsymbol{A} + \frac{1}{k^2}\nabla(\nabla\cdot\boldsymbol{A})\right]$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla\times\boldsymbol{A}$$

$$\psi_0 = iQ\exp\left(-iQ\frac{x^2+y^2}{w_0^2}\right)$$

$$E_x(x,y,z) = E_0\psi_0\exp(-ikz)$$
$$E_y(x,y,z) = 0$$
$$E_z(x,y,z) = -\epsilon_L\frac{2Qx}{l}E_x$$
$$H_x(x,y,z) = 0$$
$$H_y(x,y,z) = H_0\psi_0\exp(-ikz)$$
$$H_z(x,y,z) = -\epsilon_L\frac{2Qy}{l}H_y$$

- Paraxial APPROXIMATION: $O(s^2)$.
- This field does NOT satisfies the Maxwell equations in strict sense.
- The approximation depends on the position in the beam.
- cf. Gouesbet J. Opt. 1985 for circular Gaussian beam

EM expression of a shaped beam

EM field of an elliptical Gaussian beam:

$$\psi_0^{sh} = i\sqrt{Q_x Q_y}\exp\left(-iQ_x\frac{x^2}{w_{0x}^2} - iQ_y\frac{y^2}{w_{0y}^2}\right)$$

$$Q_x = \frac{1}{i+\frac{2z}{l_x}} \qquad Q_y = \frac{1}{i+\frac{2z}{l_y}}$$

$$E_x(x,y,z) = E_0\psi_0^{sh}\exp(-ikz)$$

$$E_y(x,y,z) = 0$$

$$E_z(x,y,z) = -\frac{2Q_x x}{l_x}E_x$$

$$H_x(x,y,z) = 0$$

$$H_y(x,y,z) = H_0\psi_0^{sh}\exp(-ikz)$$

$$H_z(x,y,z) = -\frac{2Q_y y}{l_y}H_y$$

- This is the EM field of linearly polarized (along *x* axis) Gaussian beam.
- Paraxial APPROXIMATION.
- This field does NOT satisfies the Maxwell equations in strict sense.
- The approximation depends on the position in the beam.
- cf. K.F Ren J. Opt. 1994

EM expression of a shaped beam

EM field of a high order Gaussian beam:

cf：Barton, *Appl. Opt*. 1997

$$TEM^{x}_{mn} = \frac{\partial^m \partial^n (TEM^{x}_{00})}{\partial \xi^m \partial \eta^n}, \quad TEM^{y}_{mn} = \frac{\partial^m \partial^n (TEM^{y}_{00})}{\partial \xi^m \partial \eta^n}$$

$$\xi = \frac{x}{w_0}, \ \eta = \frac{y}{w_0}.$$

-With the fundamental mode TEM_{00}:

TEM^{x}_{00}

$$\boldsymbol{E}^{(x)} = E_0 \psi_0 \exp(-ikz) \begin{pmatrix} 1 \\ 0 \\ -2sQ\frac{x}{w_0} \end{pmatrix}$$

$$\boldsymbol{H}^{(x)} = H_0 \psi_0 \exp(-ikz) \begin{pmatrix} 0 \\ 1 \\ -2sQ\frac{y}{w_0} \end{pmatrix}$$

TEM^{y}_{00}

$$\boldsymbol{E}^{(y)} = E_0 \psi_0 \exp(-ikz) \begin{pmatrix} 0 \\ 1 \\ -2sQ\frac{y}{w_0} \end{pmatrix}$$

$$\boldsymbol{H}^{(y)} = H_0 \psi_0 \exp(-ikz) \begin{pmatrix} -1 \\ 0 \\ 2sQ\frac{x}{w_0} \end{pmatrix}$$

EM expression of a shaped beam

Example: TEM_{01} and TEM_{10} mode:

$$\boldsymbol{E}^x_{10} = E_0 \exp(-ikz) \begin{pmatrix} \Omega\xi \\ 0 \\ -s\Omega(i + 2Q\xi^2) \end{pmatrix} \qquad \boldsymbol{H}^x_{10} = H_0 \exp(-ikz) \begin{pmatrix} 0 \\ \Omega\xi \\ -2s\Omega Q\xi\eta \end{pmatrix}$$

$$\boldsymbol{E}^x_{01} = E_0 \exp(-ikz) \begin{pmatrix} \Omega\eta \\ 0 \\ -2s\Omega Q\xi\eta \end{pmatrix} \qquad \boldsymbol{H}^x_{01} = H_0 \exp(-ikz) \begin{pmatrix} 0 \\ \Omega\eta \\ -s\Omega(i + 2Q\eta^2) \end{pmatrix}$$

$$\boldsymbol{E}^y_{10} = E_0 \exp(-ikz) \begin{pmatrix} 0 \\ \Omega\xi \\ -2s\Omega Q\xi\eta \end{pmatrix} \qquad \boldsymbol{H}^y_{10} = H_0 \exp(-ikz) \begin{pmatrix} -\Omega\xi \\ 0 \\ s\Omega(i + 2Q\xi^2) \end{pmatrix}$$

$$\boldsymbol{E}^y_{01} = E_0 \exp(-ikz) \begin{pmatrix} 0 \\ \Omega\eta \\ -s\Omega(i + 2Q\eta^2) \end{pmatrix} \qquad \boldsymbol{H}^y_{01} = H_0 \exp(-ikz) \begin{pmatrix} -\Omega\eta \\ 0 \\ 2\Omega Qs\xi\eta \end{pmatrix}$$

$$\Omega = -2iQ\psi_0 = 2Q^2 \exp\left[-iQ(\xi^2 + \eta^2)\right]$$

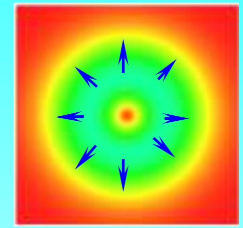
- Same comment as for Gaussian beam but here 2 polarizations (in x and y direction).
- Other polarization EM field can be constructed from these EM.

EM expression of a shaped beam

Doughnut beam:

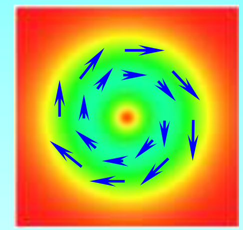
- Radial:

$$
\begin{aligned}
\boldsymbol{E}^{rad}_{dn} &= \frac{E_0}{\sqrt{2}}\exp(-ikz)\begin{pmatrix}\Omega\xi \\ \Omega\eta \\ -2\Omega s\left[i + Q(\xi^2+\eta^2)\right]\end{pmatrix} \\
\boldsymbol{H}^{rad}_{dn} &= \frac{H_0}{\sqrt{2}}\exp(-ikz)\begin{pmatrix}-\Omega\eta \\ \Omega\xi \\ 0\end{pmatrix}
\end{aligned}
$$



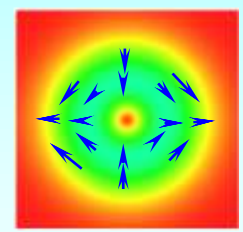
- Angular:

$$
\begin{aligned}
\boldsymbol{E}^{ang}_{dn} &= \frac{E_0}{\sqrt{2}}\exp(-ikz)\begin{pmatrix}\Omega\eta \\ -\Omega\xi \\ 0\end{pmatrix} \\
\boldsymbol{H}^{ang}_{dn} &= \frac{H_0}{\sqrt{2}}\exp(-ikz)\begin{pmatrix}\Omega\xi \\ \Omega\eta \\ -2\Omega s[i + Q(\xi^2+\eta^2)]\end{pmatrix}
\end{aligned}
$$



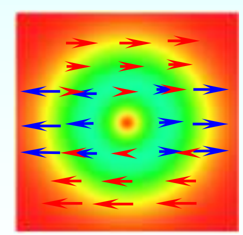
- Arc:

$$
\begin{aligned}
\boldsymbol{E}^{arc}_{dn} &= \frac{E_0}{\sqrt{2}}\exp(-ikz)\begin{pmatrix}\Omega\xi \\ -\Omega\eta \\ 2\Omega Q s(\eta^2-\xi^2)\end{pmatrix} \\
\boldsymbol{H}^{arc}_{dn} &= \frac{H_0}{\sqrt{2}}\exp(-ikz)\begin{pmatrix}\Omega\eta \\ \Omega\xi \\ -4\Omega Q s\xi\eta\end{pmatrix}
\end{aligned}
$$



- helix:

$$
\begin{aligned}
\boldsymbol{E}^{hel}_{dn} &= \frac{E_0}{\sqrt{2}}\exp(-ikz)\begin{pmatrix}\Omega(\xi+i\eta) \\ 0 \\ -\Omega s\left[i + 2Q\xi(\xi+i\eta)\right]\end{pmatrix} \\
\boldsymbol{H}^{hel}_{dn} &= \frac{H_0}{\sqrt{2}}\exp(-ikz)\begin{pmatrix}0 \\ \Omega(\xi+i\eta) \\ \Omega s\left[1 - 2Q\eta(\xi+i\eta)\right]\end{pmatrix}
\end{aligned}
$$



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EM expression of a shaped beam

EM field of a Bessel beam

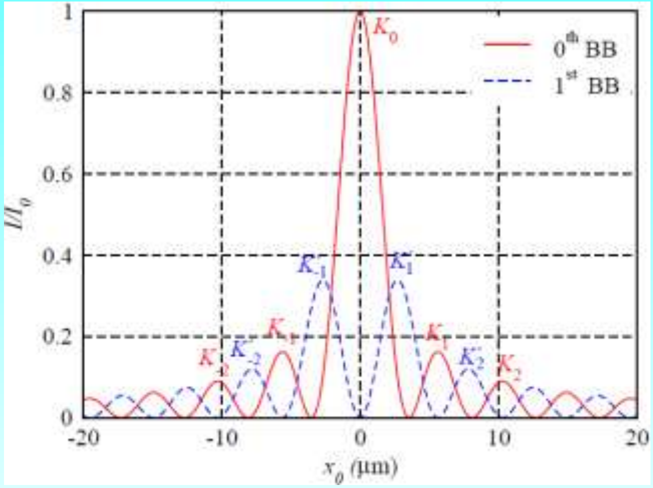
R. X . Li et al, *JQSRT* 2012

$$\mathbf{E}(\mathbf{r}) = \mathbf{e}_x E_0 J_v(k_\rho \rho_G) e^{iv\phi_G} e^{-ik_z(z-z_0)}$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{e}_y H_0 J_v(k_\rho \rho_G) e^{iv\phi_G} e^{-ik_z(z-z_0)}$$

$$\rho_G = \sqrt{\rho^2 + \rho_0^2 - 2\rho\rho_0 \cos(\phi - \phi_0)}$$

$$\phi_G = \tan^{-1}\left(\frac{\rho \sin\phi - y_0}{\rho\cos\phi - x_0}\right)$$



where $k_\rho = k \sin\alpha_0$ and $k_z = k\cos\alpha_0$ with $k = 2\pi/\lambda$

- Bessel function is a non-diffractive beam.
- α_0 is the angle of axicon.
- Amplitude is independent of *z*.:

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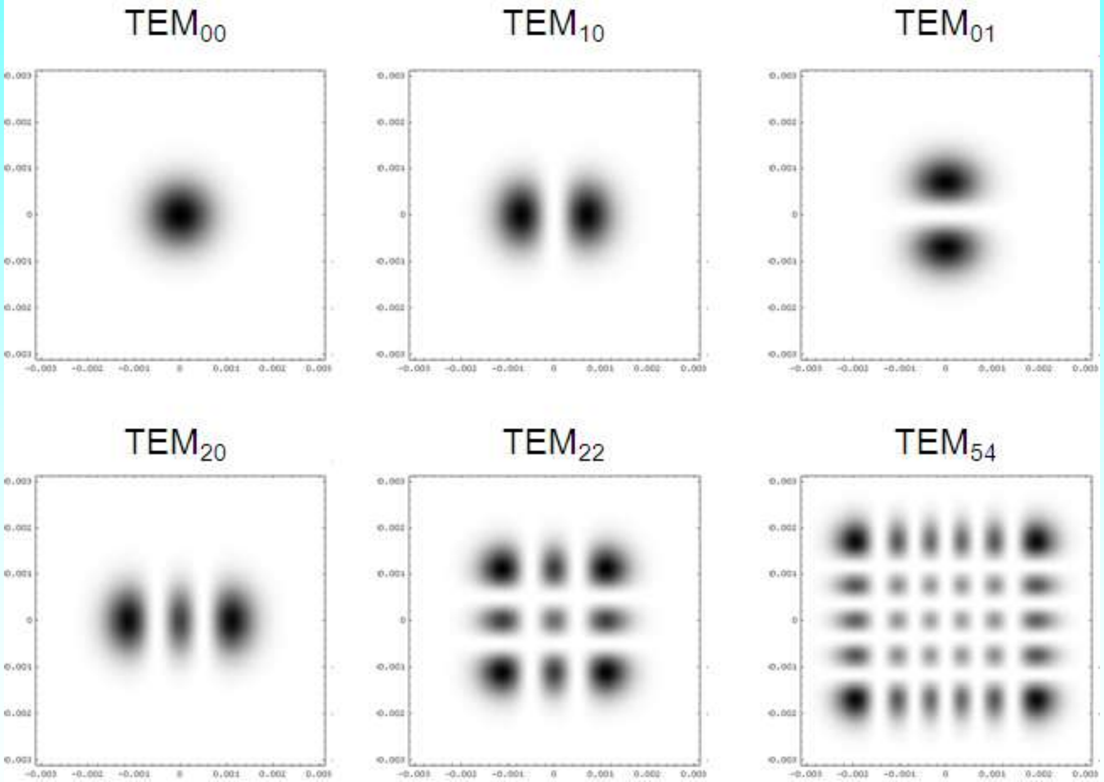
EM expression of a shaped beam

Hermite-Gauss beam:

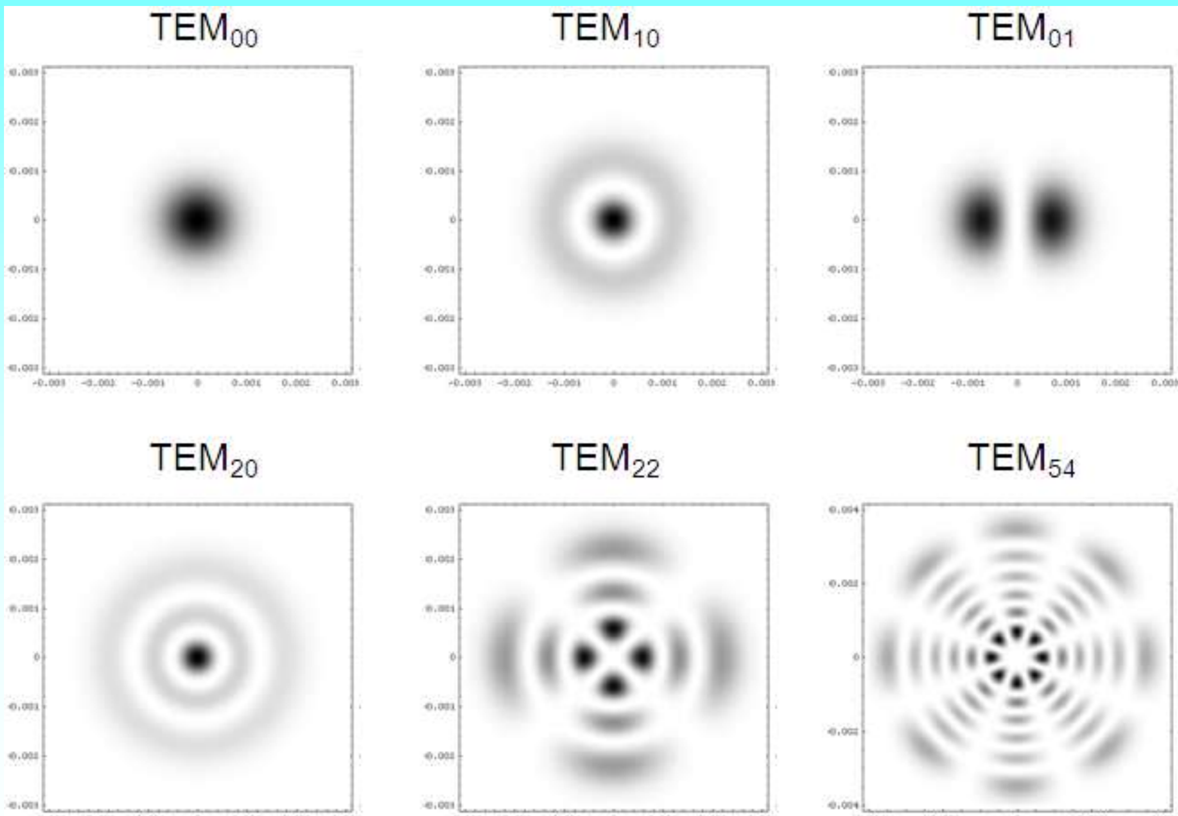
$$\Psi_{mn}(x,y,z)=\frac{w_0}{w(z)}H_m\left(\frac{\sqrt{2}x}{w(z)}\right)H_n\left(\frac{\sqrt{2}y}{w(z)}\right)\exp\left(-\frac{r^2}{w(z)^2}\right)\exp\left(-i\phi_{mn}(r,z)\right)$$



EM expression of a shaped beam

Laguerre-Gauss beam:

$$\Psi_{pl}(r,\theta,z)=\frac{w_0}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{l}L_p^l\left(\frac{2r^2}{w(z)^2}\right)\exp\left(-\frac{r^2}{w(z)^2}-i\phi_{pl}(r,z)\right)\times\begin{cases}\cos l\theta\\ \sin l\theta\end{cases}$$



Expansion of shaped beam

1. In spherical system

Solution:

$$\psi_{mn}(r,\theta,\phi) = z_n(kr) P_n^m(\cos\theta)\exp\left(-im\phi\right)$$

Any wave can be expanded as summation of these spherical functions (similar as Fourier transform).

- $z_n(kr)$ is a spherical Bessel function.

when $kr \to \infty$, $e^{\pm ikr}/kr$. So a spherical wave.

- $P_n^m(\cos\theta)$ is the associate Legendre function.

For a plane wave or an axis symmetric wave (ex. circular Gaussian beam), only m=1 is necessary, so Legendre function $P_n(\cos\theta)$.

- The index n is from 1 to infinity, describing mainly the variation in r.
- The index m from – n to n, describes the symmetry of the beam.

Therefore, for a shaped beam *m* takes not only 1 but also other values depending on its symmetry.

Expansion of shaped beam

Beam shape coefficients in spherical system:

$$\vec{E}_i = E_0 \sum_{n=1}^{\infty} \sum_{m=-n}^{n} C_n^{pw} \left[g_{n,TM}^m \vec{n}_{mn} - i g_{n,TE}^m \vec{m}_{mn} \right]$$

$$\vec{H}_i = H_0 \sum_{n=1}^{\infty} \sum_{m=-n}^{n} C_n^{pw} \left[g_{n,TE}^m \vec{n}_{mn} + i g_{n,TM}^m \vec{m}_{mn} \right]$$

$$E_r(r,\theta,\phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \frac{E_0}{kr} (-i)^{n+1} (2n+1) g_{n,TM}^m j_n(kr) P_n^{|m|}(\cos\theta) \exp(im\phi)$$

$$H_r(r,\theta,\phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \frac{E_0}{kr} (-i)^{n+1} (2n+1) g_{n,TE}^m j_n(kr) P_n^{|m|}(\cos\theta) \exp(im\phi)$$

$$\int_0^{2\pi} \exp(im\phi) \exp(-im'\phi) d\phi = \begin{cases} 2\pi & m = m' \\ 0 & m \neq m' \end{cases}$$

$$\int_0^{\pi} P_n^m(\cos\theta) P_l^m(\cos\theta) \sin\theta d\theta = \frac{2}{2n+1} \frac{(n+m)!}{(n-m)!} \delta_{nl}$$

Expansion of shaped beam

Beam shape coefficients in spherical system:

To be demonstrated by yourself

$$g_{n,TM}^{m} = \frac{kri^{n+1}}{4\pi j_n(kr)} \frac{(n-|m|)!}{(n+|m|)!} \int_0^{2\pi} \int_0^{\pi} \frac{E_r(r,\theta,\phi)}{E_0} P_n^{|m|}(\cos\theta) \exp(-im\phi) \sin\theta d\theta d\phi$$

$$g_{n,TE}^{m} = \frac{kri^{n+1}}{4\pi j_n(kr)} \frac{(n-|m|)!}{(n+|m|)!} \int_0^{2\pi} \int_0^{\pi} \frac{H_r(r,\theta,\phi)}{H_0} P_n^{|m|}(\cos\theta) \exp(-im\phi) \sin\theta d\theta d\phi$$

1. The beam shape coefficients depend explicitly on *r* but they should not.
   - When the EM field satisfies the Maxwell equations they do not depend on *r*. Ex. Plane wave.
   - The choice of *r* has nothing to do with the particle.
   - The dependence on *r* can be eliminated by integration over *r* → purification of the beam.

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Expansion of shaped beam

Beam shape coefficients in spherical system:

- Triple integration:

$$\int_0^\infty j_n(x) j_{n'}(x) dx = \frac{\pi}{2(2n+1)} \delta_{nn'}$$

$$g_{n,TM}^m = \frac{2n+1}{2\pi^2(-i)^{n+1}} \frac{(n-|m|)!}{(n+|m|)!} \times \int_0^\infty kr\psi_n^{(1)}(kr) \int_0^{2\pi} \exp(-im\phi) \int_0^\pi \frac{E_r(r,\theta,\phi)}{E_0} P_n^{|m|}(\cos\theta) \sin\theta d\theta d\phi d(kr)$$

To be demonstrated by yourself

- Axial symmetric beam:

$$g_n = \frac{i^{n+1} k a_s}{2n(n+1)\psi_n^{(1)}(ka_s)} \int_0^\pi \frac{E_r(a_s,\theta)}{E_0} P_n^1(\cos\theta) \sin\theta d\theta$$

$$g_n = \frac{(2n+1) i^{n+1}}{\pi n(n+1)} \int_0^\infty kr\psi_n^{(1)}(kr) \int_0^\pi \frac{E_r(r,\theta)}{E_0} P_n^1(\cos\theta) \sin\theta d\theta d(kr)$$

Very **stable** and **flexible** but very time consuming.

Expansion of shaped beam

Beam shape coefficients in spherical system:

Localized approximation (see van de Huslt for the principle):

E_r at the plane z=0:

To be understood: localization principle.

$$\theta = \pi/2 \text{ and } kr = n + 1/2$$

$$g_{n,TM}^{m} = \frac{Z_n^m}{2\pi E_0}\int_0^{2\pi} \overline{E}_r\left(kr = n+\frac{1}{2}, \theta = \frac{\pi}{2}, \phi'\right)\exp(-im\phi')d\phi'$$

$$g_{n,TE}^{m} = \frac{Z_n^m}{2\pi H_0}\int_0^{2\pi} \overline{H}_r\left(r = \rho_n, \theta = \frac{\pi}{2}, \phi'\right)\exp(-im\phi')d\phi'$$

$$Z_n^0 = \frac{2n(n+1)}{2n+1}$$

$$Z_n^m = \left(\frac{-2i}{2n+1}\right)^{|m|-1}, \qquad m \neq 0$$

Expansion of shaped beam

Beam shape coefficients in spherical system:

Localized approximation for Gaussian beam:

Reformulated by K. F. Ren (PPSC 1994)

$$g_{n,TM}^{m} = Z_n^m \exp(ikz_0)\overline{\psi}_0^{0sh} \frac{1}{2} \sum_{j=0}^{\infty} \frac{\overline{B}^{j+m-1}\overline{C}^{j}}{j!(j+m-1)!} \left(1 + \frac{\overline{B}^2}{(j+m)(j+m+1)}\right)$$

$$g_{n,TE}^{m} = Z_n^m \exp(ikz_0)\overline{\psi}_0^{0sh} \frac{1}{2i} \sum_{j=0}^{\infty} \frac{\overline{B}^{j+m-1}\overline{C}^{j}}{j!(j+m-1)!} \left(1 - \frac{\overline{B}^2}{(j+m)(j+m+1)}\right)$$

$$\overline{B} = \rho_n \frac{iQ}{w_0^2}(x_0 - iy_0))$$

$$\overline{C} = \rho_n \frac{iQ}{w_0^2}(x_0 + iy_0))$$

Widely used but not numerically stable.

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Expansion of shaped beam

Beam shape coefficients in spherical system:

Integral localized approximation (Introduced by Ren (JOSA A 1996)

$$
\begin{aligned}
g_{n,TM}^{m} &= \frac{Z_n^m}{2\pi E_0}\int_0^{2\pi} \overline{E}_r\left(kr = n+\frac{1}{2}, \theta = \frac{\pi}{2}, \phi'\right)\exp(-im\phi')d\phi' \\
g_{n,TE}^{m} &= \frac{Z_n^m}{2\pi H_0}\int_0^{2\pi} \overline{H}_r\left(r = \rho_n, \theta = \frac{\pi}{2}, \phi'\right)\exp(-im\phi')d\phi'
\end{aligned}
$$

Applicable to any shaped beam propagating along z axis:

Applied to Gaussian beam:

$$
\begin{aligned}
\begin{pmatrix} g_{n,TM}^{m} \\ ig_{n,TE}^{m} \end{pmatrix} &= iQ\frac{Z_n^m}{4\pi}e^{-iQ\gamma^2+ikz_0}\int_0^{2\pi} e^{2iQ\rho_n(\xi_0\cos\phi+\eta_0\sin\phi)}\left(e^{-i(m-1)} \pm e^{-i(m+1)}\right)d\phi \\
&= iQ\frac{Z_n^m}{2}e^{-iQ\gamma^2+ikz_0}\left[e^{i(m-1)\phi_0}J_{m-1}(2Q\rho_n\rho_0) \pm e^{i(m+1)\phi_0}J_{m+1}(2Q\rho_n\rho_0)\right]
\end{aligned}
$$

No problem of instability.

Expansion of shaped beam

Beam shape coefficients in spherical system:

Applied to Doughnut beam (similar for other polarizations):

$$\begin{aligned} g_{n,TM}^{m,rad} &= \frac{1}{2} Z_n^m \overline{\Omega}_n e^{im\phi_0} \left[2\rho_n J_m(x_n) - \rho_0 (J_{m-1} e^{-2i\phi_0} + J_{m+1} e^{2i\phi_0}) \right] \\ g_{n,TE}^{m,rad} &= \frac{i}{2} Z_n^m \overline{\Omega}_n e^{im\phi_0} \rho_0 (J_{m-1} e^{-2i\phi_0} - J_{m+1} e^{2i\phi_0}) \end{aligned}$$

Applied to Bessel beam:

$$\begin{aligned} g_{n,TM}^{0} &= \frac{Z_n^0}{2} \left[J_1(\varpi) J_{1-v}(\xi) e^{-i\phi_0} + J_{-1}(\varpi) J_{-1-v}(\xi) e^{i\phi_0} \right] e^{ik\cos\alpha_0 z_0} \\ g_{n,TM}^{m} &= \frac{Z_n^m}{2} \left[J_{1+m}(\varpi) J_{1+m-v}(\xi) e^{-i(1+m)\phi_0} + J_{-1+m}(\varpi) J_{-1+m-v}(\xi) e^{-i(-1+m)\phi_0} \right] e^{ik\cos\alpha_0 z_0} \\ g_{n,TE}^{0} &= \frac{Z_n^0}{2} \left[iJ_1(\varpi) J_{1-v}(\xi) e^{-i\phi_0} - iJ_{-1}(\varpi) J_{-1-v}(\xi) e^{i\phi_0} \right] e^{ik\cos\alpha_0 z_0} \\ g_{n,TE}^{m} &= \frac{iZ_n^m}{2} \left[J_{1+m}(\varpi) J_{1+m-v}(\xi) e^{-i(1+m)\phi_0} - J_{-1+m}(\varpi) J_{-1+m-v}(\xi) e^{-i(-1+m)\phi_0} \right] e^{ik\cos\alpha_0 z_0} \end{aligned}$$

Expansion of shaped beam

2. In spheroidal system:

Solution:

$$\psi_{mn}(\eta,\xi,\phi)=S_{|m|n}(c,\eta)R_{|m|n}(c,\xi)e^{im\phi}$$

Any wave can be expanded as summation of these spheroidal functions

- $S_{|m|n}(c,\eta)$ and $R_{|m|n}(c,\xi)$ are respectively the angular and radial spheroidal function.
- $c=kf$ with f being the semifocal length of the spheroid

- **It is more correct to write EM in M_{mn} and N_{mn} than odd and even separated.**
- **The computation of the spheroidal functions is much more difficulty, so application limited.**

Expansion of shaped beam

Beam shape coefficients in spheroidal system:

EM field in spherical system:

$$\mathbf{E}^{(i)} = \sum_{m=-\infty}^{+\infty} \sum_{n=|m|,n\neq 0}^{+\infty} c_{n,pw} i^{n+1} \left(i g_{n,TE}^{m} \mathbf{m}_{mn}^{(i)}(r,\theta,\phi) + g_{n,TM}^{m} \mathbf{n}_{mn}^{(i)}(r,\theta,\phi) \right),$$

$$\mathbf{H}^{(i)} = -\frac{ik_1}{\omega\mu_0} \sum_{m=-\infty}^{+\infty} \sum_{n=|m|,n\neq 0}^{+\infty} c_{n,pw} i^{n+1} \left(g_{n,TM}^{m} \mathbf{m}_{mn}^{(i)}(r,\theta,\phi) + i g_{n,TE}^{m} \mathbf{n}_{mn}^{(i)}(r,\theta,\phi) \right)$$

EM field in spheroid system:

$$\mathbf{E}^{(i)} = \sum_{m=-\infty}^{\infty} \sum_{n=|m|,n\neq 0}^{\infty} i^{n+1} \left[i G_{n,TE}^{m} \mathbf{M}_{mn}^{(i)}(c_1;\xi,\eta,\phi) + G_{n,TM}^{m} \mathbf{N}_{mn}^{(i)}(c_1;\xi,\eta,\phi) \right],$$

$$\mathbf{H}^{(i)} = -\frac{ik_1}{\omega\mu_0} \sum_{m=-\infty}^{\infty} \sum_{n=|m|,n\neq 0}^{\infty} i^{n+1} \left[G_{n,TM}^{m} \mathbf{M}_{mn}^{(i)}(c_1;\xi,\eta,\phi) + i G_{n,TE}^{m} \mathbf{N}_{mn}^{(i)}(c_1;\xi,\eta,\phi) \right]$$

Vector potential given in combined form
odd and even functions not separated.

Expansion of shaped beam

Beam shape coefficients in spheroidal system:

Relation between the vector wave functions in the two systems:

$$\mathbf{n}_{mn}^{(i)}(r,\theta,\phi)=\sum_{l=|m|,|m|+1}^{\infty}{}'\frac{2(n+|m|)!}{(2n+1)(n-|m|)!}\frac{i^{l-n}}{N_{|m|l}}d_{n-|m|}^{|m|l}\mathbf{N}_{ml}^{(i)}(c;\xi,\eta,\phi)$$

$$\mathbf{m}_{mn}^{(i)}(r,\theta,\phi)=\sum_{l=|m|,|m|+1}^{\infty}{}'\frac{2(n+|m|)!}{(2n+1)(n-|m|)!}\frac{i^{l-n}}{N_{|m|l}}d_{n-|m|}^{|m|l}\mathbf{M}_{ml}^{(i)}(c;\xi,\eta,\phi)$$

Beam shape coefficients:

$$G_{n,TE}^{m}=\frac{1}{N_{|m|n}(c_1)}\sum_{r=0,1}^{\infty}{}'g_{r+|m|,TE}^{m}\frac{2(r+2|m|)!}{(r+|m|)(r+|m|+1)r!}d_r^{|m|n}(c_1)$$

$$G_{n,TM}^{m}=\frac{1}{N_{|m|n}(c_1)}\sum_{r=0,1}^{\infty}{}'g_{r+|m|,TM}^{m}\frac{2(r+2|m|)!}{(r+|m|)(r+|m|+1)r!}d_r^{|m|n}(c_1)$$

gnm can not be calculated by Localized approximation for ***oblique incidence.***

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Expansion of shaped beam

3. In cylindrical system:

Solution:

$$\psi_{hn}(r,\phi,z) = Z_n(\rho)e^{in\phi}e^{ihz}$$

Any wave can be expanded as summation of these cylindrical wave functions (similar as Fourier transform).

- $Z_n(\rho)$ is a cylindrical Bessel function with

$$\rho = r\sqrt{k^2 - h^2} = kr\cos\alpha$$

- When $kr \to \propto$, $e^{\pm ikr}/\sqrt{kr}$. So a cylindrical wave.
- The index *n* is from 1 to infinity, describing mainly the variation in *r*.
- α can be considered as incident angle (of a plane wave) to the cylinder.

A shaped beam can be expanded in plane wave by taking α as the index *m* in the scattering of a sphere.

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Expansion of shaped beam

Beam shape coefficients in cylindrical system:

Incident field:

$$E_z^i = \frac{E_0}{\rho^2}\sum_{m=-\infty}^{+\infty}(-i)^m e^{im\phi}\int_{-1}^{1}\rho_0^2 I_{m,TM}(\delta)J_m(\rho_0)e^{i\delta\zeta}d\delta$$

$$H_z^i = \frac{H_0}{\rho^2}\sum_{m=-\infty}^{+\infty}(-i)^m e^{im\phi}\int_{-1}^{1}\rho_0^2 I_{m,TE}(\delta)J_m(\rho_0)e^{i\delta\zeta}d\delta$$

Beam shape coefficients in integral form:

$$I_{m,TM}(\delta) = \frac{i^m}{4\pi^2(1-\delta^2)J_m(\rho_0)}\int_0^{2\pi} e^{-im\phi}\int_{-\infty}^{+\infty}\frac{E_z^i}{E_0}e^{-i\delta\zeta}d\phi d\zeta$$

$$I_{m,TE}(\delta) = \frac{i^m}{4\pi^2(1-\delta^2)J_m(\rho_0)}\int_0^{2\pi} e^{-im\phi}\int_{-\infty}^{+\infty}\frac{H_z^i}{H_0}e^{-i\delta\zeta}d\phi d\zeta$$

- The beam shape coefficients depend on *z* components of *E*,*H*.
- The second index is continuous – continuous spectrum.

Expansion of shaped beam

Beam shape coefficients in cylindrical system:

Localized approximation:

$$I_{m,TM} = \frac{1}{2\pi E_0}\int_{-\infty}^{\infty} E_z^i(\phi=\frac{\pi}{2}, \rho=m)\exp(-i\delta\zeta)d\zeta$$
$$I_{m,TE} = \frac{1}{2\pi H_0}\int_{-\infty}^{\infty} H_z^i(\phi=\frac{\pi}{2}, \rho=m)\exp(-i\delta\zeta)d\zeta$$

Beam shape coefficients normal incident Gaussian beam:

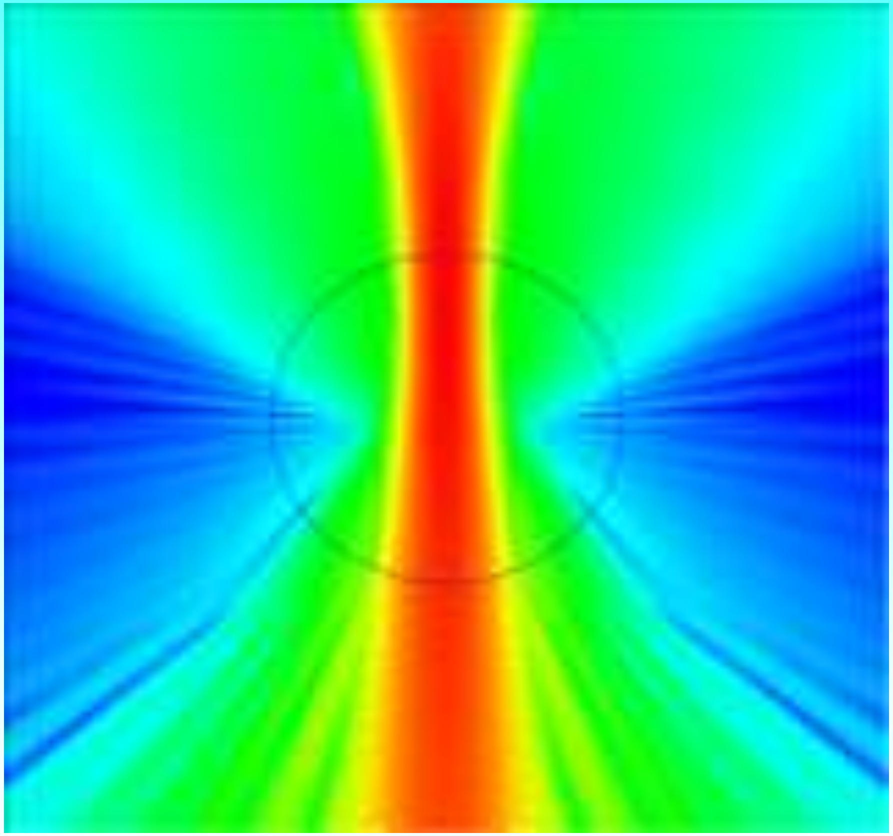
$$\begin{aligned} I_{m,TM} = I_{m,TE} &= \frac{1}{2\pi H_0}\int_{-\infty}^{\infty}\exp(-s^2m^2)\exp(-s^2\zeta^2)\exp(-i\delta\zeta)d\zeta \\ &= \frac{1}{2\sqrt{\pi}s}\exp\left[-m^2s^2-\frac{\delta^2}{4s^2}\right] \end{aligned}$$

The beam shape coefficients are Gaussian both on the discrete index m and the continuous index δ.

Formulae of physical quantities

Scattering by a sphere.

1. Internal and near fields:
   Formula can be found in the literature.



Practical consideration:
- Continuous at the surface,
- *m* must be sufficiently great.

Interesting subjects to be studied:
- Check numerically the surface wave.
- Different effect s by illuminating with strongly focus beam.

Formulae of physical quantities

Plane wave case:

Incident wave:

$$\begin{pmatrix} \psi^i_{TM} \\ \psi^i_{TE} \end{pmatrix} = \frac{1}{k^2}\sum_{n=1}^{\infty}\frac{1}{i^{n+1}}\frac{2n+1}{n(n+1)}\psi_n(kr)P_n^1(\cos\theta)\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$$

Far field:

$$E_r = H_r = 0$$

$$E_\theta = \frac{iE_0}{kr}\exp(-ikr)\cos\varphi\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}\left[a_n\tau_n(\cos\theta)+ib_n\pi_n(\cos\theta)\right] = \frac{iE_0}{kr}\exp(-ikr)\cos\varphi\, S_2$$

$$E_\varphi = \frac{-E_0}{kr}\exp(-ikr)\sin\varphi\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}\left[a_n\pi_n(\cos\theta)+ib_n\tau_n(\cos\theta)\right] = \frac{-E_0}{kr}\exp(-ikr)\sin\varphi\, S_1$$

$$H_\varphi = \frac{H_0}{E_0}E_\theta$$

$$H_\theta = -\frac{H_0}{E_0}E_\varphi$$

a_n, b_n coefficients de diffusion dépendants des propriétés de la particule

τ_n, π_n fonctions angulaire de Legendre

Formulae of physical quantities

Plane wave case:

Scattering intensities:

$I_\perp(q)=|S_1|^2$

$I_\parallel(q)=|S_2|^2$

$$S_1 = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[a_n \pi_n(\cos\theta) + i b_n \tau_n(\cos\theta)\right]$$

$$S_2 = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[a_n \tau_n(\cos\theta) + i b_n \pi_n(\cos\theta)\right]$$

Sections efficaces:

$C_{ext}=C_{sca}+C_{\mathrm{abs}}$

$$C_{sca} = \frac{\lambda^2}{2\pi} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2)$$

$$C_{ext} = \frac{\lambda^2}{2\pi} \sum_{n=1}^{\infty} (2n+1)\mathrm{Re}(a_n + b_n)$$

Pression de radiation:

$C_x=C_y=0$

$$C_{pr,z} = \frac{\lambda^2}{2\pi}\mathrm{Re}\left[\sum_{n=1}^{\infty}(2n+1)\frac{(a_n+b_n)}{2} - \frac{2n+1}{n(n+1)}a_n b_n^* - \frac{n(n+2)}{n+1}(a_n a_{n+1}^* + b_n b_{n+1}^*)\right]$$

Formulae of physical quantities

Arbitrarily shaped beam:

1. Scattered wave in field:

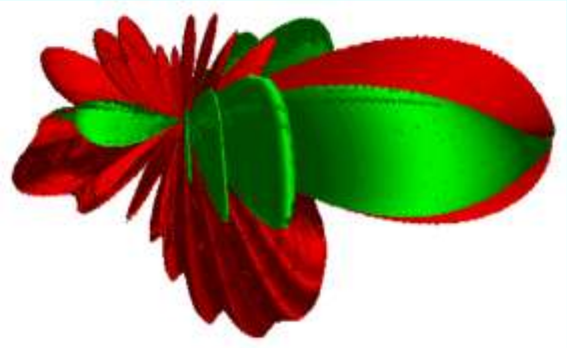
$$E_\theta^s = \frac{iE_0}{kr}\exp(-ikr)\sum_{n=1}^{\infty}\sum_{m=-m}^{n}\frac{2n+1}{n(n+1)}\left[a_n g_{n,TM}^m \tau_n^{|m|}(\cos\theta) + im\, b_n g_{n,TE}^m \pi_n^{|m|}(\cos\theta)\right]\exp(im\phi)$$

$$E_\phi^s = \frac{-E_0}{kr}\exp(-ikr)\sum_{n=1}^{\infty}\sum_{m=-m}^{n}\frac{2n+1}{n(n+1)}\left[m\, a_n g_{n,TM}^m \pi_n^{|m|}(\cos\theta) + i\, b_n g_{n,TE}^m \tau_n^{|m|}(\cos\theta)\right]\exp(im\phi)$$

$$H_\phi^s = \frac{H_0}{E_0}E_\theta^s \qquad H_\theta^s = -\frac{H_0}{E_0}E_\phi^s \qquad E_r^s = H_r^s = 0$$

These formula and those given in the following are valid for any "spherical" particle:

- Homogenous,
- Stratified,
- Spherical with inclusion
- … …



Formulae of physical quantities

Arbitrarily shaped beam:

The extinction, scattering and absorption sections:

$$C_{ext} = \frac{\lambda^2}{\pi} Re \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \frac{2n+1}{n(n+1)} \frac{(n+|m|)!}{(n-|m|)!} (a_n |g_{n,TM}^m|^2 + b_n |g_{n,TE}^m|^2)$$

$$C_{sca} = \frac{\lambda^2}{\pi} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \frac{2n+1}{n(n+1)} \frac{(n+|m|)!}{(n-|m|)!} (|a_n|^2 |g_{n,TM}^m|^2 + |b_n|^2 |g_{n,TE}^m|^2)$$

$$C_{abs} = C_{ext} - C_{sca}$$

- Double summation : $\sum_{n=1}^{\infty} \sum_{m=-n}^{n} = \sum_{m=-\infty}^{\infty} \sum_{n=m\neq 0}^{\infty}$
- The sense of the efficiency factors for shaped beam.

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Formulae of physical quantities

***Arbitrarily* shaped beam**:

$$A_n = a_n + a^*_{n+1} - 2a_n a^*_{n+1}$$
$$B_n = b_n + b^*_{n+1} - 2b_n b^*_{n+1}$$
$$C_n = -i(a_n + b^*_{n+1} - 2a_n b^*_{n+1})$$

The radiation pressure:

$$\begin{aligned} C_{pr,z} &= \frac{\lambda^2}{\pi}\sum_{n=1}^{\infty} Re\left\{\frac{1}{n+1}\left(A_n g^0_{n,TM} g^{0*}_{n+1,TM} + B_n g^0_{n,TE} g^{0*}_{n+1,TE}\right)\right. \\ &+ \sum_{m=1}^{n}\left[\frac{1}{(n+1)^2}\frac{(n+m+1)!}{(n-m)!}\left(A_n g^m_{n,TM} g^{m*}_{n+1,TM} + A_n g^{-m}_{n,TM} g^{-m*}_{n+1,TM}\right.\right. \\ &\qquad\left. + B_n g^m_{n,TE} g^{m*}_{n+1,TE} + B_n g^{-m}_{n,TE} g^{-m*}_{n+1,TE}\right) \\ &+ \left.\left. m\frac{2n+1}{n^2(n+1)^2}\frac{(n+m)!}{(n-m)!} C_n\left(g^m_{n,TM} g^{m*}_{n,TE} - g^{-m}_{n,TM} g^{-m*}_{n,TE}\right)\right]\right\} \end{aligned}$$

$$C_{pr,x} = Re(C) \qquad C_{pr,y} = Im(C)$$

$$\begin{aligned} C = \frac{\lambda^2}{2\pi}\sum_{n=1}^{\infty}\left\{-\frac{(2n+2)!}{(n+1)^2}F^{n+1}_n + \sum_{m=1}^{n}\frac{(n+m)!}{(n-m)!}\frac{1}{(n+1)^2}\left[F^{m+1}_n - \frac{n+m+1}{n-m+1}F^m_n\right.\right. \\ \left.\left. + \frac{2n+1}{n^2}\left(C_n g^{m-1}_{n,TM} g^{m*}_{n,TE} - C_n g^{-m}_{n,TM} g^{-m+1*}_{n+1,TE} + C^*_n g^{m-1}_{n,TE} g^{m*}_{n,TM} - C^*_n g^{-m}_{n,TE} g^{-m+1*}_{n,TM}\right)\right]\right\} \end{aligned}$$

$$F^m_n = A_n g^{m-1}_{n,TM} g^{m*}_{n+1,TM} + B_n g^{m-1}_{n,TE} g^{m*}_{n+1,TE} + A^*_n g^{-m}_{n+1,TM} g^{-m+1*}_{n,TM} + B^*_n g^{-m}_{n+1,TE} g^{-m+1*}_{n,TE}$$

- Rewritten for programming.

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Formulae of physical quantities

Arbitrarily shaped beam:

The radiation torque:

$$
\begin{aligned}
T_x &= \frac{4\hat{m}}{c}\frac{\pi}{k^3}\sum_{n=1}^{\infty}\sum_{m=1}^{n} C_n^m \Re(A_n^m), \\
T_y &= \frac{4\hat{m}}{c}\frac{\pi}{k^3}\sum_{n=1}^{\infty}\sum_{m=1}^{n} C_n^m \Im(A_n^m), \\
T_z &= -\frac{4\hat{m}}{c}\frac{\pi}{k^3}\sum_{n=1}^{\infty}\sum_{m=1}^{n} m C_n^m B_n^m,
\end{aligned}
$$

$$
\begin{aligned}
C_n^m &= \frac{2n+1}{n(n+1)}\frac{(n+|m|)!}{(n-|m|)!} \\
A_n^m &= A_n\left(g_{n,TM}^{m-1} g_{n,TM}^{m*} - g_{n,TM}^{-m} g_{n,TM}^{-m+1*}\right) + B_n\left(g_{n,TE}^{m-1} g_{n,TE}^{m*} - g_{n,TE}^{-m} g_{n,TE}^{-m+1*}\right) \\
B_n^m &= A_n\left(|g_{n,TM}^{m}|^2 - |g_{n,TM}^{-m}|^2\right) + B_n\left(|g_{n,TE}^{m}|^2 - |g_{n,TE}^{-m}|^2\right) \\
A_n &= \Re(a_n) - |a_n|^2 \\
B_n &= \Re(b_n) - |b_n|^2
\end{aligned}
$$

- Transversal components null for transparent sphere whatever the form and the position of the beam.

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Exampled results and conclusions

Scattering by a sphere:

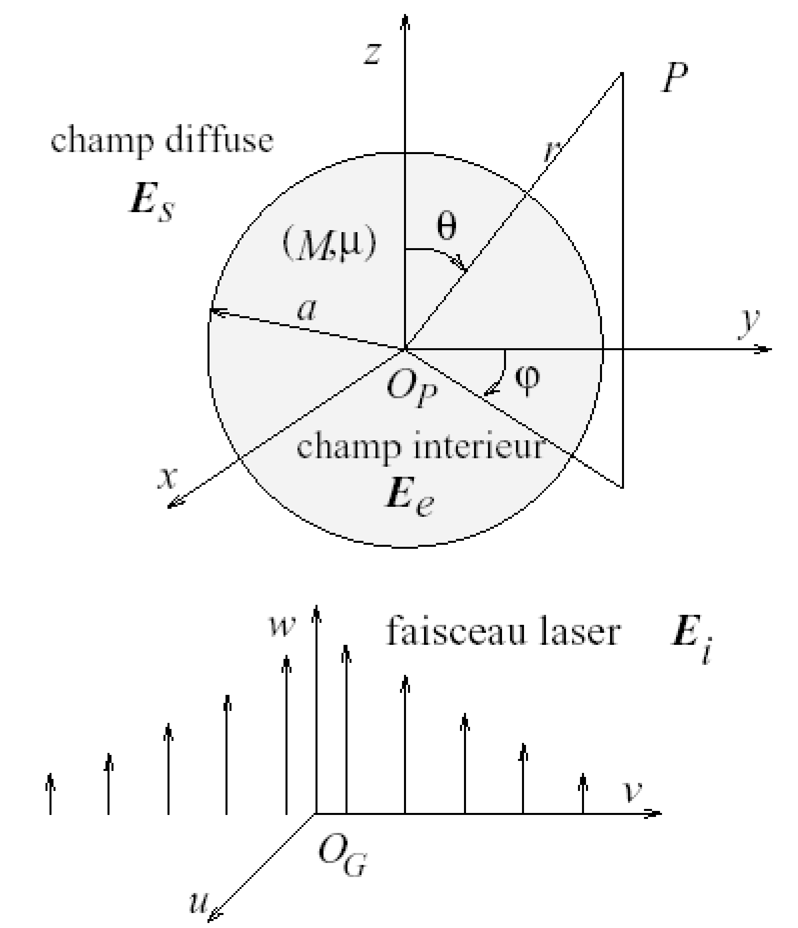
Conditions:

1. Incident beam: ***Arbitrary shape***
2. Particle :
   - Spherical
   - Homogeneous or stratified
   - Isotropic

Particularities:

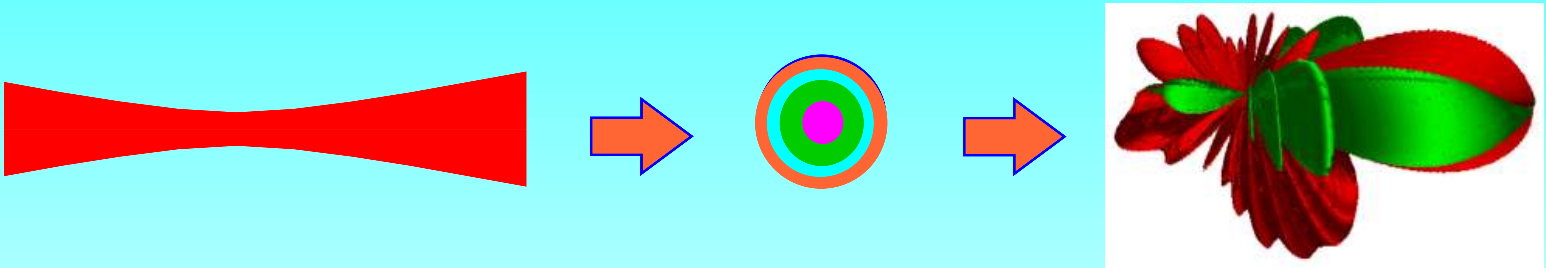
1. Illumination inhomogeneous when beam is small.
2. Incident beam is described by two series of beam coefficients :

$$g_{n,TM}^{m} \quad \text{et} \quad g_{n,TE}^{m}$$



Exampled results and conclusions

Scattering by a sphere:



E_i $\longleftrightarrow$ $g_{n,TM}^{m}$

H_i $\longleftrightarrow$ $g_{n,TE}^{m}$

$+$ a_n, b_n $\rightarrow$ E_s I H_s P

$$a_n \rightarrow a_n g_{n,TM}^{m}$$
$$b_n \rightarrow b_n g_{n,TE}^{m}$$

$$\pi_n(\cos\theta) \rightarrow \pi_n^{m}(\cos\theta)$$
$$\tau_n(\cos\theta) \rightarrow \tau_n^{m}(\cos\theta)$$

$$\sum_{n=1}^{\infty} \rightarrow \sum_{n=1}^{\infty}\sum_{m=-n}^{m=+n}$$

Exampled results and conclusions

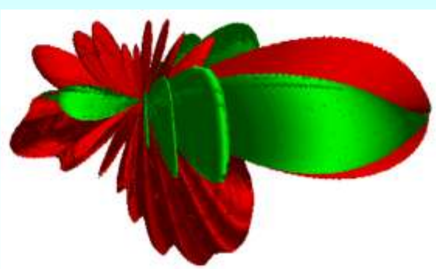
Particule :
a = 5 µm
m = 1.33

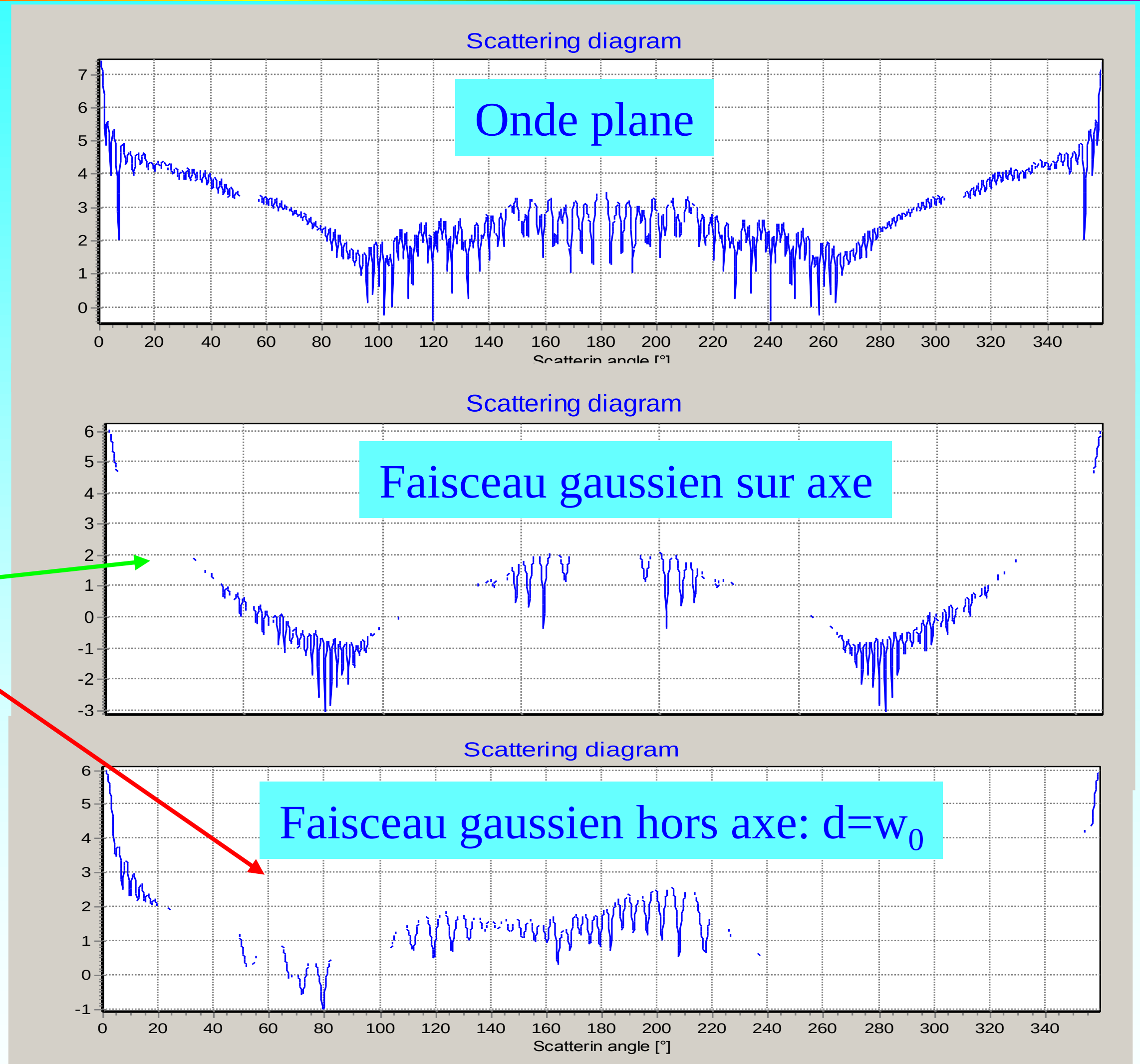
Faisceau gaussien:
λ = 0.6328 µm
w_0 = 5 µm

Diagramme de diffusion:

Sur axe - symétrique

Hors axe - non-symétrique





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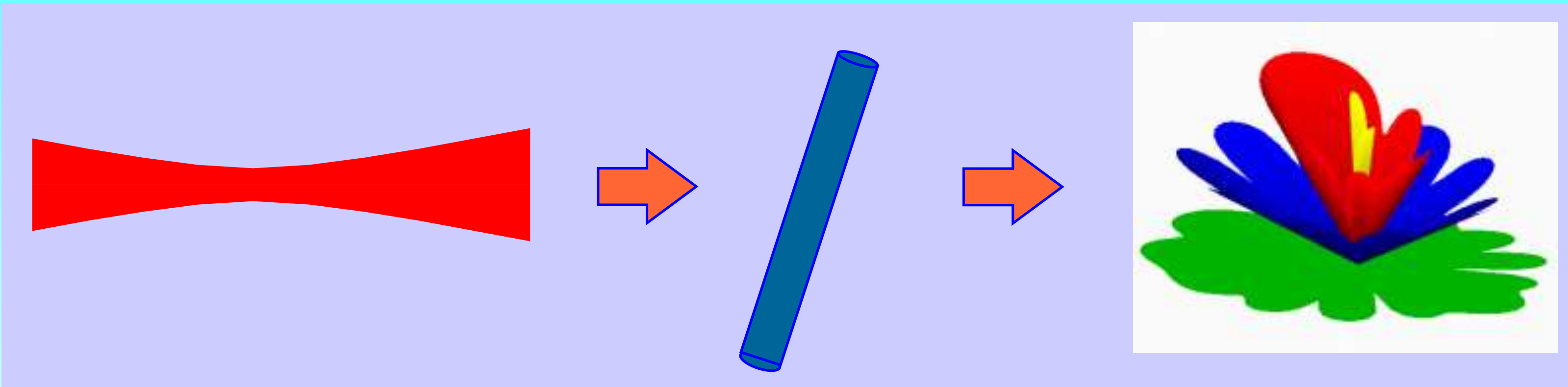
Exampled results and conclusions

Scattering by s infinite cylinder:



$$\begin{matrix} E_i \\ H_i \end{matrix} \leftrightarrow \begin{matrix} I_{n,TM}(\gamma) \\ I_{n,TE}(\gamma) \end{matrix} + \begin{matrix} a_{n\mathrm{I}}, a_{n\mathrm{II}} \\ b_{n\mathrm{I}}, b_{n\mathrm{II}} \end{matrix} \rightarrow \begin{matrix} E_s \\ H_s \end{matrix} \quad I$$

Spectral of plane wave

Exampled results and conclusions

Scattering by spheroid:

$$\mathbf{E}^{(i)} = \sum_{m=-\infty}^{\infty} \sum_{n=|m|, n\neq 0}^{\infty} i^{n+1} \left[iG_{n,TE}^{m} \mathbf{M}_{mn}^{(i)}(c_1;\xi,\eta,\phi) + G_{n,TM}^{m} \mathbf{N}_{mn}^{(i)}(c_1;\xi,\eta,\phi) \right],$$

$$\mathbf{H}^{(i)} = -\frac{ik_1}{\omega\mu_0} \sum_{m=-\infty}^{\infty} \sum_{n=|m|, n\neq 0}^{\infty} i^{n+1} \left[G_{n,TM}^{m} \mathbf{M}_{mn}^{(i)}(c_1;\xi,\eta,\phi) + iG_{n,TE}^{m} \mathbf{N}_{mn}^{(i)}(c_1;\xi,\eta,\phi) \right]$$

$$G_{n,TE}^{m} = \frac{1}{N_{|m|n}(c_1)} \sum_{r=0,1}^{\infty}{}' g_{r+|m|,TE}^{m} \frac{2(r+2|m|)!}{(r+|m|)(r+|m|+1)r!} d_r^{|m|n}(c_1)$$

$$G_{n,TM}^{m} = \frac{1}{N_{|m|n}(c_1)} \sum_{r=0,1}^{\infty}{}' g_{r+|m|,TM}^{m} \frac{2(r+2|m|)!}{(r+|m|)(r+|m|+1)r!} d_r^{|m|n}(c_1)$$

- The vector potential given in combined form not separate odd and even function
- gnm can not be calculated by Localized approximation for oblique incidence

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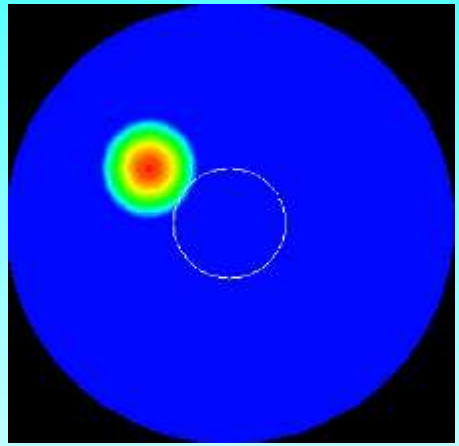
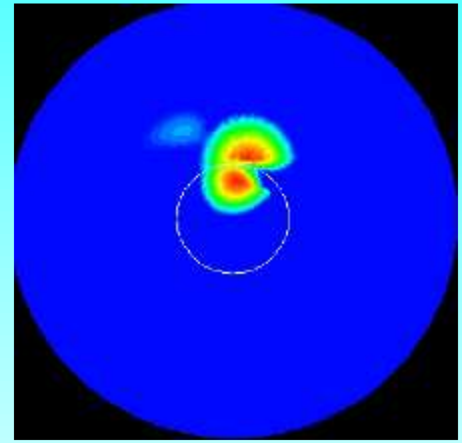
Exampled results and conclusions

Scattering of a pulse beam by a sphere:

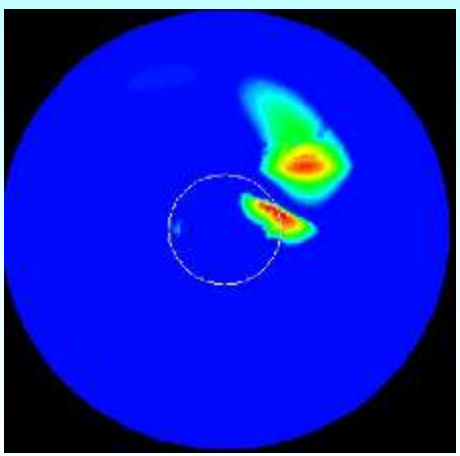
Internal field
Homogeneous sphere
d=40 µm, τ=50 fs
Gaussian beam



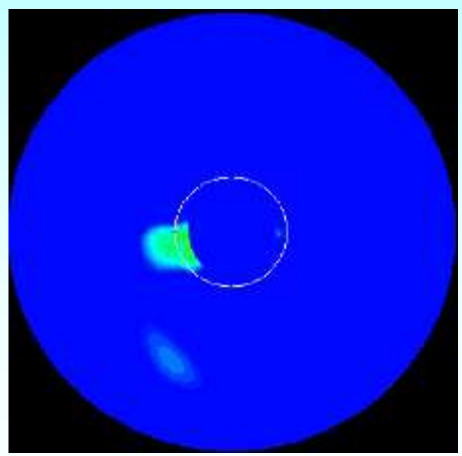
t = – 120



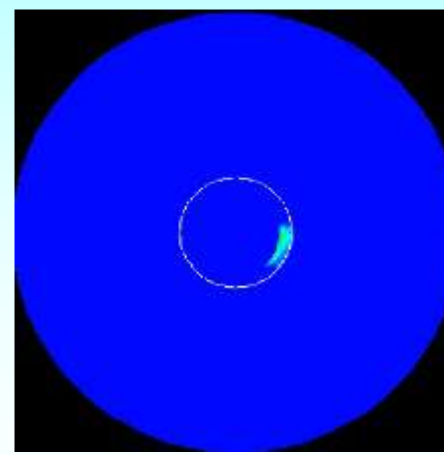
t = 20



t = 120

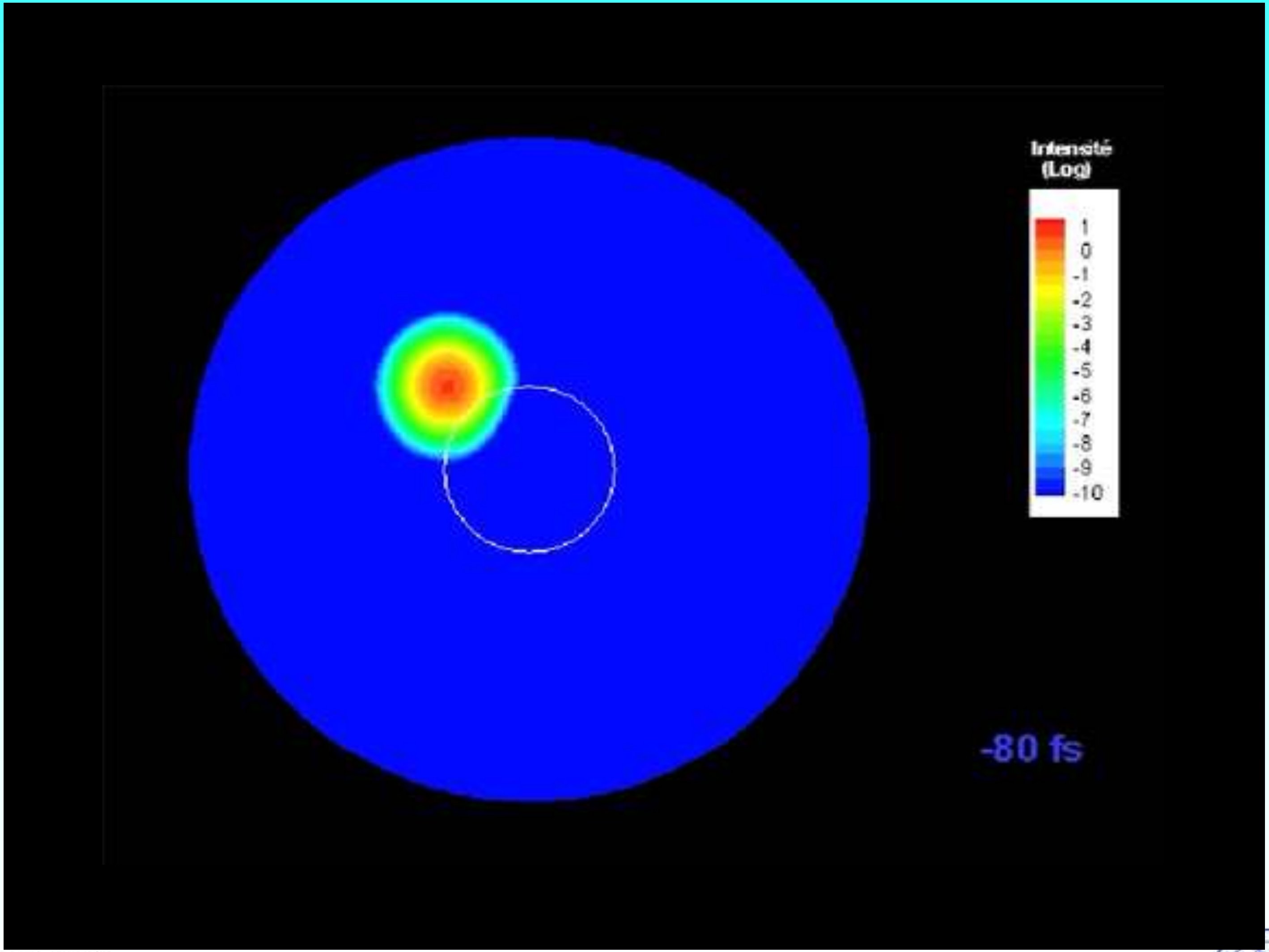


t = 1080



t = 3400

Exampled results and conclusions



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